Math 170A, Fall 2022 HOMEWORK #3 due Thursday, Oct 20

Homework problems that will be graded (Q1 - Q5, 30pts in total):

Q1. a) (This is the first part of Question 3.1.5 from the textbook.) Consider the following data:

t_i	1.0	1.5	2.0	2.5	3.0
y_i	1.1	1.2	1.3	1.3	1.4

Set up the problem of minimization associated with least squares for a best fit line for this data.

b) Let

$$w_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, w_2 = \begin{bmatrix} 3 \\ 3 \\ -1 \\ -1 \end{bmatrix}, \text{ and } w_3 = \begin{bmatrix} 6 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

be vectors in $\mathbb{R}^4$. Apply the classical Gram-Schmidt process to find an orthonormal basis for the subspace spanned by w_1, w_2, and w_3.

Q2. Let $A = (a_{ij})$ be a 10×10 positive definite matrix with Cholesky factor $R = (r_{ij})$. Suppose that $a_{11} = 9$, $a_{12} = 12$, $a_{17} = 3$, $a_{22} = 25$, and $a_{27} = 7$. Solve for r_{27}.

Q3. Let A be an $n \times n$ positive definite and symmetric matrix, and X an $n \times n$ invertible matrix. Show that $B = X^T AX$ is positive definite and symmetric.

Q4. Show that the function from $\mathbb{R}^{n\times n}$ to $\mathbb{R}$ given by

$$||A||_{\max} = \max_{i,j} |a_{ij}|$$

is not a norm by stating which of the 4 matrix norm conditions it fails to satisfy, and by demonstrating this fact via an example.

Q5. Write MATLAB functions classicalGS.m and modifiedGS.m, implementing the codes given in class to orthogonalize the columns of a matrix A, and output the factors Q and R (Q has orthonormal columns, and R is an upper triangular matrix with positive diagonal).

Once you saved the function scripts, go back to the Command Window and type

$$A = hilb(8);$$

This makes A a particularly ill-conditioned 8×8 matrix form MATLAB's matrix library (we will learn the concept of ill-conditioning later in the course).

Then, call

$$\begin{aligned}[Q1, \sim] &= classicalGS(A);\\ [Q2, \sim] &= modifiedGS(A);\end{aligned}$$

to get the computed factors $Q1$ and $Q2$ with the classical, respectively, modified Gram-Schmidt algorithms.

Finally, let us now test the quality of the $Q1$ and $Q2$ factors. At the prompt, type

$$\text{norm}(Q1' * Q1 - eye(8)),$$

followed by

$$\text{norm}(Q2' * Q2 - eye(8))$$

These commands let you know how close to orthogonal the matrices $Q1$, respectively $Q2$, are.

What do you notice? Comment on it. Take screenshots of the two codes and the Command Window, making sure the outputs to the last two lines are visible.