MSIN0041 – Individual Coursework 2

Last update: 3 November 2022

General instructions: Please submit your work in a PDF file to the designated submission box on Moodle

Problem 1. Regression-Based Demand Model

Use `data()` to load the `tuna` dataset from the `bayesm` package. You can view the documentation of this dataset by running `?tuna` in R. From the documentation, you will see the correspondence between each column and the different brand names.

a. (2 marks) Regress the log demand of Chicken of the Sea tuna on the log prices of all seven brands. Show your R codes and use `summary()` to show the regression output.

b. (2 marks) Interpret the coefficient on `LPRICE1`.

c. (2 marks) Interpret the coefficient on `LPRICE2`.

d. (2 marks) Suppose the regression estimation is unbiased. What is the optimal cost markup for Chicken of the Sea tuna?

Problem 2. Ad Effectiveness Measurement

Angel Hotel Group shares its customer information with Google so that whenever a user is using Google's search engine, the hotel would know whether the user is a registered customer at the hotel.

Suppose the hotel's marketing department mainly targets the hotel's registered customers that are searching for travel related keywords such as "hotel" on Google. For each registered customer i, let $X_i = 1$ if the customer is searching for a travel-related keyword and $X_i = 0$ if otherwise. For each registered customer doing a travel-related search, the hotel is able to win the bid to push its ads to the customer with a probability of 40%. For those registered customers not searching for travel-related keywords, they see the hotel's ads with a probability of 5% through internet traffic outside Google. From Google Ad's analytics data, the proportion of registered customers' searching for hotels on Google is about 20%. Let $W_i \in \{0, 1\}$ denote a registered customer's treatment assignment such that $W_i = 1$ means the customer sees the hotel's ad whereas $W_i = 0$ when the customer does not see the hotel's ad.

Let $Y_i \in \{0, 1\}$ be the observed outcome where $Y_i = 1$ if the customer i makes a booking and $Y_i = 0$ if otherwise. Each customer i is associated with the two potential outcomes $Y_i(0) \in \{0, 1\}$ and $Y_i(1) \in \{0, 1\}$, such that $Y_i = Y_i(W_i)$. We will be looking at the estimation

of treatment effect from an omniscient view. Below is the distribution of $Y_i(0)$ and $Y_i(1)$ conditional on X_i:

$$\Pr(Y_i(0) = 1|X_i = 1) = 0.3, \quad \Pr(Y_i(1) = 1|X_i = 1) = 0.4,$$
$$\Pr(Y_i(0) = 1|X_i = 0) = 0.01, \quad \Pr(Y_i(1) = 1|X_i = 0) = 0.02.$$

In practice, the above distributional information is not available to the firm, but some can be measured through data. Here is a quick summary of the data about registered customers that can be collected by the hotel:

- Each registered customer's observed outcome $Y_i \in \{0, 1\}$.
- Each registered customer's treatment assignment $W_i \in \{0, 1\}$.
- Each registered customer's search pattern $X_i \in \{0, 1\}$.

Assume that for each registered customer i, $(Y_i(0), Y_i(1)) \perp W_i|X_i$, ie conditional on the customer's search pattern, whether the customer sees the hotel's ad is independent of the two potential outcomes. Now the hotel is interested in knowing how effective is its advertising to its registered customers.

a. (2 marks) Use the distributional information. Conditional on $W_i = 1$, what is the probability that $X_i = 1$? What is the probability of $X_i = 1$ conditional on $W_i = 0$? (*Hint: Use Bayes' rule.*)

b. (2 mark) Calculate the average treatment effect on the treated with the distributional information. Why cannot the hotel directly measure ATT through the data that it can collect?

c. (4 marks) Due the fundamental challenge to measuring ATT, the company decides to use $E(Y_i(1)|W = 1) - E(Y_i(0)|W = 0)$ as a proxy measure for ATT. Using the distributional information, calculate the bias of this proxy measure. (*Hint: Use the law of iterated expectations.*)

d. (2 marks) Explain the intuition of the bias.

e. (3 marks) Construct an unbiased estimator of ATT that is feasible given the data that can be collected by the hotel. Remember to explain why it is unbiased.

f. (5 marks) Suppose now due to privacy regulation, the hotel is not allowed to share its customer information with Google. Consequently, when a customer does not see the hotel's ad, the hotel does not know whether the customer was searching for travel-related keywords, ie X_i is unobserved when $W_i = 0$. For your information, here is a quick summary of data observable by the hotel after the privacy regulation:

    - Each registered customer's observed outcome $Y_i \in \{0, 1\}$.

Passenger / carriage	Economy	First
Less Well-Off	£1.5	£2
Well-Off	£2.5	£6

Table 1: WTP of passengers

- Each registered customer's treatment assignment $W_i \in \{0, 1\}$.
- Each registered customer's search pattern conditional on $W_i = 1$.

Propose an experimental design that could help the company collect the data for an unbiased estimator of ATT. Explain how the estimator is constructed. (*Hint: In the reading for Lecture 4, what did Rocket Fuel do to measure the advertising effectiveness for its clients?*)

Problem 3. Second-Degree Price Discrimination

Suppose you are in the 19th century and you are working for a railway company called Monopoly Rail. The company runs a monopoly railway service connecting Liverpool and Manchester. Currently, the company has two types of carriages for its trains: first class and economy class. The economy class offers an okay ride between the two cities, whereas the first class offers an extravagant one. For simplicity, assume the cost of operating each type of carriage is 0.

Your marketing department has learned that the company has two types of passengers: the well-off and the less well-off. The sizes of these two segments are approximately the same. Assume the total market size is 100. Through a conjoint analysis, the company learned about each passenger type's willingness-to-pay for each type of carriage. The estimates are presented in Table 1. Given a menu of options, each customer chooses the option that gives the customer the highest surplus. If multiple options maximize the customer's surplus, the customer chooses among these surplus-maximizing options the one that gives the highest WTP. Assume the firm has no supply constraint: the firm is able to service the entire market with each type of carriage.

a. (2 marks) If the company only operates one type of carriage, which type should the company operate and what are the optimal price and profit?

b. (4 marks) What are the optimal prices and profit if the company offers both types of carriages?

c. (2 marks) Based on your results so far, which carriage(s) should the company offer and at what price(s)?

d. (6 marks) The company has been thinking about new ways to improve profitability.

Passenger/ carriage	Economy without roofs	Economy with roofs	First
Less Well-Off	£0.6	£1.5	£2
Well-Off	£0.6	£2.5	£6

Table 2: WTP of passengers including the roofless economy

Towards this objective, the company hired a consultant to analyse its business. After careful analysis, the consultant proposed that the company should dismantle the roofs of its economy carriages. Understandably, the company's executives were not convinced of the counter-intuitive proposal at all. To show this proposal's merits, the consultant worked with the company's marketing department to estimate each passenger type's WTP for each type of carriage. The results are presented in Table 2.

Riding in an economy carriage without a roof becomes very miserable when it rains. In this case, the well-off and the less well-off suffer equally from being drenched in rain. If the company goes ahead with dismantling the roof of the economy-class carriages, which type(s) of carriages should the company offer and at what price(s)? Would it be profit-improving for the company to remove the roofs?[1] (Note that the company can sell both roofed and roofless economy-class carriages)

[1] Assume social and legal factors such as customer backlash and regulatory response, can be ignored and only profits are considered here.