UNIVERSITY OF LONDON

BSc/MSci/MSc EXAMINATION 2017

For internal students
of Royal Holloway

MT3470: MATHEMATICS OF FINANCIAL MARKETS
MOCK EXAM

Attempt all questions.

MT and Type A Calculators are permitted / Statistical Tables are provided.

Spares not allowed.

Section A (60 marks)

1. (a) An investor enters into a short forward contract on a certain asset where the strike price is £50. How much does the investor gain or lose if the asset price at the end of the contract is £60? Explain your answer.

   (b) Consider a one year forward contract on a stock with no income when the spot price is $S(0) = £100$ and the risk-free interest rate with continuous compounding is 5% per annum. Calculate the forward price F_0.

   (c) Let F_0 be the forward price for the forward contract in Part (c).
   Suppose a zero value forward contract with the same maturity and delivery price of
   i) $F_0 + £1$, ii) $F_0 - £1$,
   is available on the same stock. Describe a strategy by which you could achieve arbitrage in these situations.

(10 marks)

2. (a) What is a European call option? What is a European put option?

   (b) Consider two European call options written on the same asset, with the same maturity, but different strike prices $K_1 < K_2$. Which of two options is more expensive? Justify your answer.

   (c) At time $t = 0$ an investor holds a portfolio consisting of one European put option, valued at P, one unit of the underlying asset, valued at S, but is short one European call option, valued at C, on the same underlying asset. Both options have the same expiry date T and exercise price K. What is the pay-off of this portfolio at expiry? Justify your answer. Using this information, establish the put-call parity theorem for European options on a non-dividend paying underlying asset, i.e. $C + Ke^{-rT} = P + S$, where r is the risk-free rate of return.

(10 marks)

3. (a) State the model equations and assumptions of the single-index model.

   (b) Suppose the risk-free rate is 10% and the expected return of the market is 6%. Suppose that an asset has beta $\beta = 0.4$. Find the asset expected return under CAPM model.

   (c) Suppose that the expected return of the market index is $\mathsf{E}(R_M) = 0.20$ and the market risk (variance) is $\mathsf{Var}(R_M) = 0.02$. Consider asset S_1 and S_2 such that $\alpha_1 = 0.1, \beta_1 = 0.7, \sigma^2_{a_1} = 0.06$, and $\alpha_2 = 0.03$, $\beta_2 = 1.2, \sigma^2_{a_2} = 0.04$. Under the single-index model find the expected return and risk of the portfolio P, which has equal holdings in assets S_1 and S_2.

(10 marks)

4. (a) In the Markowitz portfolio model define:
   (i) the feasible set, (ii) the efficient portfolio and (iii) the efficient frontier.

   (b) Consider two assets S_1 and S_2 with expected returns $r_1 = 0.15$ and $r_2 = 0.3$, and standard deviations of returns $\sigma_1 = 0.01$ and $\sigma_2 = 0.05$ respectively. Also, the correlation coefficient is $\rho = -0.2$.

   (i) Calculate the expected return and variance of the portfolio, where 5/6 of wealth are invested in S_1 and 1/6 in S_2.

   (ii) Find the minimum-risk portfolio in case short sales are allowed and if they are forbidden.

(10 marks)

5. Suppose a stock price $S(t)$ follows a two-step binomial model specified by the following parameters $u = 1.2, d = 0.8, p = 0.5$ and the risk free interest rate $r = 5\%$ (i.e. $\alpha = 1.05$). Suppose that $S(0) = £10$.

   (a) Consider a European call option on the asset with maturity $T = 2$ and strike price $£8$.

      (i) Using the principle of risk-neutrality find the no-arbitrage price of the call option.

      (ii) Find the no-arbitrage price of the call option by constructing a replicating portfolio.

   (b) What is a risk-neutral probability that a European put option with maturity $T = 2$ and strike price $£8$ will be exercised?

(10 marks)

6. Suppose an asset with no costs or dividends is now on sale at a price $£35$. Suppose the expected return of the stock is $\mu = 15\%$, volatility of the asset is $\sigma = 20\%$ (with time measured in years), and the risk-free interest rate is 9% per annum compounded continuously.

   (a) Calculate the expected price of the stock in 3 months.

   (b) Using the Black-Scholes formula nd the price of a European call option on the asset with strike price $£40$ and maturity 6 months from now.

(10 marks)

Section B (40 marks)

7. (a) Consider n risky assets S_i with returns $R_i, i = 1, \ldots, n$. Let $\underline{r} = (r_1, \ldots, r_n)^T$ be the vector of their expected returns and $C = (\mathsf{Cov}(R_i, R_j))_{i,j=1}^n$ be the covariance matrix of their returns. Let S_0 be a risk free asset with return r_0. Consider the tangency portfolio $T = \underline{w} = (w_1, \ldots, w_n)^T$ with w_i holding of S_i.

(i) Show that T obeys the formula

$$r_i - r_0 = \frac{\sigma_{i,T}}{\sigma_T^2}(\mathsf{E}(R_T) - r_0),$$

where $\sigma_{i,T} = \mathsf{Cov}(R_i, R_T)$, $\mathsf{E}(R_T)$ and $\sigma_T^2 = \mathsf{Var}(R_T)$ are the expected return and the variance of T respectively.

You may assume that for a portfolio $P = \underline{w}$ on the efficient frontier the following formula holds:

$$\underline{w} = tC^{-1}(\underline{r} - r_0\underline{e}),$$

where $\underline{e} = (1, \ldots, 1)^T$ and t is a real valued parameter.

(ii) Use results of Part (a)(i) to derive the CAPM equation. Explain all terms and state any facts that you use.

(b) Consider a risky asset S with return R_S, expected return r_S and variance σ_S^2. Consider a portfolio P where a fraction x of S is held and a fraction $1 - x$ of the risk free asset S_0 with return r_0.

(i) Derive formulas for the average return and variance of portfolio P.

(ii) Show that the expected return $\mathsf{E}(R_P)$ of portfolio P is a linear function of the portfolio volatility $\sigma_P = \sqrt{\mathsf{Var}(R_P)}$.

(20 marks)

8. Suppose that a stock price $S(t)$ follows the log-normal process with expected return μ and volatility σ, that is

$$S(t) = S(0)e^{(\mu-\frac{\sigma^2}{2})t+\sigma W_t},\ t \geq 0,$$

where W_t is the standard Brownian motion.

(a) Show that given $S(0)$ the expected value of $S(t)$ is equal to $S(0)e^{\mu t}$ and the standard deviation of $S(t)$ is $S(0)e^{\mu T}\sqrt{(e^{\sigma^2 T}-1)}$. State any facts that you use.

(b) What is the probability distribution of $\log(S(1)/S(0))$? State any facts that you use.

(c) Suppose that parameters $S(0) = £35$, $\mu = 15\%$ and $\sigma = 20\%$. Calculate the probability that a stock price does not exceed $£40$ in 3 months time.

(d) Let r be the risk free interest rate compounded continuously. Using the principle of risk neutrality show that the no arbitrage price of a binary call option with the strike price K and the expiration date T is

$$C_{bin} = e^{-rT}\Phi(x_0),$$

where

$$x_0 = \frac{\log(S(0)/K) + (r-\sigma^2/2)T}{\sigma\sqrt{T}},$$

$\Phi(x)$ is the cdf of the standard normal distribution.

The option pays £1, if $S(T) \geq K$ and zero otherwise.

(20 marks)

END