OMIS 4000

Assignment 1

Due: October 6, 11:59 PM EST

Question 1. [23 Points] – Binary Program

Luke's diner is a family-owned restaurant with 6 employees; $i=1,...,6$. Sally and Henry are managers of the restaurant while George, Harmony, Mike, and Maria are regular employees. The restaurant opens five days per week (Wednesday through Sunday) at 7 AM and closes at 11 PM. Shifts are broken down into two 8-hour time blocks per day, i.e., 7 AM – 3 PM and 3 PM – 11 PM. As a result, there are $j=1,...,10$ possible shifts per week with odd numbers representing morning shifts and even numbers representing evening shifts (e.g., $j=1$ represents Wednesday morning). As demonstrated in the table below, the cost of scheduling employee i to shift j depends on whether it is a weekday or weekend.

Employee Name	Weekday Cost Per Shift ($)	Weekend Cost Per Shift ($)
Sally	160	184
Henry	160	184
George	144	166
Harmony	128	147
Mike	128	147
Maria	112	128

For the upcoming week, formulate and solve (with Python) a binary linear programming model to minimize the total cost of scheduling employees to shifts such that the following restrictions are adhered to:

1. At least 3, but no more than 5, employees can be assigned to the same shift.
2. Each employee must work between 32 and 48 hours per week.
3. Each employee must be assigned to at least one weekend shift (i.e., Saturday or Sunday).
4. At least one manager must be assigned to each one of the ten weekly shifts.
5. Employees cannot work back-to-back shifts (i.e., 7 AM – 11 PM) on the same day of the week.

Question 2. [43 Points] – Mixed-Integer Linear Program

The UNICEF Supply Community is at the forefront of UNICEF's humanitarian work. Currently, the community is in charge of preparing a loading plan for a cargo ship destined for Ghana. The following four products need to be transported aboard the ship:

Commodity	Amount Available (tons)	Volume per Ton (cubic feet)	Profit per Ton ($)
Nutrition Supplies	4,800	40	70
Medical Supplies	2,500	25	50
Pharmaceuticals	1,200	60	60
Sanitation Supplies	1,700	55	80

Any and/or all of the available commodities can be loaded. However, the ship has three cargo holds with the following capacity restrictions:

Cargo Hold	Weight Capacity (tons)	Volume Capacity (cubic feet)
Forward	3,000	145,000
Center	6,000	180,000
Rear	4,000	155,000

Only one type of commodity can be placed into any cargo hold. However, because of balance considerations, the weight in the forward cargo hold must be within 10% of the weight in the rear cargo hold. Further, the weight in the center cargo hold must be between 40% and 60% of the total weight on board.

a. Formulate a Mixed-Integer Linear programming (MILP) model for this problem.

b. Implement your model in Python and solve it.

c. What is the optimal solution?

Question 3. [24 Points] – Nonlinear Program

The year is 2024 and the U.S. presidential campaign is well underway. It has become evident that the race is going to be close. As such, one of the presidential candidates has hired Adrian Monk, a prescriptive analytics consultant, to estimate the percentage of votes the campaign will receive in each of the states based on the amount of money the campaign spends and the number of times the candidate visits each state during the final two weeks before the election. The results of Adrian's analysis provided the following function:

Percentage of votes in state k $= (a_k V^2_k - b_k D^2_k)$

where:

V_k = the number of times the candidate visits state k in the last two weeks of the campaign, and
D_k = the amount of money (in \$1,000,000s) the campaign spends on advertising in state k in the last two weeks of the campaign.

The following table summarizes Adrian's estimates of the parameters a and b for each state, along with the number of electoral votes at stake in each state:

State	a	b	Electoral Votes
Florida	0.145	0.31	25
Georgia	0.117	0.27	13
California	0.123	0.21	54
Texas	0.125	0.28	32
Illinois	0.128	0.26	22
New York	0.121	0.22	33
Virginia	0.134	0.24	13
Michigan	0.121	0.38	18

The candidate believes that he can make 21 campaign stops in the next two weeks, and there is \$15 million left in the campaign budget available for advertising. He wants to spend at least \$500,000 in each of these states in the next two weeks. He also wants to make at least one, but no more than five, campaign stops in each of these states. Within these constraints, Adrian wants to allocate the available resources to maximize the number of electoral votes the candidate can receive. Assume the candidate needs 51% of the vote to win in each state.

a. Formulate a Nonlinear Programming (NLP) model for this problem to determine *i*) how much money the candidate should spend on advertising in each state; *ii*) how many campaign stops the candidate should make in each state.

b. Implement your model in Python and solve it. What is the expected number of electoral votes generated by this solution?