

1. Finding stationary points: *from Problem 4.26 (2nd or 3rd edition)* [10pt]
Use the First Order Necessary Condition (FONC) to find all stationary points of the following function, and then use the Second Order Sufficiency Condition (SOSC) to classify those points as local maxima, minima, or saddle points. Show your work (this should be done by hand).

$$f = x_1 + x_1^{-1} + x_2 + x_2^{-1}$$

Hint: There are 4 stationary points.

2. Solving with optimality conditions: *from Problem 4.12 (2nd or 3rd edition)* [10pt]
Find the point on the plane $x_1 + 2x_2 + 3x_3 = 1$ in \mathbb{R}^3 that is nearest to the point $(-1, 0, 1)^T$. To do so, first formulate it as an optimization problem, and then use the FONC and SOSC to find the optimum and optimizer. Show your work (this should be done by hand), and provide both the minimum f^* and minimizer \mathbf{x}^* .

Hint: Since the distance d between two points is always positive, you can minimize the square of the distance d^2 to find the same optimizer.

3. Solving using gradient-based methods: *from Problem 4.20 (2nd or 3rd edition)* [15pt]
Minimize the following function using 3 different approaches, showing your work for each:

$$f = x_1^2 + 12x_1x_2 + 4x_1x_3 + 6x_1 + 2x_2x_3 + 6x_3^2$$

- a. Use the FONC and SOSC. Show the minimum and minimizer(s).
 - b. Use the gradient descent algorithm, with the optimal step size from Lecture 5 Slide 29, and show \mathbf{x} and f for at least 4 iterations. Use the starting point $(1, 1, 1)^T$.
 - c. Use Newton's method with the starting point $(1, 1, 1)^T$. Continue until it converges. Show \mathbf{x} and f at each iteration.
4. Solving using gradient-based methods: *from Problem 4.25 (2nd or 3rd edition)* [15pt]
Minimize the following function using 3 different approaches, showing your work for each:

$$f = x_1^2 + x_1^4x_2^2 + x_3^2$$

- a. Use the FONC and SOSC. Show the minimum and minimizer(s).
- b. Use the gradient descent algorithm with the optimal step size and show at least 4 iterations. Use the starting point $(1, 1, 1)^T$. Show \mathbf{x} and f at each iteration.
- c. Use Newton's method with start point $(0.2, 0.2, 0.2)^T$ and show \mathbf{x} and f for 3 iterations.
- d. Use MATLAB's fmincon function. Since this is an unconstrained problem, you can enter empty brackets [] in the places where the constraints and lower and upper bounds go. Test this out and write down your solutions (f^* and \mathbf{x}^*) using three algorithm options: 'sqp', 'interior-point', and 'active-set'.