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MGMTMFE-431-1 Credit Markets
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Credit Scoring, Credit Migration, and Actuarial Approaches: Problems

1. (Linear Discriminant Analysis) Using the data in Table 1, implement linear discriminant analysis (LDA) for default prediction:

Table 1: Sample Firms and Characteristics

Firm #	Current Ratio (%)	Debt-to-Asset (%)	Bankruptcy (Yes =1)	Z-Score	Predicted Bankruptcy
1	3.84	68	0	?	?
2	3.13	20	0	?	?
3	3.14	65	0	?	?
4	2.85	75	1	?	?
5	2.35	41	0	?	?
6	2.65	48	1	?	?
7	3.13	68	1	?	?
8	2.07	40	1	?	?
9	2.6	63	0	?	?
10	2.35	25	0	?	?
11	1.76	40	0	?	?
12	2.38	50	0	?	?
13	1.08	67	1	?	?
14	1.77	21	0	?	?
15	1.58	40	1	?	?
16	1.04	42	1	?	?
17	0.65	25	0	?	?
18	1.12	38	1	?	?
19	1.3	56	1	?	?

   (a) Provide a scatter plot of the data. Illustrate the observations of survivor firms using blue dots, and those of bankrupt firms using red dots. Discuss how the two financial ratios that are provided in Table 1 are correlated with the observed bankruptcy outcomes. Do these correlations make sense to you?

   (b) Assuming equal priors, compute the Z-scores and fill in the corresponding missing values in Table 1.

   (c) Compute the zones of discrimination and fill in the predicted bankruptcy outcomes in Table 1. How many observations are misclassified? Discuss! [Note: 1 denotes bankruptcy, 0 denotes survival.]

   (d) (*Optional*) Implement the analysis using an alternative classification scheme based on machine learning algorithms. You can consider support vector machine (SVM), K-nearest neighbors (KNN), decision trees or random forest algorithms, among many others.

2. (Logistic Regression) Using the data from Table 1 in Question 1, answer the following questions:

   (a) Estimate the probability of bankruptcy using the linear probability model. Compute the predicted probability and discuss the potential caveats of this linear model. You can consider drawing similar plots as in the lecture slide titled "Linear Probability Model vs. Logit Model – Example."

   (b) Repeat the exercise in (a) using the logistic regression model.

   (c) Prepare a confusion matrix using the estimated outcome from the logistic model. What is the false positive rate, the false negative rate, the true positive rate (a.k.a. sensitivity), the false positive rate (a.k.a. specificity), respectively?

   (d) Construct the ROC curve of your logistic regression and discuss the model performance based on the AUC.

   (e) Compare the in-sample performance of the prediction accuracy between the logistic regression model and LDA in Question 1.

3. (CVaR using CreditMetrics) Consider the information in Tables 2 and 3 for answering the following questions. Table 2 summarizes 1-year transition probabilities. Table 3 provides 1-year forward rates for year by year cash flows by ratings. Using these rates, one can construct forward discount factors for the cash flow terms of interest by ratings:

Table 2: 1-Year Rating Transition Matrix

	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81%	8.33%	0.68%	0.06%	0.12%	0.00%	0.00%	0.00%
AA	0.70%	90.65%	7.79%	0.64%	0.06%	0.14%	0.02%	0.00%
A	0.09%	2.27%	91.05%	5.52%	0.74%	0.26%	0.01%	0.06%
BBB	0.02%	0.33%	5.95%	86.93%	5.30%	1.17%	0.12%	0.18%
BB	0.03%	0.14%	0.67%	7.73%	80.53%	8.84%	1.00%	1.06%
B	0.00%	0.11%	0.24%	0.43%	6.48%	83.46%	4.07%	5.20%
CCC	0.22%	0.00%	0.22%	1.30%	2.38%	11.24%	64.86%	19.79%

Table 3: 1-year Forward Rate Curve by Ratings

	Terms (Years)				
	0	1	2	3	4
AAA	N.A.	3.10%	3.67%	4.23%	4.62%
AA	N.A.	3.15%	3.72%	4.28%	4.67%
A	N.A.	3.22%	3.82%	4.43%	4.82%
BBB	N.A.	3.60%	4.17%	4.75%	5.13%
BB	N.A.	5.05%	5.52%	6.28%	6.77%
B	N.A.	5.55%	6.52%	7.53%	8.02%
CCC	N.A.	14.55%	14.52%	13.53%	13.02%

   (a) Consider a 5-year senior unsecured bond issued by an A-rated obligor today. The bond has a face value of $100 million and an annual coupon rate of 5%. Over 1 year, the obligor's

rating could migrate to new ratings. The average recovery rate upon default for the senior unsecured claim is 51.13%. What is the bond's expected value (and its standard deviation) in 1 year? Also determine the 1-year 99.7% CVaR of this single debt position.

(b) Now consider a portfolio of two bonds. The first bond is the bond introduced in (a) above. The second bond is the 3-year senior unsecured bond issued by a BB-rated obligor. The obligors' assets are correlated with a correlation parameter of 30%. What is the bond portfolio's expected value (and its standard deviation) in 1 year? Also determine its 1-year 99.7% CVaR.

(c) (Optional) For a general portfolio of N$\gg 2$ bonds, one has to simulate correlated standard normal variables, and then use the ratings' cutoffs to determine the new rating that is expected for each obligor in 1 year. To become familiar with this simulation approach, consider the same case as in (b), i.e., N=2, and illustrate how you can generate two correlated standard normal variables using the Cholesky decomposition of the covariance matrix, $V = \begin{bmatrix} 1 & 0.30 \\ 0.30 & 1 \end{bmatrix}$. Simulate a sample of 10,000 draws and determine the 1-year 99.7% CVaR using the simulated portfolio values. [Note: A Cholesky decomposition yields a lower triangular matrix L such that $L^T L = V$.]