Final Exam

Question 1 (40 points)

Consider the following neoclassical growth model. The equilibrium conditions of the model are given by

$$K_{t+1}=I_t+K_t(1-\delta)$$

$$Y_t=K_t^{\alpha}L^{1-\alpha}$$

$$C_t^{-\sigma}=\beta C_{t+1}^{-\sigma}\left[\alpha K_{t+1}^{\alpha-1}L^{1-\alpha}+(1-\delta)\right]$$

$$I_t=Y_t-C_t$$

- Log-linearize these four equations around the steady state. Note that "L" is a constant, not a variable.

Question 2 (40 points)

Consider a version of the New Keynesian model with the following preference. The model is identical to the baseline model discussed in the lecture, except that the household's per period utility is given by

$$\sum_{t=1}^{\infty}\beta^{t-1}\left[\frac{1}{1-\gamma}\left(C_t-\frac{N_t^{1+\chi_n}}{1+\chi_n}\right)^{1-\gamma}\right]$$

- Derive the private-sector equilibrium conditions of the model.
- Assuming that (i) the policy rate is determined by the standard Taylor rule, (ii) the inflation target is zero (that is, $\Pi^{targ}=1$), and (iii) $\tau=\frac{1}{\theta-1}$, analytically compute the standard steady state of the model.

Question 3 (40 points)

A two-period model with a static PC with the ELB. There is a government spending shock financed by a consumption tax at time one:

At t=1:

$$y_1 - g_1 = (y_2 - g_2) - \sigma(\tau_{c,1} - \tau_{c,2}) - \sigma(i_1 - \pi_2 - r_1^n)$$

$$\pi_1 = \kappa(y_1 - \Gamma_g g_1)$$

$$g_1 = \tau_{c,1}$$

$$i_1 \geq 0$$

At t=2:

$$y_2 - g_2 = -\sigma \tau_{c,2} - \sigma(i_2 - r^{¿})$$

$$\pi_2 = \kappa(y_2 - \Gamma_g g_2)$$

$$g_2 = \tau_{c,2}$$

$$i_2 \geq 0$$

with $g_1 > 0$ and $g_2 = 0$. The payoff function for the central bank at each time is given by the standard quadratic objective function. That is,

$$u(\pi_t, y_t) = \frac{-1}{2}[\pi_t^2 + \lambda y_t^2]$$

for each t=1,2.

Part A:

Assume that the policy rate is determined by the truncated Taylor rule:

$$i_t = max[r^{¿} + \phi_\pi \pi_t, 0]$$

Assume also that r_1^n is sufficiently small so that $i_1 = 0$. Solve the model analytically.

Part B:

Now, assume that the government is optimizing under discretion.

- Formulate the optimization problem(s) of the central bank.
- Define the Markov-Perfect equilibrium.

Part C:

Assume that the government is optimizing under commitment.

- Formulate the optimization problem of the central bank.
- Define the Ramsey equilibrium.

<u>Question 4 (20 points)</u>

Consider the following two-period loglinearized New Keynesian model with a <u>static</u> Phillips curve and with a time-one demand shock ($r_1^n<0$ and $r_2^n=r^{¿}>0$). The policy rule is given by a price-level targeting rule.

At t=1,

$$y_1=y_2-\sigma\left(i_1-\pi_2-r^{¿}\right)$$

$$\pi_1=\kappa\, y_1$$

$$p_1=p_0+\pi_1$$

$$i_1=max\left[0,r^{¿}+\phi\, p_1\right]$$

At t=2,

$$y_2=-\sigma\left(i_2-r^{¿}\right)$$

$$\pi_2=\kappa\, y_2$$

$$p_2=p_1+\pi_2$$

$$i_1=max\left[0,r^{¿}+\phi\, p_2\right]$$

- Assuming that (i) $p_0=0$ and and (ii) the shock size is such that the policy rate is zero and positive at time one and two, respectively, solve the model analytically.

<u>Question 5 (20 points)</u>

Consider the following two-period loglinearized New Keynesian model with a <u>static</u> Phillips curve and with a time-one demand shock ($r_1^n<0$ and $r_2^n=r^{¿}>0$). The policy rule is given by a Reifschneider-Williams rule.

At t=1,

$$y_1 = y_2 - \sigma\left(i_1 - \pi_2 - r^{¿}\right)$$

$$\pi_1 = \kappa\, y_1$$

$$i_1 = max\left[0, i_1^{¿} - Z_1\right]$$

$$Z_1 = Z_0 + \left(i_0 - i_0^{¿}\right)$$

$$i_1^{¿} = r^{¿} + \phi\, \pi_1$$

At t=2,

$$y_2 = -\sigma\left(i_2 - r^{¿}\right)$$

$$\pi_2 = \kappa\, y_2$$

$$i_2 = max\left[0, i_2^{¿} - Z_2\right]$$

$$Z_2 = Z_1 + \left(i_1 - i_1^{¿}\right)$$

$$i_2^{¿} = r^{¿} + \phi\, \pi_2$$

- Assuming that (i) $i_0 = i_0^{¿} = r^{¿}$ and $Z_0 = 0$ and (ii) the shock size is such that the policy rate is zero and positive at time one and two, respectively, solve the model analytically.