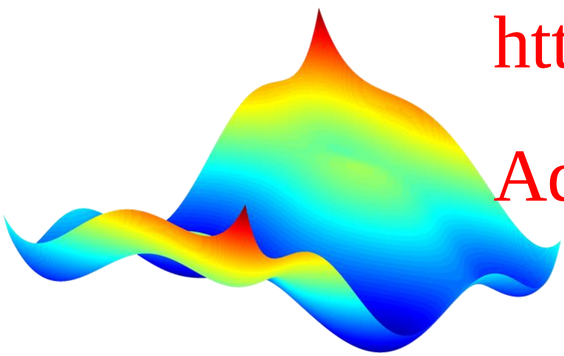
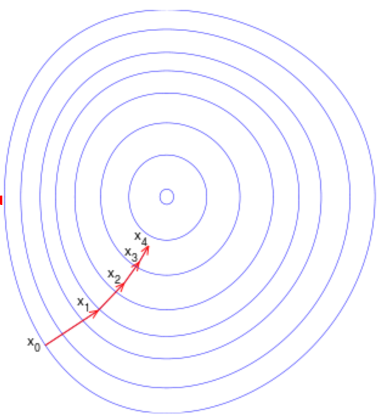
Introduction to design optimization



ME 564/SYS 564

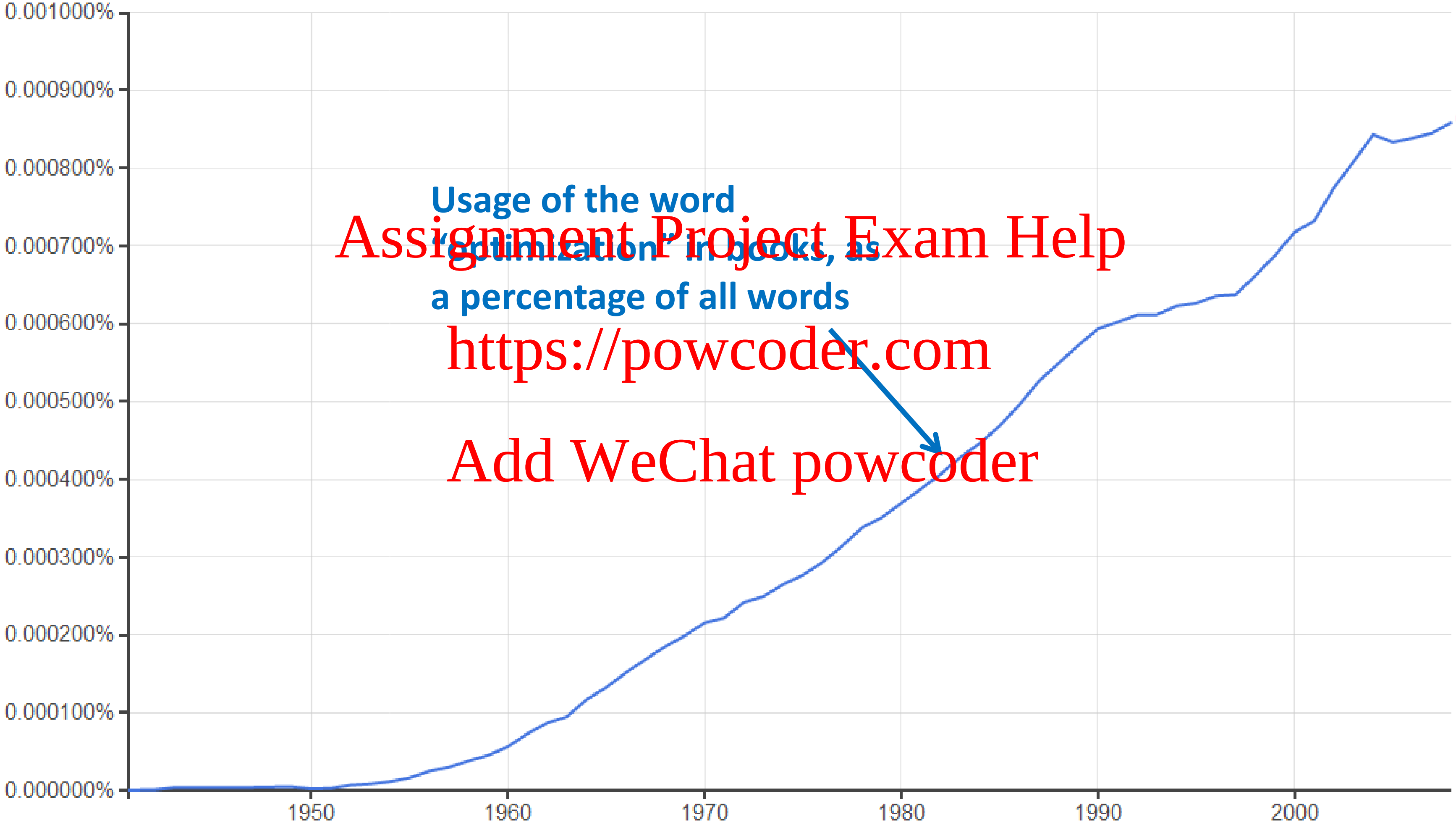
Wed Aug 29, 2018

Steven Hoffenson



<u>Goal of Week 1</u>: To become familiar with the concept of mathematical optimization, see some applications, & begin forming teams and topics

Optimization is trendy



Source: Google ngrams

What is design optimization?

Improving a design

Finding a solution that satisfies everyone

Increasing efficiency

Finding the best possible solution

Strictly speaking, **design optimization** is about **mathematically** finding the **best possible** design solution for **given models and an objective**

Every real-life problem

is an optimization problem

Design an optimal automobile for you

- Speed
- Efficiency
- Safety
- Capacity (people)
- Capacity (cargo)
- Sales

How to optimize

1. **Formulate** the problem
   a) Define system boundaries
   b) Develop analytical models
   c) Explore/reduce the problem space
   d) Formalize optimization problem

(Weeks 1-2, 4, 9-12)

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}, \mathbf{p}) \\ \text{subject to} \quad & \mathbf{g}(\mathbf{x}, \mathbf{p}) \leq 0 \\ & \mathbf{h}(\mathbf{x}, \mathbf{p}) = 0 \end{aligned}$$

2. **Solve** the problem
   a) Choose the right approach/algorithm
   b) Solve (by hand, code, or software)
   c) Interpret the results
   d) Iterate if needed

(Weeks 3, 5-8, 12)

$$\mathbf{x}_{k+1} = \mathbf{x}_k - [\mathbf{H}(\mathbf{x}_k)]^{-1} \nabla f(\mathbf{x}_0)$$

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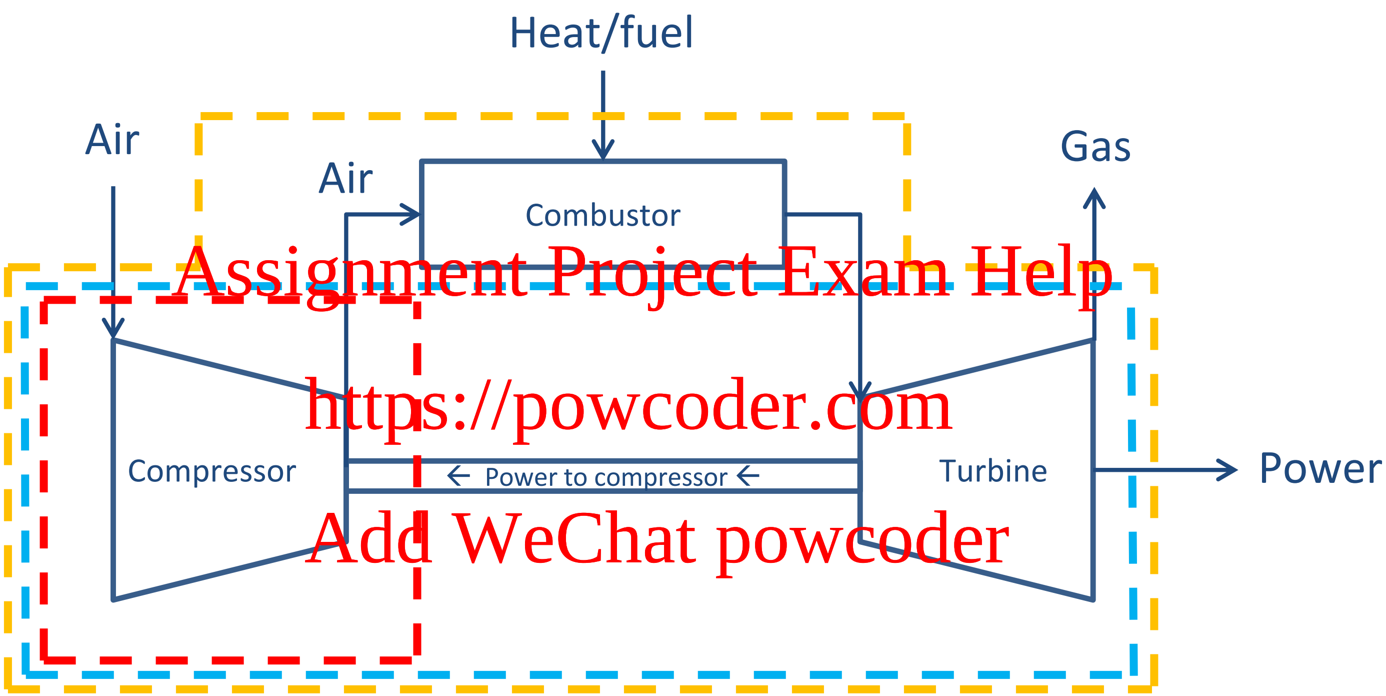
Write it out mathematically. Then, ask and adjust based on: Is it multi-disciplinary? Is it multi-objective? Is there uncertainty?

a) Define system boundaries



Where you draw your box defines the problem space and ultimately the design solution

Objectives, constraints, variables, parameters

Managers might say…	Designers might say…	What it means	Car examples
Key performance indicators (KPIs/KPPs)	Objectives	What we want to maximize/minimize	Seek best possible cost or performance (e.g., speed, efficiency)
Requirements	Hard constraints	Must-haves, with specific thresholds	Must pass FMVSS government crash test
Desirements, Targets	Soft constraints	Wants, with specific thresholds	At least 36 miles per gallon (35 wouldn't invalidate the project)
Decisions	Variables	Things we can change and want the optimizer to change	Sizes, material choices, layout, capacity
Environment	Parameters	Quantities that we can't or won't change	Material properties, e.g., strength of steel

Example: Stigler diet

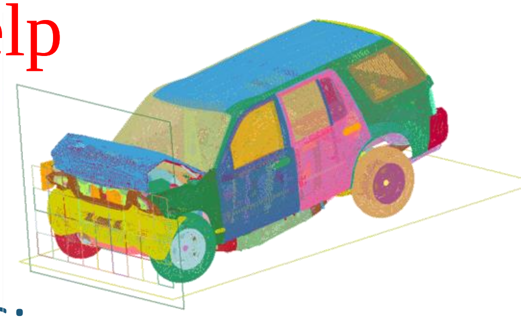
What is the lowest possible cost of a diet for a moderately-active, 154-pound male, that meets the National Research Council's 1943 Recommended Dietary Allowances (RDA) of 9 nutrients?

- **Objective:** Minimize cost
- **Constraints:** Meet 9 nutrients' RDAs
- **Variables:** Amounts of each food
- **Parameters:** 77 foods included; nutrient content and cost of each food; moderately active 154-lb man
- **Models:** Linear equations of nutrients and costs per unit of food

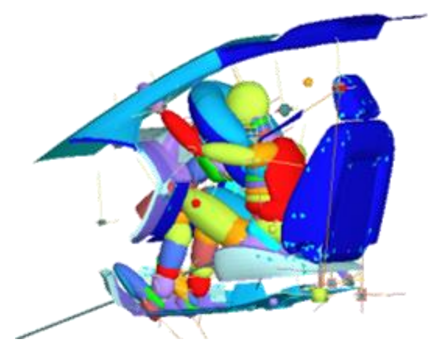
Stigler, G. J. (1945). The cost of subsistence. *Journal of farm economics*, *27*(2), 303-314.

Example: Crash safety

What is the lowest probability of serious injury that we can achieve through structural and restraint system design for a mid-sized male crash test dummy in a 35-mph crash with a rigid barrier?



- **Objective:** Minimize injury probability
- **Constraints:** Pass FMVSS tests
- **Variables:** Thicknesses of structural elements; stiffness of seat belt; airbag inflation rate
- **Parameters:** Vehicle shape; material properties; size of mid-size male dummy; definition of "serious injury" on Abbreviated Injury Scale (AIS); crash test specs
- **Models:** Physics-based simulations

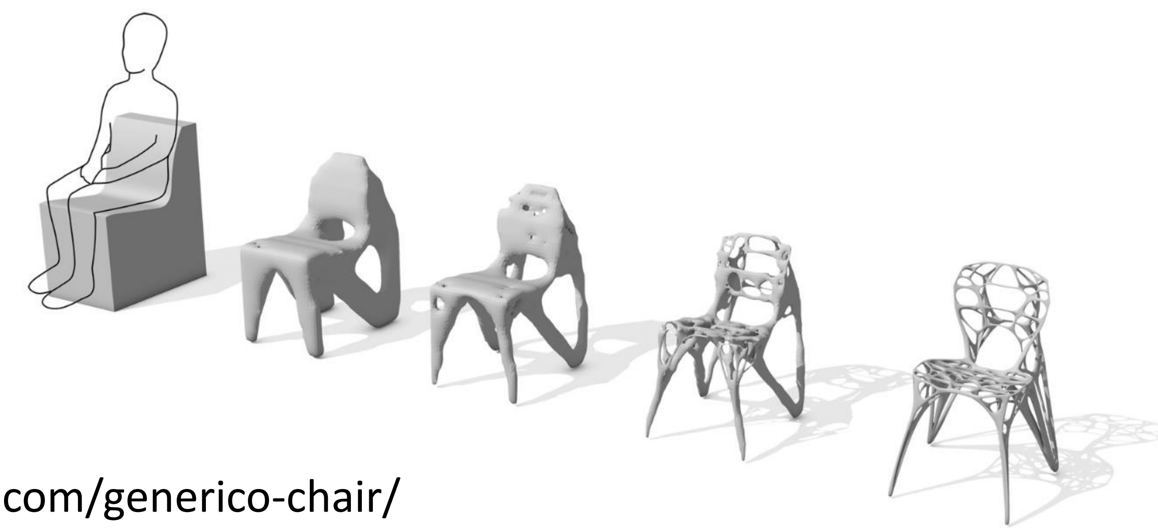


Hoffenson, S., Reed, M. P., Kaewbaidhoon, Y., & Papalambros, P. Y. (2013). On the impact of the regulatory frontal crash test speed on optimal vehicle design and road traffic injuries. *International Journal of Vehicle Design*, *63*(1), 39-60.

Example: Topology of a chair

How can we design the shape of a fixed-mass chair to hold the maximum weight possible?

- **Objective:** Maximize stiffness
- **Constraints:** Mass
- **Variables:** Material in each coordinate position (yes/no)
- **Parameters:** Material properties; loading direction of weight
- **Model:** Finite element simulation



http://mocosubmit.com/generico-chair/

Exercise

In groups, come up with an objective, constraints, variables, and parameters for designing a battery pack for an electric car

Quantity	What it means	Battery examples
Objectives	What we want to maximize/minimize	Maximize capacity in kWh
Hard constraints	Must-haves, with specific thresholds	Must meet safety standards
Soft constraints	Wants, with specific thresholds	Weigh no more than 200 lb; Capacity of at least 30 kWh; Volume no more than 15 ft^3; Cost no more than $3,000
Variables	Things we can change	Dimensions, material choice, layout
Parameters	Quantities that we can't or won't change	Material properties, e.g., density of a particular lithium-ion battery; thresholds of soft constraints

Note: This is what you need to do for the "optimization in the real world" assignment!

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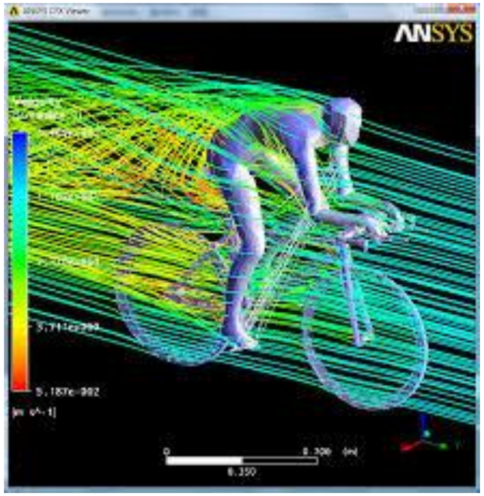
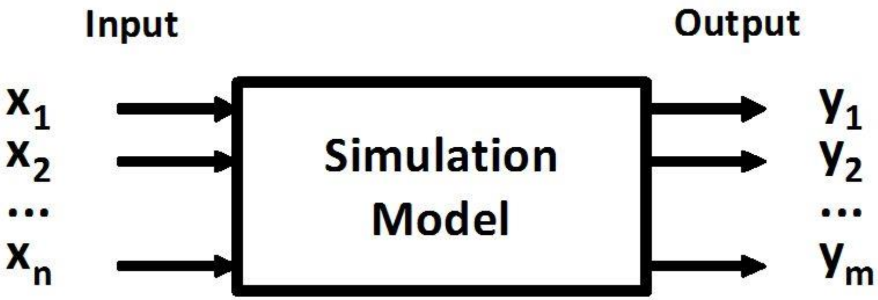
b) Develop analytical models

- How do we represent our system mathematically?
  - Inputs: Variables & parameters
  - Outputs: Objectives & constraints
- Three ways to do this
  1. Chemical/physical/mathematical equations
  2. Simulation models (FEA, CFD, etc.)
  3. Experimental data
     - Design of experiments
     - Metamodeling





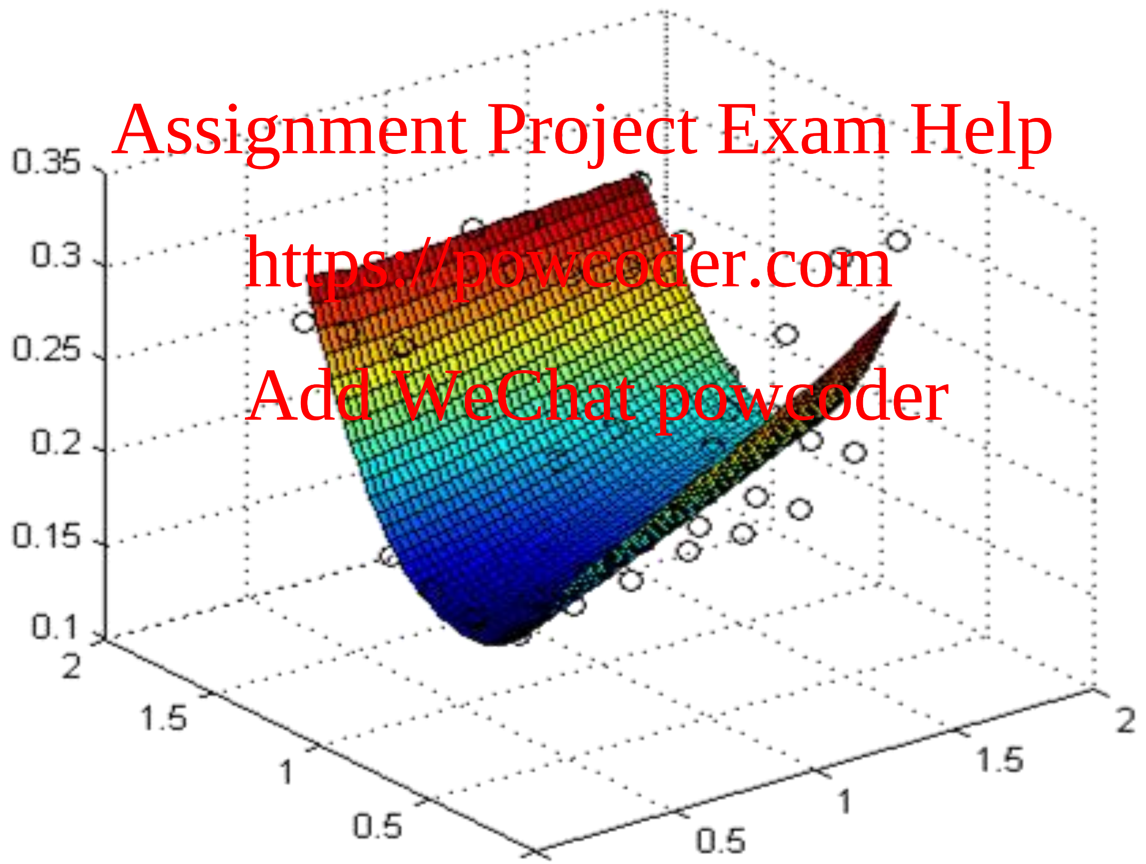
Design of Experiments (DOE)

When we have experimental or computationally expensive simulation data, we need to sample the space efficiently

RunOrder	Pressure	Speed	Temp	Output
1	10	50	45	
2	20	50	45	
3	10	100	45	
4	20	100	45	
5	10	50	65	
6	20	50	65	
7	10	100	65	
8	20	100	65	

Metamodeling

Fit an analytical model to data



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c) Explore/reduce the problem space

Once we've framed the problem and defined the models, we can ask:

Week 2

- Does an optimal solution exist?
- Is the problem well-bounded?
- Are the constraints active?
- Are the functions monotonic?
- Are the functions differentiable?
- Are the functions convex?
- Can the formulation be simplified?

Answering these questions can help detect formulation errors, save time, and potentially find the solution!

1. Formulate the problem

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d) Formulate optimization problem

Objective function

Parameters

$$\underset{\mathbf{x}}{\text{minimize}} \quad f(\mathbf{x},\mathbf{p})$$

Variables

$$\text{subject to} \quad \mathbf{g}(\mathbf{x},\mathbf{p}) \leq 0$$

$$\mathbf{h}(\mathbf{x},\mathbf{p}) = 0$$

Constraints

"negative null" form

2. Solve the problem

a) Choose the right approach/algorithm

What are the different types of algorithms (pattern search, gradient-based, population-based)? How do they work? When do we apply each?

b) Solve (by hand, code, or software)

Apply the chosen algorithm to the formulated problem

c) Interpret the results

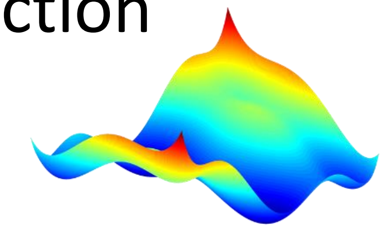
Do the outputs make sense? How do we choose among multi-objective results?

d) Iterate if needed

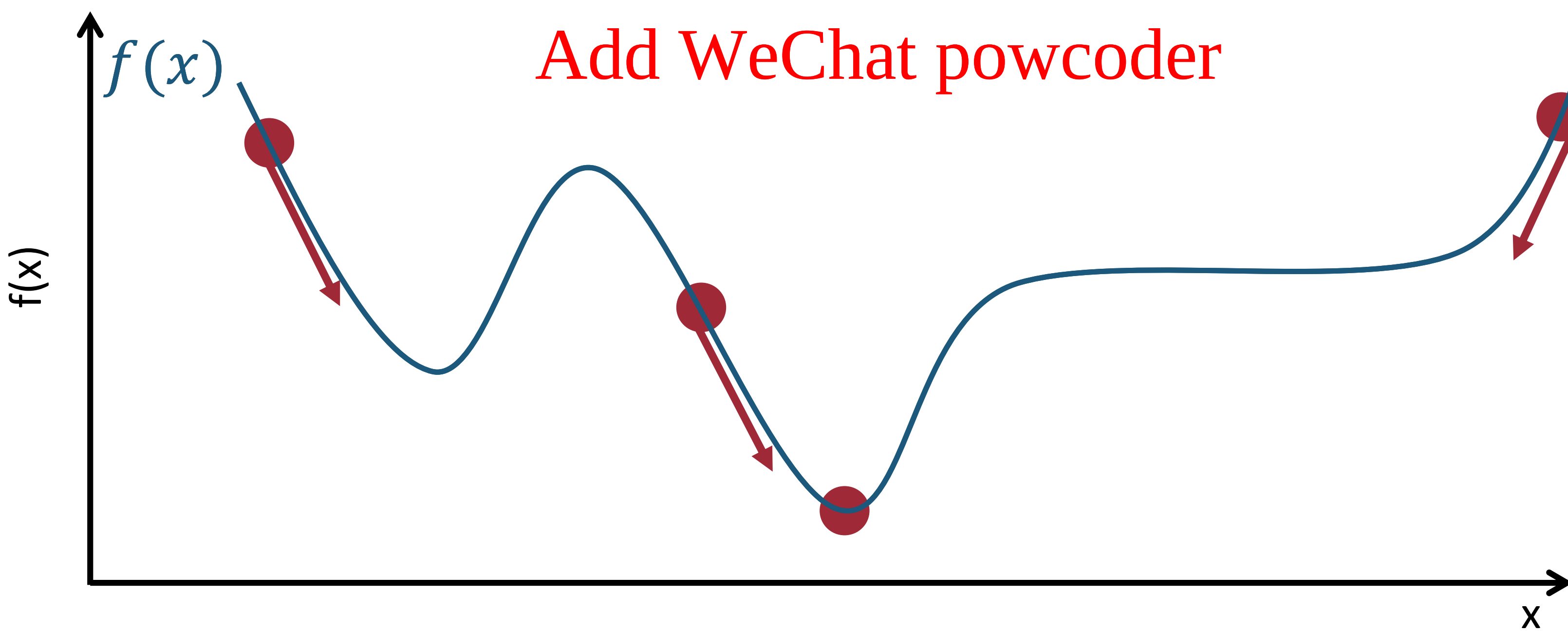
Use findings to update the formulation or algorithm

Why do we need algorithms?

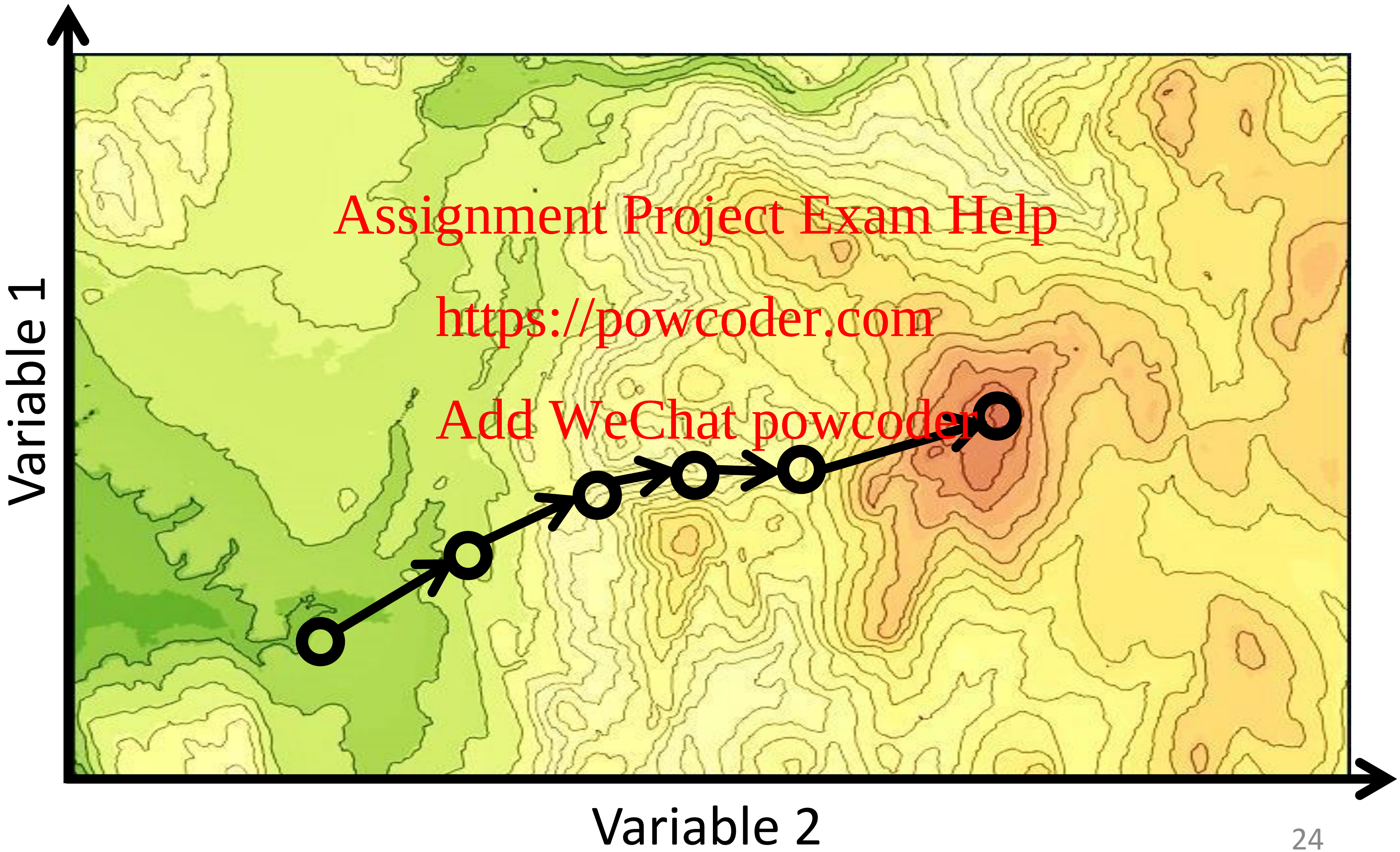
- We don’t always know the shape of a function
  - Too many dimensions to visualize
  - Not enough data points



- Most algorithms take us from a starting point or points, and then move in directions of improvement



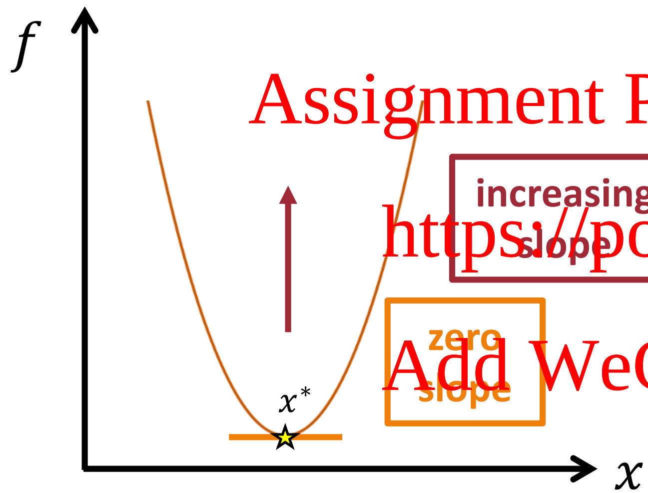
Two-variable contour map

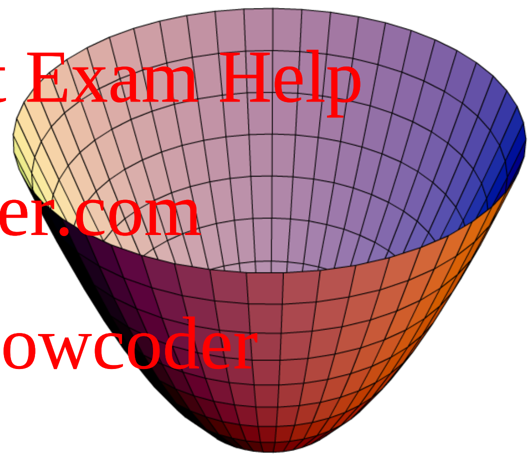


Weeks 5-7

Gradient-based algorithms

Use derivatives to find the optimal solution





Optimality conditions (min)

First-order: $\frac{\partial f}{\partial x}(x^*) = 0$

Second-order: $\frac{\partial^2 f}{\partial x^2}(x^*) > 0$

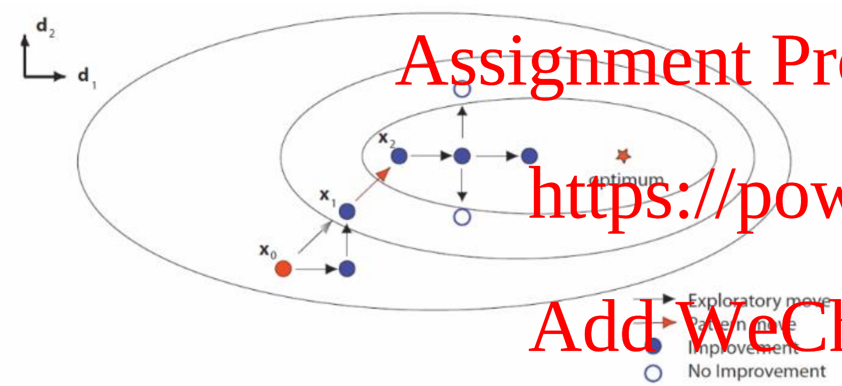
Extension to multi-variable problems:

First-order: $\nabla f(x^*) = \mathbf{0}$

Second-order: $\mathbf{H}(x^*)$ is positive definite

Gradient-free algorithms

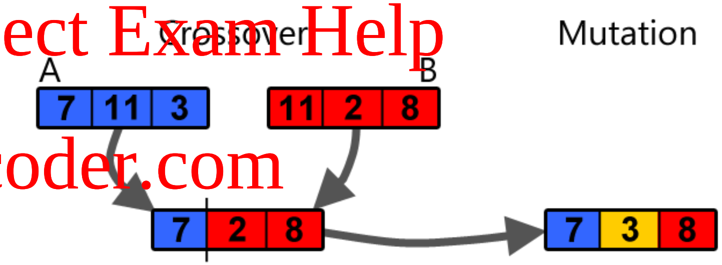
Pattern search



e.g., Hooke-Jeeves direct search, DIRECT, Nelder-Meade

Week 3

Population-based



e.g., genetic/evolutionary algorithms, particle swarm, ant colony

Week 8

Business-oriented optimization

What is the most common objective in design?

We need 2 models:

1. **cost, *C***
2. **demand, *Q***

product attributes

$$\max_{\alpha,P} \pi = Q(\alpha,P)\big(P - C(\alpha)\big)$$

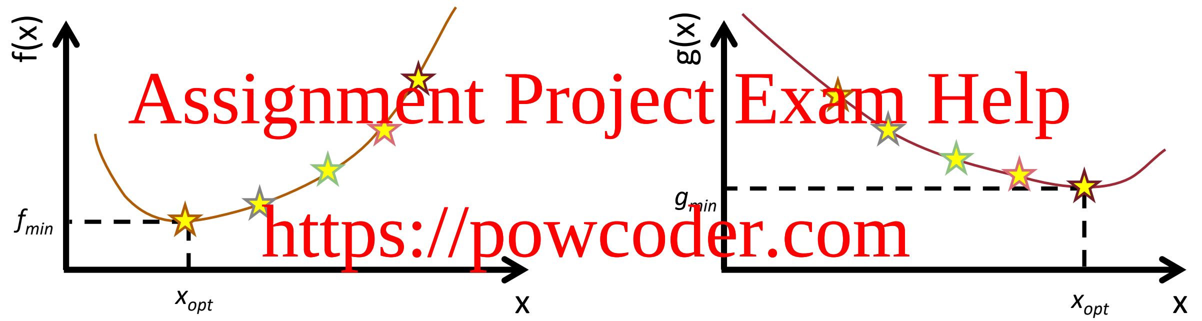
profit

sales quantity

price

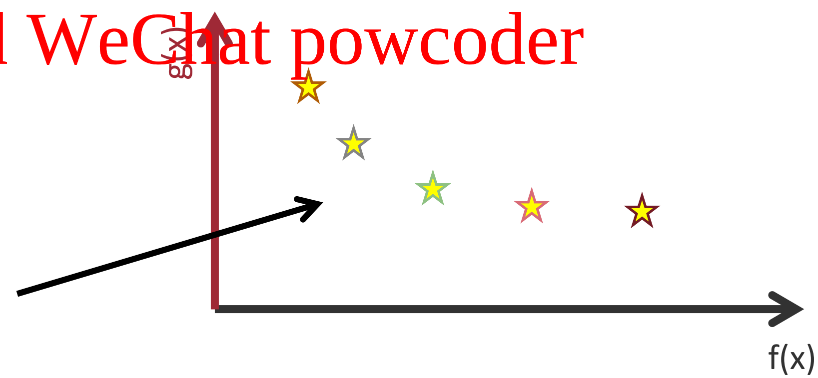
cost

Multi-objective optimization



$$\min_{x} f(x)$$

Pareto frontier



This is useful for trade-off analysis

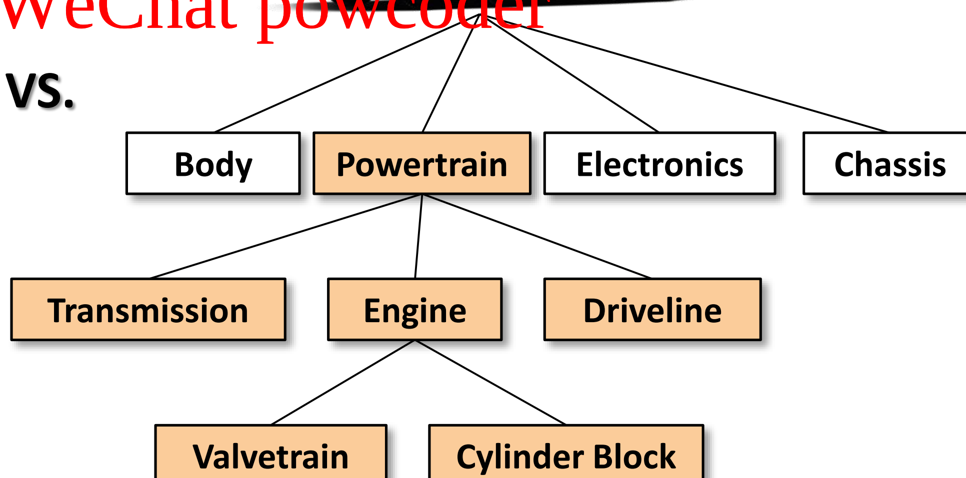
System design

If the system-level problem is difficult to solve all at once, you may need to decompose the problem into subsystems

All In One (AIO)

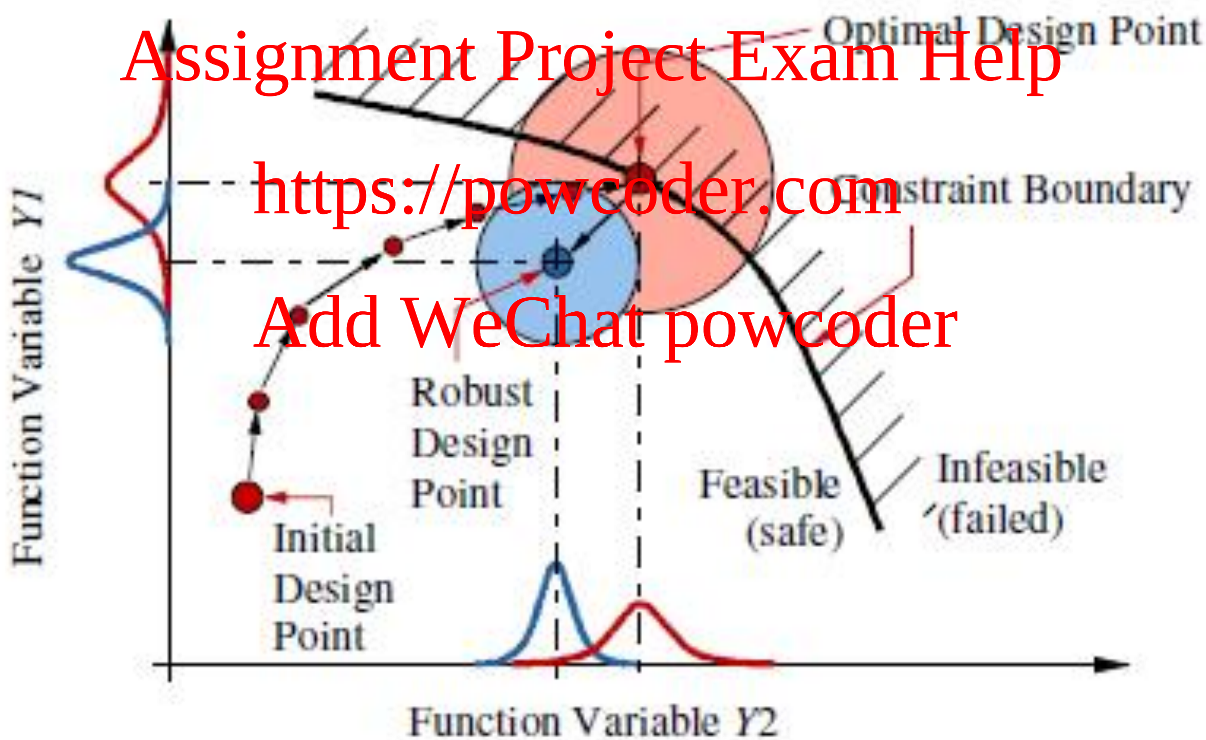
vs.

Decomposition-based Strategy (Partitioning and Coordination)



Handling uncertainty

Ensure the solution isn't too close to a constraint



Software support

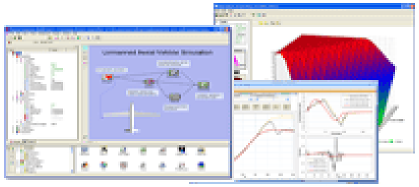
Week 12

Excel **MATLAB** **Specialty optimization packages**

Isight

Design for real Optimus®

PHOENIX INTEGRATION



modeFRONTIER

Commercial CAD software integration

CATIA

ANSYS

NX

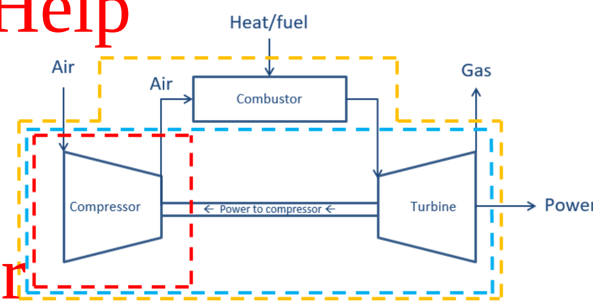
PTC

AUTODESK

Altair HyperWorks

Important takeaways

- What is design optimization?
- What are the major steps of formulating and solving an optimization problem?
- How do we set system boundaries?
  - What are objectives and constraints?
  - What are variables and parameters?



- How do we write a formal optimization problem?

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}, \mathbf{p}) \\
\text{subject to} \quad & \mathbf{g}(\mathbf{x}, \mathbf{p}) \leq 0 \\
& \mathbf{h}(\mathbf{x}, \mathbf{p}) = 0
\end{aligned}
$$

Questions