OLET1638 Sample Exam - Solution

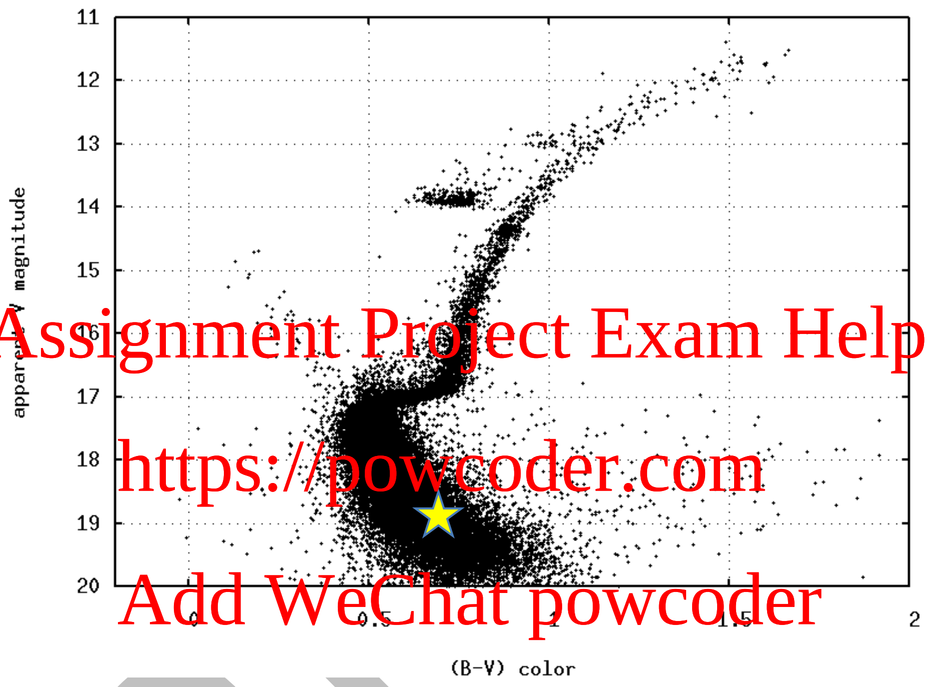
Question 1

(a)

In a cluster, all stars are at essentially the same distance – hence apparent magnitude is a valid proxy for luminosity

2 marks

(b)
Approx. position of the sun:



B-V = 0.66, so apparent magnitude in the cluster would be ~19 - on the main sequence.

1 mark

The Sun is a middle-aged main sequence star.

1 mark

(c)
Types of star plotted:
(description requested but naming is sufficient)

- main sequence (dwarfs class V)
- sub-giants (class IV)
- giants (class III-RGB)

3 marks

One or more of:

- blue stragglers
- Horizontal Branch (HB) stars
- some non cluster stars

1 mark

(d)

Diagram shows an evolved population, implying an old cluster >10 by

1 mark

Indicated by the low turn-off pt

1 mark

Total - 10 marks

Question 2

(a)
(i)
Internal changes during main sequence lifetime of a 1 solar mass star:

- fusion of hydrogen into helium means hydrogen is slowly used up and there is a decrease in the number of atoms, which leads to a drop in pressure in the core.
- ~12% of its total hydrogen converted to helium, but much more in the core
- to maintain hydrostatic equilibrium, the core of the star gradually contracts, becoming hotter and denser, which causes an increase in the nuclear reaction rates and hence an increase in luminosity.

2 marks

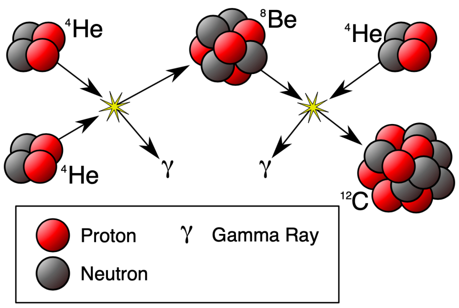
(ii)
External appearance change during main sequence lifetime of a 1 solar mass star:

- gradual changes
  - increasing luminosity
  - surface T changes but not very much
  - outer layers expand

- more details:
  - the Sun is currently 10% larger in radius and 30% more luminous than when it started on the main sequence 4.5 Gyr ago.
  - By the end of its main-sequence phase, in about 5.5 Gyr it will be ~25% larger in radius and twice as luminous as it was at zero-age main sequence.

3 marks

(b)
(i)
triple-alpha reactions occur when the helium core contracts sufficiently to raise temperature to ~100 million K:



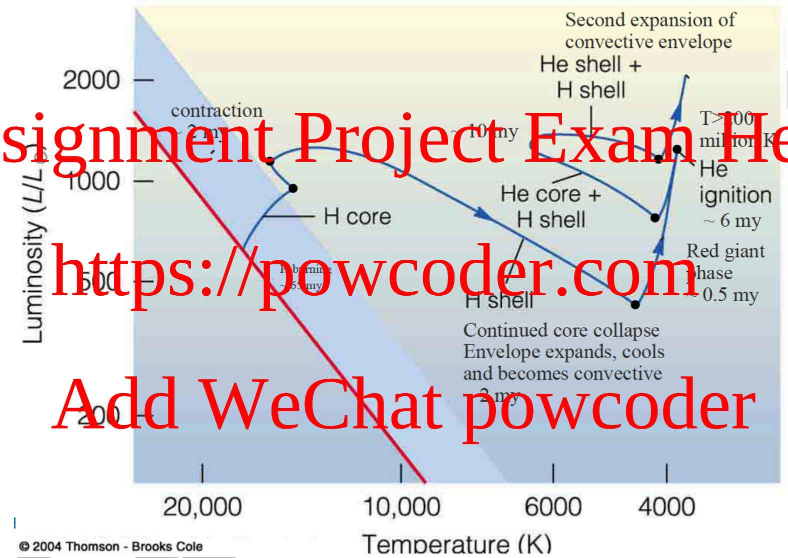
- so-called because 3 helium nuclei (alpha particles) combine to produce one carbon nucleus
- two steps: 2 helium nuclei fuse to produce an unstable beryllium nucleus, which then fuses with a third helium nucleus to form carbon. The second step must happen quickly because the beryllium nucleus decays in only 10^{-16} seconds.
- Some of the carbon nuclei fuse with a helium nucleus to produce oxygen, so the core slowly turns from helium into a mixture of carbon and oxygen.

2 marks

(ii)

- Triple-alpha process starts in the core when the star reaches the top of the red giant branch.
- The star then reduce in size as it moves across the horizontal branch
- Once all the helium in the core has been consumed, a shell of helium around the core will start to undergo fusion. If the star is sufficiently massive, triple-alpha will continue in a shell as higher temperature reactions occur later.
- More massive stars will increase in size again as it climbs the asymptotic giant grant

Example of a 5 $M_\odot$ star:



3 marks

Total - 10 marks

Question 3

(a)

- Stars with masses above ~8 to 10 $M_\odot$ leave the main sequence as supergiants with lifetimes of just a few million years.
- the cores of supergiants become hot enough for fusion to produce elements all the way up to iron, after which there are no more nuclear fusion reactions available to *supply* energy.
- Without an energy source, the core of the stars collapses under gravity in less than a second.
- During the collapse, the electrons in the core are forced to combine with protons to form neutrons, which also releases tremendous amounts of energy.
- If enough material falls back onto the neutron star, the neutron star will collapse to form a black hole.

- a shockwave is produced that travels outwards, compressing and heating the outer layers of the star and blowing them off in a massive supernova explosion

3 marks

(b)

- During the collapse, the electrons in the core combine with protons to form neutrons.
- Matter made of neutrons is incredibly dense so that the core that was about the size of the Earth collapses to form a neutron star ~ 20 km across, but with approximately the same mass as the Sun.

2 marks

(c)

- The pulse rate reflects the very rapid rotation of the neutron star.
- rapid initial rotation (<<1 rotation per second) is due to conservation of angular momentum, where a rotating object spins faster as it contracts.
- Most pulsars have rotation periods between ~0.05 and 2 s, but millisecond pulsars have rotation periods of ~ 0.001 to 0.01 s because they have been 'spun up' by accretion of material from a companion star.

3 marks

(d)

- Pulsars loose energy (by mechanisms that are only partially understood) and slow their rotation over time.
- When it slows to more than a few seconds the radio emission turns off
- Pulsar lifetime is ~10^7 years.

2 marks

Total - 10 marks

Question 4

The equation for the Schwarzschild radius of a non-rotating black hole is

$$R_{Schwarzschild} = \frac{2GM}{c^2}$$

where:

G is the gravitational constant = 6.67 x 10^{-11} $m^3.kg^{-1}.s^{-2}$
M is the mass of the black hole = (m solar masses) x (1.99 x 10^{30} kg per solar mass)
c is the speed of light = 3.00 x 10^8 $m.s^{-1}$

(a)

$$R_{Schwarzschild} = \frac{2GM}{c^2}$$

$$44 \times 10^3 = \frac{2\left(6.67 \times 10^{-11}\right) M}{\left(3.00 \times 10^8\right)}$$

So M = 14.9 $M_\odot \approx 15$ $M_\odot$

1 mark

(b)
The maximum stable masses for white dwarfs is ~1.4 $M_\odot$, and for neutron stars is ~ 3 $M_\odot$.

Values from Tutorial 4 on Black Holes:

Object	**Mass ($M_\odot$)**	**R_S (km)**
Earth	3×10^{-6} $M_\odot$	0.89 cm
Sun	1 $M_\odot$	3.0
Massive star	15 $M_\odot$	44
Intermediate mass black hole	10^4 $M_\odot$	30,000
MW supermassive black hole	4×10^6 $M_\odot$	1.2×10^7

3 marks

(c)
A black hole is a region of space in which the attractive force of gravity is so strong that nothing can escape, not even light. The size of this region is defined by the event horizon, the distance R called the Schwarzschild radius at which the escape velocity is equal to the speed of light *c*.

1 mark

(d)
The photon sphere is a theoretical sphere around a black hole. Photons that move on exact tangents to the photon sphere would be trapped in a circular orbit about the black hole.

1 mark

(e)
The innermost stable circular orbit (ISCO) is a theoretical sphere around a black hole. The radius of the innermost stable circular orbit is the smallest circular orbit in which a particle with mass (not a photon) can stably orbit a black hole. However, any infinitesimal perturbations to a circular orbit will lead to spiral into the black hole.

1 mark

(f)
Clearly all of these dimensions are too small to be seen over interstellar distances with available techniques.

- Until recently, *stellar mass* black holes were only detected by emission from accretion disks and jets around the holes, with material supplied by a close companion star.
- Some *stellar mass* black holes have now been detected by gravitational waves emitted during black hole-black holes or black-hole-neutron star mergers.
- In addition, supermassive black holes have been detected in the centre of galaxies by emission from accretion disks and jets around the holes. The black hole in the centre of the Milky Way has also been studied from its gravitational effect on the orbits of nearby stars.

3 marks

Total - 10 marks