

Lecture 7:

MIP Models for Scheduling

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Outline

- Time-indexed Formulation for JSP
- Disjunctive Formulation for JSP

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Readings

- Presentations by Stefan Heinz & Timo Berthold, Zuse Institute Berlin
- Applegate & Cook, A Computational Study of the Job-Shop Scheduling Problem, *ORSA Journal on Computing*, 3(2), 149-156, 1991
- Ku & Beck, Revisiting Off-the-Shelf Mixed Integer Programming and Constraint Programming Models for Job Shop Scheduling, *Computers & Operations Research*, 73, 165-173, 2016

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MILP Basics

$$\min \sum_{i \in V} c_i x_i$$

Objective function

$$\text{s. t. } \sum_{i \in V} \sum_{j \in C} a_{ij} x_i \leq b_j$$

Could be \leq , $=$, or \geq

Constraints

$$V = V_I \cup V_R$$

$$x_i \mathbb{Z}, \forall i \in V_I$$

Integer variables

$$x_i \in \mathbb{R}, \forall i \in V_R$$

Continuous variables

Continuous (linear) relaxation:
poly-time soluble!

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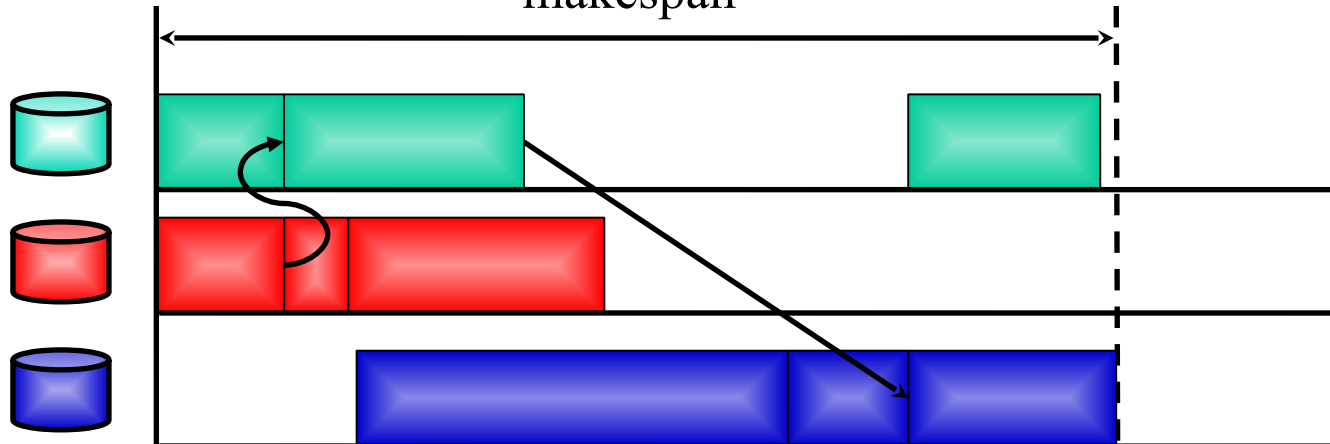
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Job Shop Scheduling



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makespan



Where Do We Start?

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Time-Indexed MIP

- The main decision variable represents whether a job starts at time t or not
- Variables are “indexed” by time

Notation

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- \mathcal{H} is the set of all time-points
- $T_{jt} = \{t - p_j + 1, \dots, t\}$ Set of time points defined by t and j
- \mathcal{J}_k is the set of operations on resource $k \in \mathcal{K}$
- \mathcal{E} is the set of all operation pairs (j, i) s.t. i is constrained to be after j
- $x_{jt} = 1$ iff operation j starts at time t

What Constraints Do We Need?

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Where:

- \mathcal{H} is the set of all time-points
- $T_{jt} = \{t - p_j + 1, \dots, t\}$
- \mathcal{J}_k is the set of operations on resource $k \in \mathcal{K}$
- \mathcal{E} is the set of all operation pairs (j, i) s.t. i is constrained to be after j
- $x_{jt} = 1$ iff operation j starts at time t

Time-Indexed JSP MIP

$$\min C_{max}$$

$$\text{s. t. } \sum_{t \in \mathcal{H}} x_{jt} = 1$$

All operations start only once

$$\sum_{j \in \mathcal{J}_k} \sum_{t' \in T_{jt}} x_{jt'} \leq 1$$

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Resource constraints

$$\sum_{t \in \mathcal{H}} (t + p_j) x_{jt} \leq C_{max}$$

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C_{max} is the largest end-time

$$\sum_{t \in \mathcal{H}} (t + p_j) x_{jt} \leq \sum_{t \in \mathcal{H}} (t) x_{it}$$

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Precedence constraints

$$x_{jt} \in \{0, 1\}$$

$$\forall j \in \mathcal{J}, \forall t \in \mathcal{H}$$

Where:

- \mathcal{H} is the set of all time-points
- $T_{jt} = \{t - p_j + 1, \dots, t\}$
- \mathcal{J}_k is the set of operations on resource $k \in \mathcal{K}$
- \mathcal{E} is the set of all operation pairs (j, i) s.t. i is constrained to be after j
- $x_{jt} = 1$ iff operation j starts at time t

$$\sum_{j \in \mathcal{J}_k} \sum_{t' \in T_{jt}} x_{jt'} \leq 1$$

$$\forall k \in \mathcal{K}, \forall t \in \mathcal{H}$$

$$T_{jt} = \{t - p_j + 1, \dots, t\}$$

Goal: Make sure we are not over-capacity on the blue resource at $t = 10$

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$$p_1 = 5$$



$$p_2 = 3$$



$$p_3 = 6$$



$t = 10$
 $k = \text{blue}$

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Time-Indexed JSP MIP

$$\min C_{max}$$

$$\text{s. t. } \sum_{t \in \mathcal{H}} x_{jt} = 1$$

All activities start only once

$$\sum_{j \in \mathcal{J}_k} \sum_{t' \in T_{jt}} x_{jt'} \leq 1$$

Resource constraints

$$\sum_{t \in \mathcal{H}} (t + p_j) x_{jt} \leq C_{max}$$

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$$\sum_{t \in \mathcal{H}} (t + p_j) x_{jt} \leq \sum_{t \in \mathcal{H}} (t) x_{it}$$

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Precedence constraints

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Where:

- \mathcal{H} is the set of all time-points
- $T_{jt} = \{t - p_j + 1, \dots, t\}$
- \mathcal{J}_k is the set of operations on resource $k \in \mathcal{K}$
- \mathcal{E} is the set of all operation pairs (j, i) s.t. i is constrained to be after j
- $x_{jt} = 1$ iff operation j starts at time t

Questions?





Disjunctive MIP

- The main decision variable represents the sequence between each pair of operations

- A before B or B before A

Notation

- \mathcal{J}_k is the set of operations on resource $k \in \mathcal{K}$
- \mathcal{E} is the set of all operation pairs (j, i) s.t. i is constrained to be after j
- M a big constant
- S_j is the start-time of operation j
- $z_{ji} = 1$ iff operation i is scheduled after operation j

What Constraints Do We Need?

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Where:

- \mathcal{J}_k is the set of operations on resource $k \in \mathcal{K}$
- \mathcal{E} is the set of all operation pairs (j, i) s.t. i is constrained to be after j
- M a big constant
- S_j is the start-time of operation j
- $z_{ji} = 1$ iff operation i is scheduled after operation j

Disjunctive JSP MIP

$$\min \quad C_{max}$$

$$\text{s. t.} \quad S_j + p_j \leq C_{max}$$

C_{max} is the largest end-time

$$S_j + p_j \leq S_i$$

Precedence constraints

$$S_j \geq S_i + p_i - M \cdot z_{ji}$$

$$S_i \geq S_j + p_j - M \cdot (1 - z_{ji})$$

Resource constraints

$$0 \leq S_j \leq |\mathcal{H}| \quad \forall j \in \mathcal{J}$$

$$z_{ji} \in \{0, 1\}$$

$$\forall j, i \in \mathcal{J}_k, \forall k \in \mathcal{K}$$

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Where:

- \mathcal{J}_k is the set of operations on resource $k \in \mathcal{K}$
- \mathcal{E} is the set of all operation pairs (j, i) s.t. i is constrained to be after j
- M a big constant
- S_j is the start-time of operation j
- $z_{ji} = 1$ iff operation i is scheduled after operation j

$$S_j \geq S_i + p_i - M \cdot z_{ji} \quad j, i \in \mathcal{J}_k, j < i, \forall k \in \mathcal{K}$$

$$S_i \geq S_j + p_j - M \cdot (1 - z_{ji}) \quad j, i \in \mathcal{J}_k, j < i, \forall k \in \mathcal{K}$$

- $z_{ji} = 1 \rightarrow$ operation i is after operation j
 - $S_j \geq S_i + p_i - M$: redundant since $S_j \geq 0$
 - $S_i \geq S_j + p_j - M \times 0$: precedence constraint
- $z_{ji} = 0 \rightarrow$ operation j is after operation i
 - $S_j \geq S_i + p_i - M \times 0$: precedence constraint
 - $S_i \geq S_j + p_j - M$: redundant since $S_i \geq 0$

Disjunctive JSP MIP

$$\min \quad C_{max}$$

$$\text{s. t.} \quad S_j + p_j \leq C_{max}$$

$$S_j + p_j \leq S_i$$

$$S_j \geq S_i + p_i - M \cdot z_{ji}$$

$$S_i \geq S_j + p_j - M \cdot (1 - z_{ji})$$

$$0 \leq S_j \leq |\mathcal{H}|$$

$$z_{ji} \in \{0, 1\}$$

C_{max} is the largest end-time

Precedence constraints

Resource constraints

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$$\forall j \in \mathcal{J}$$

$$\forall j, i \in \mathcal{J}_k, \forall k \in \mathcal{K}$$

Questions?



Where:

- \mathcal{J}_k is the set of operations on resource $k \in \mathcal{K}$
- \mathcal{E} is the set of all operation pairs (j, i) s.t. i is constrained to be after j
- M a big constant
- S_j is the start-time of operation j
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Which is Best?

So is time-indexed
useless?

Problem	Disjunctive		Time-Indexed	
	Time (gearithh)	Opt	Time (gearithh)	Opt
3×3	0.00 / 0.00	10	0.02 / 0.01	10
4×3	0.01 / 0.00	10	0.04 / 0.03	10
5×3	0.01 / 0.01	10	0.17 / 0.17	10
3×6	0.01 / 0.01	10	0.18 / 0.18	10
3×8	0.01 / 0.01	10	0.44 / 0.42	10
3×10	0.01 / 0.01	10	0.94 / 0.85	10
8×8	0.59 / 0.58	10	3001.69 / 2478.13	2
10×10	5.95 / 5.30	10	~ 10	-
15×15	-	-	#	#
20×15	-	-	#	#

Results with CPLEX (default parameters)



What Happened Today?

- MIP models for JSP
 - Time-indexed model
 - Disjunctive model

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