Last time

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

This time

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

A suggestive notation

$$A \to B$$
$$\forall \alpha . A$$
$$\exists \alpha . A$$

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

A suggestive notation

Assignment Project Exam Help

$$A \to B$$
$$\forall \alpha . A$$
$$\exists \alpha . A$$
$$A \times B$$
$$A + B$$

https://powcoder.com

Add WeChat powcoder

A suggestive notation

Assignment Project Exam Help

$$A \to B$$
$$\forall\alpha.A$$
$$\exists\alpha.A$$
$$A \wedge B$$
$$A \vee B$$

https://powcoder.com

Add WeChat powcoder

A suggestive notation

Assignment Project Exam Help

$$A \rightarrow B$$
$$\forall\alpha.A$$
$$\exists\alpha.A$$
$$A \wedge B$$
$$A \vee B$$

https://powcoder.com

Types *correspond to* **propositions**

Add WeChat powcoder

A suggestive notation

Assignment Project Exam Help

$$A \rightarrow B$$
$$\forall\alpha.A$$
$$\exists\alpha.A$$
$$A \wedge B$$
$$A \vee B$$

https://powcoder.com

Types *correspond to* **propositions**

Add WeChat powcoder

(Part 1 of the **Curry-Howard** correspondendence)

What logic?

$\lambda^{\rightarrow}$

Assignment Project Exam Help

B $\quad A \rightarrow B \quad A \wedge B \quad A \vee B$

https://powcoder.com

Add WeChat powcoder

What logic?

$\lambda^{\rightarrow}$ corresponds to **propositional logic**

Assignment Project Exam Help

$B \qquad A \rightarrow B \qquad A \land B \qquad A \lor B$

https://powcoder.com

Add WeChat powcoder

What logic?

$\lambda^{\to}$ corresponds to **propositional logic**

$$\beta \qquad A \to B \qquad A \wedge B \qquad A \vee B$$

System F

$$\forall \alpha . A \qquad \exists \alpha . A$$

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

What logic?

$\lambda^{\rightarrow}$ corresponds to **propositional logic**

Assignment Project Exam Help

$$\beta \qquad A \rightarrow B \qquad A \wedge B \qquad A \vee B$$

System F corresponds to **second-order propositional logic**

https://powcoder.com

$$\forall \alpha . A \qquad \exists \alpha . A$$

Add WeChat powcoder

What logic?

$\lambda^{\rightarrow}$ corresponds to **propositional logic**

Assignment Project Exam Help

$$B \qquad A \rightarrow B \qquad A \wedge B \qquad A \vee B$$

System F corresponds to **second-order propositional logic**

https://powcoder.com

$$\forall \alpha . A \qquad \exists \alpha . A$$

System Fω

Add WeChat powcoder

$$\lambda \alpha . A \qquad A\ B$$

What logic?

$\lambda^{\rightarrow}$ corresponds to **propositional logic**

Assignment Project Exam Help

$$B \qquad A \rightarrow B \qquad A \wedge B \qquad A \vee B$$

System F corresponds to **second-order propositional logic**

https://powcoder.com

$$\forall \alpha . A \qquad \exists \alpha . A$$

System Fω corresponds to **higher-order propositional logic**

Add WeChat powcoder

$$\lambda \alpha . A \qquad A\ B$$

What logic?

$\lambda^{\rightarrow}$ corresponds to **propositional logic**

Assignment Project Exam Help

$$B \qquad A \rightarrow B \qquad A \wedge B \qquad A \vee B$$

System F corresponds to **second-order propositional logic**

https://powcoder.com

$$\forall\alpha.A \qquad \exists\alpha.A$$

System Fω corresponds to **higher-order propositional logic**

Add WeChat powcoder

$$\lambda\alpha.A \qquad A\ B$$

What about **first-order logic**?

Propositional vs predicate

Assignment Project Exam Help

Propositional logic	**Predicate logic (FOPL)**
$P \to Q$	$P(x)$
$(\forall P.P \to P) \to (\exists Q.Q \to Q)$	$\forall x \in A.P(x)$

https://powcoder.com

Add WeChat powcoder

Assignment Project Exam Help

$$\begin{tikzcd}
& F\omega \arrow[rr] & & \lambda C \\
F \arrow[ur] \arrow[rr] & & \lambda P2 \arrow[ur] & \\
& \lambda\omega \arrow[uu] \arrow[rr] & & \lambda P\omega \arrow[uu] \\
\lambda^{\to} \arrow[uu] \arrow[ur] \arrow[rr] & & \lambda P \arrow[uu] \arrow[ur] &
\end{tikzcd}$$

https://powcoder.com

Add WeChat powcoder

Lambda and logic cubes

Assignment Project Exam Help

$$\begin{tikzcd}
 & F\omega \arrow[rr] & & \lambda C \\
F \arrow[ur] \arrow[rr] & & \lambda P2 \arrow[ur] & \\
 & \lambda\underline{\omega} \arrow[uu] \arrow[rr] & & \lambda P\underline{\omega} \arrow[uu] \\
\lambda^{\to} \arrow[uu] \arrow[ur] \arrow[rr] & & \lambda P \arrow[uu] \arrow[ur] &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & \mathrm{PROP}\omega \arrow[rr] & & \mathrm{PRED}\omega \\
\mathrm{PROP}2 \arrow[ur] \arrow[rr] & & \mathrm{PRED}2 \arrow[ur] & \\
 & \mathrm{PROP}\underline{\omega} \arrow[uu] \arrow[rr] & & \mathrm{PRED}\underline{\omega} \arrow[uu] \\
\mathrm{PROP} \arrow[uu] \arrow[ur] \arrow[rr] & & \mathrm{PRED} \arrow[uu] \arrow[ur] &
\end{tikzcd}$$

https://powcoder.com

Add WeChat powcoder

More suggestive notation

Assignment Project Exam Help

$$\frac{\Gamma \vdash M : A \to B \qquad \Gamma \vdash N : A}{\Gamma \vdash M\ N : B}$$

https://powcoder.com

Add WeChat powcoder

More suggestive notation

Assignment Project Exam Help

$$\frac{\Gamma \vdash M : A \to B \qquad \Gamma \vdash N : A}{\Gamma \vdash M\ N : B} \qquad\qquad \frac{\Gamma \vdash A \to B \qquad \Gamma \vdash A}{\Gamma \vdash B}$$

https://powcoder.com

Add WeChat powcoder

More suggestive notation

Assignment Project Exam Help

$$\frac{\Gamma \vdash M : A \to B \qquad \Gamma \vdash N : A}{\Gamma \vdash M\ N : B} \qquad\qquad \frac{\Gamma \vdash A \to B \qquad \Gamma \vdash A}{\Gamma \vdash B}$$

https://powcoder.com

Terms *correspond to* **proofs**

Add WeChat powcoder

More suggestive notation

Assignment Project Exam Help

$$\frac{\Gamma \vdash M : A \to B \qquad \Gamma \vdash N : A}{\Gamma \vdash M\ N : B} \qquad\qquad \frac{\Gamma \vdash A \to B \qquad \Gamma \vdash A}{\Gamma \vdash B}$$

https://powcoder.com

Terms *correspond to* **proofs**

Add WeChat powcoder

(Part 2 of the **Curry-Howard** correspondence)

Inference rules for $\rightarrow$

Assignment Project Exam Help

$$\frac{x : A \in \Gamma}{\Gamma \vdash x : A}\ \text{tvar} \qquad\qquad \frac{A \in \Gamma}{\Gamma \vdash A}$$

https://powcoder.com

$$\frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x : A.M : A \rightarrow B}\ \rightarrow\text{-intro} \qquad\qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B}$$

Add WeChat powcoder

$$\frac{\Gamma \vdash M : A \rightarrow B \qquad \Gamma \vdash N : A}{\Gamma \vdash M\ N : B}\ \rightarrow\text{-elim} \qquad\qquad \frac{\Gamma \vdash A \rightarrow B \qquad \Gamma \vdash A}{\Gamma \vdash B}$$

Inference rules for $\times$

Assignment Project Exam Help

$$\frac{\Gamma \vdash M : A \qquad \Gamma \vdash N : B}{\Gamma \vdash \langle M, N \rangle : A \times B} \times\text{-intro} \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge\text{-intro}$$

https://powcoder.com

$$\frac{\Gamma \vdash M : A \times B}{\Gamma \vdash \mathsf{fst}\ M : A} \times\text{-elim-1} \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \wedge\text{-elim-1}$$

Add WeChat powcoder

$$\frac{\Gamma \vdash M : A \times B}{\Gamma \vdash \mathsf{snd}\ M : B} \times\text{-elim-2} \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \wedge\text{-elim-2}$$

Classical vs intuitionistic logic

Assignment Project Exam Help

Classical logic

Emphasis on **truth**

https://powcoder.com

Truth values: $\top$, $\bot$

$A \vee \neg A$ always holds

Add WeChat powcoder

Intuitionistic logic

Emphasis on **proof**

Proofs inhabit propositions

$A \vee \neg A$ doesn't hold in general

Brouwer-Heyting-Kolmogorov (BHK) interpretation

A proof of $A \to B$:

a function that builds a proof of B from a proof of A.

A proof of $A \land B$:

a pair of a proof of A and a proof of B.

$\neg A$

means $A \to \bot$

$\bot$

has no proof

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Continuing the correspondence

Assignment Project Exam Help

Types *correspond to* **propositions**

Programs *correspond to* **proofs**

https://powcoder.com

Add WeChat powcoder

Continuing the correspondence

Assignment Project Exam Help

Types *correspond to* **propositions**

Programs *correspond to* **proofs**

https://powcoder.com

Evaluation *corresponds to* **proof simplification**

Add WeChat powcoder

Continuing the correspondence

Assignment Project Exam Help

Types *correspond to* **propositions**

Programs *correspond to* **proofs**

https://powcoder.com

Evaluation *corresponds to* **proof simplification**

Add WeChat powcoder

(The three-part **Curry-Howard** correspondence)

Who should care?

Language designers

e.g. *linear logic*: restrictions on structural rules
corresponds to a language with resource management guarantees

Logicians

since results about programming languages transfer "for free"
e.g. strong normalization implies consistency

Authors (and users) of proof assistants

e.g. Coq and other tools based on type theory

Programmers?

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Logical equivalences

Assignment Project Exam Help

$$\forall\beta.(\forall\alpha.(P\alpha \to \beta)) \to \beta \quad\leftrightarrow\quad \exists\alpha.P\alpha$$

$$\forall\beta.(P \to \beta) \wedge (Q \to \beta) \to \beta \quad\leftrightarrow\quad P \vee Q$$

https://powcoder.com

Proof: we must show

Add WeChat powcoder

$$\forall\beta.(\forall\alpha.(P\alpha \to \beta)) \to \beta \vDash \exists\alpha.P\alpha$$

$$\exists\alpha.P\alpha \vDash \forall\beta.(\forall\alpha.(P\alpha \to \beta)) \to \beta$$

etc.

A proof

Let $\Gamma = \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

Assignment Project Exam Help

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\ \forall\text{-elim} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\ \exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\ \to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\ \forall\text{-intro}}{\Gamma \vdash \exists\alpha.P\alpha}\ \to\text{-elim}$$

https://powcoder.com

Add WeChat powcoder

A program from a proof

Let $\Gamma = \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

$$\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\ \forall\text{-elim} \quad \dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\ \exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\ \to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\ \forall\text{-intro}$$
$$\overline{\qquad\qquad\Gamma \vdash \exists\alpha.P\alpha\qquad\qquad}\ \to\text{-elim}$$

Right subtree:

$$\dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\ \exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\ \to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\ \forall\text{-intro}$$

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

A program from a proof

Let $\Gamma = \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

$$
\dfrac{
  \dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\ \forall\text{-elim}
  \qquad
  \dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\ \exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\ \to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\ \forall\text{-intro}
}{\Gamma \vdash \exists\alpha.P\alpha}\ \to\text{-elim}
$$

Right subtree:

$$
\dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, v : P\alpha \vdash \text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : \exists\alpha.P\alpha}\ \exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\ \to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\ \forall\text{-intro}
$$

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

A program from a proof

Let $\Gamma = \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

$$
\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\,\forall\text{-elim}
\qquad
\dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\,\exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\,\to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\,\forall\text{-intro}
$$

$$
\dfrac{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha \qquad \Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}{\Gamma \vdash \exists\alpha.P\alpha}\,\to\text{-elim}
$$

Right subtree:

$$
\dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, v : P\alpha \vdash \text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : \exists\alpha.P\alpha}\,\exists\text{-intro}}{\Gamma, \alpha \vdash \lambda v : P\alpha.\, \text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : P\alpha \to \exists\alpha.P\alpha}\,\to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\,\forall\text{-intro}
$$

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

A program from a proof

Let $\Gamma = \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\;\forall\text{-elim} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\;\exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\;\to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\;\forall\text{-intro}}{\Gamma \vdash \exists\alpha.P\alpha}\;\to\text{-elim}$$

Assignment Project Exam Help

Right subtree:

https://powcoder.com

$$\dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, v : P\alpha \vdash \text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : \exists\alpha.P\alpha}\;\exists\text{-intro}}{\Gamma, \alpha \vdash \lambda v : P\alpha.\text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : P\alpha \to \exists\alpha.P\alpha}\;\to\text{-intro}}{\Gamma \vdash \Lambda\alpha.\lambda v : P\alpha.\text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\;\forall\text{-intro}$$

Add WeChat powcoder

A program from a proof

Let $\Gamma = \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\ \forall\text{-elim} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\ \exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\ \to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\ \forall\text{-intro}}{\Gamma \vdash \exists\alpha.P\alpha}\ \to\text{-elim}$$

Assignment Project Exam Help

Right subtree:

https://powcoder.com

$$\dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, v : P\alpha \vdash \mathsf{pack}\ \alpha, v\ \mathsf{as}\ \exists\alpha.P\alpha : \exists\alpha.P\alpha}\ \exists\text{-intro}}{\Gamma, \alpha \vdash \lambda v : P\alpha.\mathsf{pack}\ \alpha, v\ \mathsf{as}\ \exists\alpha.P\alpha : P\alpha \to \exists\alpha.P\alpha}\ \to\text{-intro}}{\Gamma \vdash \Lambda\alpha.\lambda v : P\alpha.\mathsf{pack}\ \alpha, v\ \mathsf{as}\ \exists\alpha.P\alpha : \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\ \forall\text{-intro}$$

Add WeChat powcoder

Left subtree:

$$\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\ \forall\text{-elim}$$

A program from a proof

Let $\Gamma = H : \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\;\forall\text{-elim} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\;\exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\;\to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\;\forall\text{-intro}}{\Gamma \vdash \exists\alpha.P\alpha}\;\to\text{-elim}$$

Right subtree:

$$\dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, v : P\alpha \vdash \mathsf{pack}\ \alpha, v\ \mathsf{as}\ \exists\alpha.P\alpha : \exists\alpha.P\alpha}\;\exists\text{-intro}}{\Gamma, \alpha \vdash \lambda v : P\alpha.\mathsf{pack}\ \alpha, v\ \mathsf{as}\ \exists\alpha.P\alpha : P\alpha \to \exists\alpha.P\alpha}\;\to\text{-intro}}{\Gamma \vdash \Lambda\alpha.\lambda v : P\alpha.\mathsf{pack}\ \alpha, v\ \mathsf{as}\ \exists\alpha.P\alpha : \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\;\forall\text{-intro}$$

Left subtree:

$$\dfrac{\dfrac{}{\Gamma \vdash H : \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\;\forall\text{-elim}$$

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

A program from a proof

Let $\Gamma = H : \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

$$
\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\ \forall\text{-elim} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\ \exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\ \to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\ \forall\text{-intro}}{\Gamma \vdash \exists\alpha.P\alpha}\ \to\text{-elim}
$$

Assignment Project Exam Help

Right subtree:

https://powcoder.com

$$
\dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, v : P\alpha \vdash \text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : \exists\alpha.P\alpha}\ \exists\text{-intro}}{\Gamma, \alpha \vdash \lambda v : P\alpha.\text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : P\alpha \to \exists\alpha.P\alpha}\ \to\text{-intro}}{\Gamma \vdash \Lambda\alpha.\lambda v : P\alpha.\text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\ \forall\text{-intro}
$$

Add WeChat powcoder

Left subtree:

$$
\dfrac{\dfrac{}{\Gamma \vdash H : \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash H\,[\exists\alpha.V\alpha] : (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\ \forall\text{-elim}
$$

A program from a proof

Let $\Gamma = H : \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\;\forall\text{-elim} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\;\exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\;\to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\;\forall\text{-intro}}{\Gamma \vdash \exists\alpha.P\alpha}\;\to\text{-elim}$$

Right subtree:

$$\dfrac{\cdots}{\Gamma \vdash \Lambda\alpha.\lambda v : P\alpha.\text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\;\forall\text{-intro}$$

Left subtree:

$$\dfrac{\cdots}{\Gamma \vdash H\,[\exists\alpha.V\alpha] : (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\;\forall\text{-elim}$$

Finally:

$$\dfrac{\begin{array}{c}\Gamma \vdash H\,[\exists\alpha.V\alpha] : (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha \\ \Gamma \vdash \Lambda\alpha.\lambda v : P\alpha.\text{pack } \alpha, v \text{ as } \exists\alpha.P\alpha : \forall\alpha.P\alpha \to \exists\alpha.P\alpha\end{array}}{\Gamma \vdash \exists\alpha.P\alpha}\;\to\text{-elim}$$

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

A program from a proof

Let $\Gamma = H : \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta$. Then

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall\beta.(\forall\alpha.P\alpha \to \beta) \to \beta}}{\Gamma \vdash (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\;\forall\text{-elim} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \alpha, P\alpha \vdash \exists\alpha.P\alpha}\;\exists\text{-intro}}{\Gamma, \alpha \vdash P\alpha \to \exists\alpha.P\alpha}\;\to\text{-intro}}{\Gamma \vdash \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\;\forall\text{-intro}}{\Gamma \vdash \exists\alpha.P\alpha}\;\to\text{-elim}$$

Right subtree:

$$\dfrac{\cdots}{\Gamma \vdash \Lambda\alpha.\lambda v : P\alpha.\mathsf{pack}\ \alpha, v\ \mathsf{as}\ \exists\alpha.P\alpha : \forall\alpha.P\alpha \to \exists\alpha.P\alpha}\;\forall\text{-intro}$$

Left subtree:

$$\dfrac{\cdots}{\Gamma \vdash H\ [\exists\alpha.V\alpha] : (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha}\;\forall\text{-elim}$$

Finally:

$$\dfrac{\begin{array}{c}\Gamma \vdash H\ [\exists\alpha.V\alpha] : (\forall\alpha.P\alpha \to \exists\alpha.P\alpha) \to \exists\alpha.P\alpha \\ \Gamma \vdash \Lambda\alpha.\lambda v : P\alpha.\mathsf{pack}\ \alpha, v\ \mathsf{as}\ \exists\alpha.P\alpha : \forall\alpha.P\alpha \to \exists\alpha.P\alpha\end{array}}{\Gamma \vdash H\ [\exists\alpha.V\alpha]\ (\Lambda\alpha.\lambda v : P\alpha.\mathsf{pack}\ \alpha, v\ \mathsf{as}\ \exists\alpha.P\alpha) : \exists\alpha.P\alpha}\;\to\text{-elim}$$

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Is it useful?

Assignment Project Exam Help

$$\forall \beta.(P \to \beta) \wedge (Q \to \beta) \to \beta \quad \leftrightarrow \quad P \vee Q$$

https://powcoder.com

These type equivalences can be useful in constructing programs.

The data type encodings we saw last week can be derived this way.

Add WeChat powcoder

Closing thoughts

Assignment Project Exam Help

The correspondence suggests a way of thinking about programming
— and a way of systematically constructing (some) programs

https://powcoder.com

However propositional logic is quite weak
(and our types are often uninformative)

We'll have richer types available later (GADTs, monads),
at which point we'll revisit the question of usefulness

Add WeChat powcoder

Next time

Assignment Project Exam Help

Abstraction

https://powcoder.com

Add WeChat powcoder