Exercise 3

(a)

Let y_i be the measurement at time t_i.

Let m be the number of measurements. In this case, $m = 100$.

The residual at time t_i is

$$r_i = (x_1 + x_2 t_i^2)\exp(-x_3 t_i) - y_i$$

The objective function is

$$f(x) = \frac{1}{2}\sum_{i=1}^{m} r_i^2(x) = \frac{1}{2}\sum_{i=1}^{m}((x_1 + x_2 t_i^2)\exp(-x_3 t_i) - y_i)^2$$

Our goal is to minimize $f(x)$, it is a least-squares problem.

$$\frac{\partial r_i}{\partial x_1} = \exp(-x_3 t_i)$$

$$\frac{\partial r_i}{\partial x_2} = t_i^2 \exp(-x_3 t_i)$$

$$\frac{\partial r_i}{\partial x_3} = -t_i \exp(-x_3 t_i)$$

The Jacobian matrix is

$$J(x) = [\frac{\partial r_i}{\partial r_j}]_{ij} = \begin{pmatrix} \exp(-x_3 t_1) & t_1^2 \exp(-x_3 t_1) & -t_1 \exp(-x_3 t_1) \\ \exp(-x_3 t_2) & t_2^2 \exp(-x_3 t_2) & -t_2 \exp(-x_3 t_2) \\ \dots & \dots & \dots \\ \exp(-x_3 t_{200}) & t_i^2 \exp(-x_3 t_{200}) & -t_i \exp(-x_3 t_{200}) \end{pmatrix}$$

(b)

Gauss-Newton

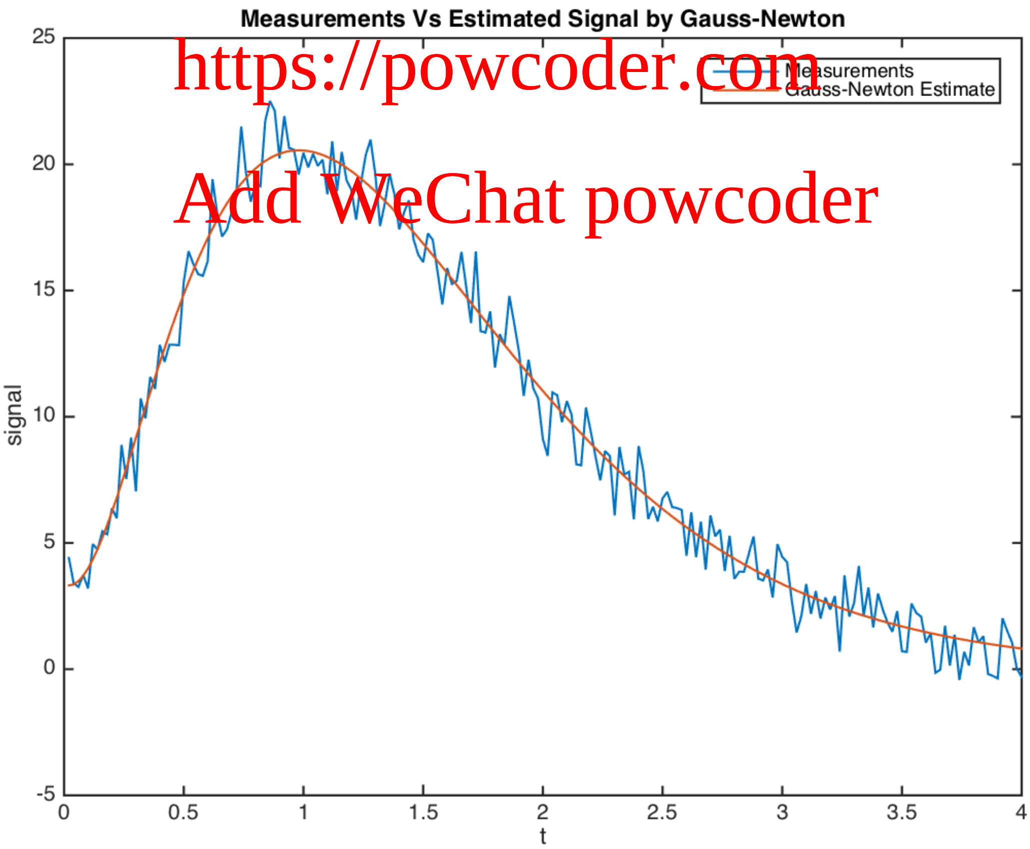
Parameters

Name	Value
x0	[1,1,1]'
descent	'gauss'
alpha0	0.05
tol	0.00001
maxIter	10000

Result

x_1	x_2	x_3	f
3.3976	147.2555	1.9922	88.0913

Plot



Levenberg-Marquardt

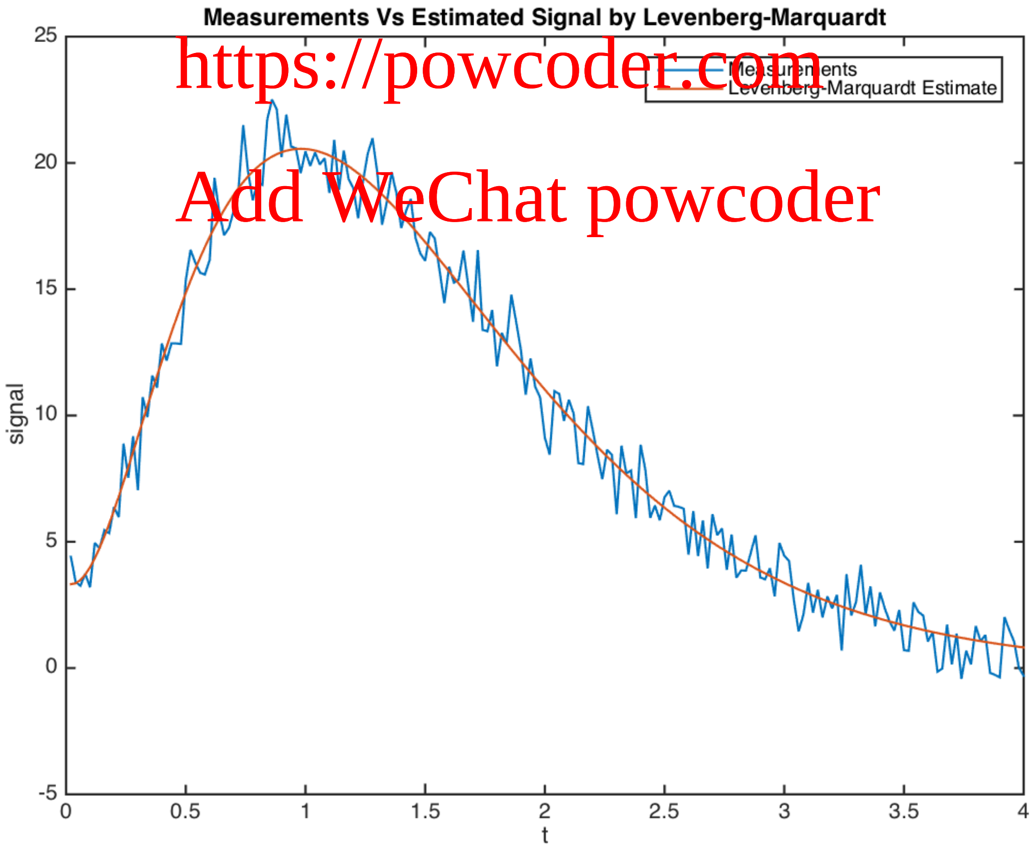
Parameters

Name	Value
x0	[1,1,1]'
Delta	1
eta	0.001
tol	0.00001
maxIter	10000

Result

x_1	x_2	x_3	f
3.3984	147.2763	1.9922	88.0908

Plot



Discussion

We can see that the parameters estimated by Gauss-Newton and Levenberg-Marquardt are very similar. The objective value achieved by Levenberg-Marquardt is a little lower than Gauss-Newton (88.0908 compared with 88.0913).

From the fit plots, we also can see their estimation have no obvious difference, both are good fit the noisy measurements. The estimated paraeters x are close to the actual value.