Tutorial 2 solutions

January 2021

1. 5

   (a)

   (b) We have $\sum_{k=0}^{n} a_k b_{n-k} = 3n$ if $n \geq 1$, and 1 for $n = 0$, and so

   $$A(x)B(X) = 1 + \sum_{n\geq 1} 3nx^n = 1 + 3x \sum_{n\geq 0}(n+1)x^n = 1 + \frac{3x}{(1-x)^2}.$$

   (c) We have $\sum_{k=0}^{n} a_k b_{n-k} = \lceil \frac{n}{2} \rceil$ when n is odd and 0 when n is even.

   From here finding the closed form is a bit tricky. Probably the easiest way to go about doing this is to notice that $\sum_{n\geq 0}(n+1)x^{2n} = (\frac{1}{1-x^r})^2$, but proving this isn't really any different than just solving the problem as in 6 part (c)...

2. 6

   (a)

   (b) We see that $A(x)\sum_{n\geq 0}(n+1)x^n = \frac{1}{(1-x)^2}$ and $B(x) = 1 + x + x^2$. Consequently

   $$A(X)B(X) = \frac{1+x+x^2}{(1-x)^2}.$$

   (c) Note that $A(x) = \sum_{n\geq 0} x^{2n} = \frac{1}{1-x^2}$ and $B(x) = \sum_{n\geq 0} x^{2n+1} = x\sum_{n\geq 0} x^{2n} = \frac{x}{1-x^2}$. Consequently,

   $$A(x)B(x) = \frac{x}{(1-x^2)^2}$$