Homework 4
ME/SYS 564-A
Fall 2018

1. Solving an equality-constrained problem in multiple analytical ways: [20pt]
   Find the point on the plane $2x_1 + 3x_2 - 5x_3 = 12$ in $\mathbb{R}^3$ that is nearest to the point $(1,1,1)^T$, using each of the following methods:
   a) Direct elimination and FONC. Confirm that it is a minimizer with the SOSC.
   b) Reduced gradient. (Only find stationary points.)
   c) Lagrange multipliers. Confirm that it is a minimizer with the Lagrangian SOSC.

   Note: You should not need code for this problem; i.e., write this out and show your work.

   Bonus (+2pt): Plot the distance versus the two variables (after you've eliminated one variable in part a), and indicate on the plot where the optimum is located. Attach your code.

2. Solving an inequality-constrained problem with KKT conditions: [20pt]
   Find the KKT points of the following optimization problem (there are 8 scenarios to examine), and verify their nature using the second-order conditions.

$$\begin{aligned} \min_{\mathbf{x}} \quad & (x_1-1)^2+(x_2-2)^2 \\ \text{s.t.} \quad & x_1 + x_2 \le 2 \\ & x_2 \ge 0 \\ & -x_1 + 2x_2 \le 1 \end{aligned}$$

   Note: You should not need code for this problem.

   Bonus (+3pt): Plot the objective and constraints, and indicate on the plot where the optimum is located. Attach your code.

3. Solving with algorithms: [10pt]
   Solve the optimization problem above in Problem 2 using MATLAB algorithms (do not use the optimtool). Submit your code along with a table of the solutions and number of function evaluations in each algorithm. (Hint: Use the options `'Display'`, `'iter'`.) Use the start point (10, 10) where relevant.
   a) Sequential Quadratic Programming (using `fmincon`)
   b) Active Set (using `fmincon`)
   c) Interior Point (using `fmincon`)
   d) Genetic Algorithm (using `ga` with default options; if the function doesn't work, check that the "Global Optimization Toolbox" is installed)
   e) Briefly (in 4-6 sentences), discuss the differences and pros/cons among the algorithms, including our observations from HW3.