Final Exam

<u>Question 1</u>

Consider the following neoclassical growth model. The equilibrium conditions of the model are given by

$$K_{t+1} = I_t + K_t(1-\delta)$$

$$Y_t = A_t K_t^{\alpha} L^{1-\alpha}$$

$$(1+\tau_{c,t})^{-1} C_t^{-\sigma} = (1+\tau_{c,t+1})^{-1} \beta C_{t+1}^{-\sigma} [A_{t+1} \alpha K_{t+1}^{\alpha-1} L^{1-\alpha} + (1-\delta)]$$

$$C_t + I_t + G_t = Y_t$$

- Log-linearize the second, third, and fourth equations around the steady state. Note that "L" is a constant, not a variable.

<u>Question 2</u>

Consider a version of the New Keynesian model with the following preference. The model is identical to the baseline model discussed in the lecture, except that the household's per period utility is given by

$$\sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{1}{1-\gamma} \left(C_t - \frac{N_t^{1+\chi_n}}{1+\chi_n} \right)^{1-\gamma} \right]$$

- Derive the private-sector equilibrium conditions of the model.
- Assuming that (i) the policy rate is determined by the standard Taylor rule, (ii) the inflation target is zero (that is, $\Pi^{targ} = 1$), and (iii) $\tau = \frac{1}{\theta - 1}$, analytically compute the standard steady state of the model.

<u>Question 3</u>

Consider the following two-period model with a static PC with the ELB. There is a labor income tax that is distributed to households as a lump-sum transfer at time one. Key equilibrium conditions of this model are given by the following equations.

At t=1:

$$y_1 = y_2 - \sigma \left(i_1 - \pi_2 - r_1^n \right)$$

$$\pi_1 = \kappa \left(y_1 - \Gamma \tau_{w,1} \right)$$

$$i_1 \geq 0$$

At t=2:

$$y_2 = -\sigma \left(i_2 - r^{¿} \right)$$

$$\pi_2 = \kappa \left(y_2 - \Gamma \tau_{w,2} \right)$$

$$i_2 \geq 0.$$

The utility function for the government at each time is given by the standard quadratic objective function. That is,

$$u(\pi_t, y_t) = \frac{-1}{2} [\pi_t^2 + \lambda y_t^2]$$

for each t=1,2.

<u>Part A:</u>

Now, assume that the government is optimizing under discretion.

Assume also that the value of r_1^n is such that $i_1 = 0$ but $i_2 > 0$.

Assume that the government can optimally choose labor income taxes at time one and time two.

- Formulate the optimization problem(s) of the central bank.
- Define the Markov-Perfect equilibrium.
- Solve the model analytically.

<u>Part B:</u>

Assume that the government is optimizing under commitment.

Assume also that the value of r_1^n is such that $i_1 = 0$ but $i_2 > 0$.

Assume that the government can optimally choose labor income taxes at time one and time two.

- Formulate the optimization problem of the central bank.
- Define the Ramsey equilibrium.
- Solve the model analytically.

<u>Question 4</u>

Consider the following two-period loglinearized New Keynesian model with a <u>static</u> Phillips curve and with a time-one demand shock ($r_1^n < 0$ and $r_2^n = r^{¿} > 0$). The policy rule is given by a nominal-income targeting rule.

At t=1,

$$y_1 = y_2 - \sigma\left(i_1 - \pi_2 - r_1^n\right)$$

$$\pi_1 = \kappa\, y_1$$

$$p_1 = p_0 + \pi_1$$

$$i_1 = max\left[0, r^{¿} + \phi\left(p_1 + y_1\right)\right]$$

At t=2,

$$y_2 = -\sigma\left(i_2 - r^{¿}\right)$$

$$\pi_2 = \kappa\, y_2$$

$$p_2 = p_1 + \pi_2$$

$$i_2 = max\left[0, r^{¿} + \phi\left(p_2 + y_2\right)\right]$$

- Assuming that (i) $p_0 = 0$ and and (ii) the shock size is such that the policy rate is zero and positive at time one and two, respectively, solve the model analytically.