Math 484 QZ Duality and Rates of Change

$$LP_1 \qquad\qquad \text{where}$$

$$\min z = c\cdot x \qquad c = (-3,4) \quad x = (x_1, x_2)$$

$$g_1(x) \ge b_1 \qquad g_1(x) = 3x_1 - 4x_2 \qquad b_1 = -12$$

$$g_2(x) \ge b_2 \qquad g_2(x) = -5x_1 + 2x_2 \qquad b_2 = -20$$

$$g_3(x) \ge b_3 \qquad g_3(x) = x1 + x2 \qquad b_3 = 10$$

$$x \ge 0$$

1. Solve LP_1 & its dual by pivoting in Mathematica and plot also the feasible set.

2. For each constraint i active in the solution, verify that $y_i^* = \dfrac{\partial z^*}{\partial b_i}$ by explicitly computing Δz for three values of $\Delta b_i = \{1, -1, 2\}$ by three different methods:

a) By putting the new b_i value in the tableau and resolving

b) Using the fact the new optimal x^* will be at the intersection of the same constraint boundaries, and therefore a function $x^* = x^*(b_i)$, express z^* as an explicit function of b_i; i.e. $z^* = z(x^*) = z(x^*(b_i)) = z^*(b_i)$.

Use the formulation of $z^*(b_i)$ to compute $\dfrac{\partial z^*}{\partial b_i}$

c) Use the dual solution $y^* = (y_1^*, y_2^*, y_3^*)$ to express

$$\Delta z = \nabla z\cdot \Delta x = \left(\sum_i y_i^* \nabla g_i\right)\cdot \Delta x = \sum_i y_i^* (\nabla g_i \cdot \Delta x) = \sum_i y_i^* (\Delta g_i) = \sum_i y_i^* \Delta b_i$$

At each step of this derivation, plug in the numerical values of $\nabla g_i, \Delta x, \Delta g_i$

3.

a) If b_2 remains fixed at -20, in what range of values (min and max) for b_3 does $y_3^* = \dfrac{\partial z^*}{\partial b_3}$?

b) If b_3 remains fixed at 10, in what range of values (min and max) for b_2 does $y_2^* = \dfrac{\partial z^*}{\partial b_2}$?