MSc in Financial Mathematics, FM50/2018

Negative rates and portfolio risk management

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This document describes one of the available topics for the MSc-project in Financial Mathematics. The focus is on an investor who holds a portfolio of assets and who wants to compute and interpret certain risk measures in order to guide future actions in an environment with negative rates.

The project has three parts: first part is a literature review that would describe and explain the contracts in the portfolio, the models used for equity/ interest rates, the methods available for the modelling of the default, the methods used to estimate the parameters, and the most common risk measures. The second part consists of a numerical analysis of a specified portfolio, where the models would be implemented on a practical level. The third part treats more advanced issues related to this topic, with a view of enhancing the understanding of the problem, the model and the results.

Implementing such a project in real life would require at a minimum identifying the risk factors and appropriate models for them, checking if counterparty credit risk is present, and how to model it if necessary, identifying which real market data to be used for parameter estimation and how long the historical time series should be.

To facilitate the analysis we provide guidance for some of the steps mentioned above. The risk factors are modelled with stochastic models that have been introduced in previous modules, and the parameters of the models are estimated using real data from Bloomberg over the specified time horizon. Future paths are generated according to these models, and the possible future values are incorporated in a risk analysis through the computation of risk measures for the portfolio.

The investor is subject to credit risk, where the counterparty of a certain contract in the portfolio can default before the maturity of the contract, thereby affecting the payoff of the contract. For our portfolio we consider a reduced-form model with constant intensity of default that will model the occurrence of the default. The parameters for this model of default can be calibrated to market data (e.g. CDS market prices), but we will assume specific values for them.

Part 1: Literature review

In the first part the student is asked to write a literature review that should include a description of the contracts in the portfolio, particularly the EONIA-based interest rate swap, a brief outline of the models used for equity/ interest rates, the methods available for the modelling of the default (i.e. structural vs reduced form models, advantages and disadvantages of each class), a brief outline of the methods used to estimate the parameters, and a review of the most common risk measures.

The student is invited to consult a number of publications on EONIA/ECB rates, Credit Risk modeling, and on Value at Risk and risk measures in general. We present some suggestions below as a starting point:

- For EONIA and EONIA based contracts see the European Money Markets Institute (EMMI) website `https://www.emmi-benchmarks.eu`
- For ECB deposit rate see the European Central Bank (ECB) website `http://www.ecb.europa.eu`

- ”Getting started guide”, Bloomberg, 2012 (available at: https://www.kcl.ac.uk/nms/depts/mathematics/research/finmath/ bloomberg/docs/education-userguide-a4.pdf)

The references at the end of this document are classical books on risk management, interest rate models, least squares parameter estimation and related topics, and give good starting points to the literature. The student should be proactive in researching the literature, which involves published journal papers and books. Working papers should be used mostly for orientation, given that their content has not been peer reviewed.

It is particularly important that the student synthesizes the information gathered from these sources and presents it as a flowing story that is consistent both in terms of notation and mathematical and financial content.

Part 2: Numerical analysis

This part applies the theoretical notions from Part 1 on an analysis of a specific portfolio. The assets in the portfolio are:

1. Equity: 1 share of the Deutsche Boerse AG German Stock Index (Bloomberg ticker DAX INDEX)
2. EONIA based interest rate swap with a counterparty that is default free. The maturity is one month, the notional amount is 10 million, and the swap rate can be obtained from Bloomberg market data on the day $t = 0$. The investor is the swap payer, i.e. pays the fixed rate and receives the floating rate pegged to daily EONIA values. The formula for the EONIA swap variable rate can be found in EMMI publications.

3. European call option (long position) with a counterparty that may default. The underlying is the equity above (DAX), the strike is 12,450 and the maturity is 50 days from $t = 0$. Initial price **of 235.1 at** $t = 0$ comes from Bloomberg. For later prices we use a pricing measure where the equity follows a Black-Scholes model with drift equal to the simulated EONIA rate at 30 days from $t = 0$, and the volatility is the square root of the element of Σ corresponding to equity. The counterparty of the option can default with zero recovery rate (in case of default the entire option becomes worthless). The default is modelled by a reduced form model with constant annual intensity of default 0.12.

The goal of the project is to analyse the risk and return characteristics of the portfolio using a stochastic model for the underlying risk factors.

Stochastic model

Consider the risk factors to be the equity (DAX) and the EONIA spread over the ECB deposit rate:

$$X_t = (\log Y_t \;\; \log S_t)',$$

and assume they follow under the subjective measure $\mathbb{P}$ a discretized version of a stochastic differential equation (SDE) of the type

$$\Delta X_t = (AX_{t-\Delta t} + b)\Delta t + \varepsilon, \;\; \varepsilon \sim N(0, \Sigma),$$

where $\Delta t = 1$ day (for tractability make the simplifying assumption that weekends or holidays are equivalent to 1 day periods).

- Estimate model parameters: A, b, Σ using two years of historical daily data from Bloomberg (see, for instance, the packages *lm, dynlm* in R). Fix the date of the analysis ($t_0 = 0$) as 27/04/2017. Assume the data spikes at the end of most months (colloquially called the beat/the pulse) are caused by expired regulatory requirements, so we exclude all end of the month observations from the data. Plot the data with and without the beats.

- If a parameter has a significance level above 5%, then temporarily set it to zero (we will use alternative information to historical data to estimate them). Write the resulting discrete dynamics of $\log Y_t$ and $\log S_t$.

- Identify b so that the log equity has an annual drift of 5%, and log EONIA spread over ECB satisfies $E(\log S_{t_0+1} - \log S_{t_0}) = 0$. Assume the central bank deposit rate remains constant at its last observed level at $t = 0$.

- The covariance matrix Σ is derived as the cross-product of the residuals, normalized appropriately with respect to the number of observations.

Computational results

- Analyse the distribution of the potential losses incurred by the portfolio over a 30 days holding period, and use VaR and ES (CVaR) to quantify them with confidence level 99%. The relative losses are defined wrt to the value V_1 of the portfolio in 30 days from $t = 0$ and the value V_0 at time $t = 0$ (you can consider the percentage change in the value of the portfolio for instance). Analyse the numerical accuracy of the results.

- To compute the value V_0 find the equity price, the option price and the swap rate on the date $t = 0$ in the Bloomberg terminal, and explain how you obtained them.

- For V_1 simulate 30 days for the process Y, and simulate also the default of the counterparty of the call. Plot one future sample path together with the historical path for each component of the process X.

- Include in your analysis the histogram of the relative losses, a plot of the return distribution of the portfolio, computation of VaR and ES, and the expected and median returns of the portfolio.

- Analyse the impact of credit risk by repeating the calculations without credit risk (zero intensity) and comparing the results. Discuss.

Summarize your answers by completing a table in the format shown in Table 1.

Portfolio	Expected returns	Median returns	V@R	CV@R
credit risk				
no credit risk				

Table 1: Format to use for the display of the results

Part 3: Advanced issues (to be done after part 2)

This part should include any pertinent analysis that would contribute to enhancing the understanding of the topic. Ideally this would be focused on negative rates and portfolio risk management. Among the possible extensions that could be studied in relation to the proposed topic we mention (but these are just suggestions, and the list is not comprehensive):

- Analyse the impact that negative rates have on the portfolio. You can try to repeat the calculations, but with a constant zero ECB deposit rate instead of the real market value, and then compare the results. Or you can propose alternative ways to analyse this, but explain your reasoning.

- Give histograms of the residuals and perform statistical tests to check for their normality. Test also for serial autocorrelation, and comment on any GARCH implications.
- Use variance reduction techniques to simulate future scenarios and improve numerical accuracy.
- Analyze the impact of # of scenarios on the accuracy of the results.
- Change the weights of the assets, starting with same initial portfolio value, to find a better portfolio (i.e. higher expected return and smaller CVaR).

References

- D. Montgomery, E. Peck, G. Vining, "Introduction to linear regression analysis", 5th Ed, Wiley, 2012.
- D. Filipovic, "Term structure models: a graduate course", Springer, 2009.
- M. Crouhy, D. Galai, and R. Mark, "A Comparative Analysis of Current Credit Risk Models", available at http://www.defaultrisk.com/pp_model_12.htm
- CreditMetrics Technical Document, available at defaultrisk.com
- D. Brigo, F. Mercurio, "Interest Rate Models: Theory and Practice", 2nd Edition, 2006, Springer Verlag.
- T. Bielecki and M. Rutkowski, "Credit risk: Modeling, Valuation and Hedging", Springer Verlag, 2002. This book has a rigorous treatment of credit risk.
- P. Jorion, "Value at Risk", 3rd ed, McGraw Hill. This covers VaR at elementary level.
- For models of default see the numerous papers by M. Jeanblanc; or the paper by Jarrow and Protter "Structural versus reduced form models: a new information based perspective", J of Inv. Manag., 2(2), 1-10, 2004.

Important: General notes on writing a Master Dissertation for King's Financial Mathematics

All students on the MSc course in Financial Mathematics are required to submit the dissertation. The following describes what this involves.

The MSc dissertation consists of a review of a suggested area of currently active research in mathematical finance, with the addition of some specific application and with possible developments. It would be good if a student could produce some original research in the final part of the dissertation, although this is not always possible in the short time available. What the faculty is hoping to see is a critical understanding of the literature, including acquired results, open problems, current difficulties, past importance and potential for future applications, and the ability to apply what has been understood in a practical case. When reading the dissertation, the examiners will try to see whether the student has absorbed and understood the material while presenting it in her own way, and whether the student has been able to implement the ideas effectively.

Important: The student should not copy whole sentences or even paragraphs directly from published papers, and should write her own code. Violations are easily noticed by markers and would be penalized. Furthermore, all sources must be properly referenced.
The dissertation must be an independent piece of work. While it can be helpful to talk with other students who are writing the dissertation, the final work produced by the student should not be collaborative.

Before resorting to advisors for help the students are encouraged to be proactive in solving their problems. It it expected that students resort to advisor help for substantial matters and only as a last resort. Advisors should not be contacted for trivial suggestions.