PHIL1012 Introductory Logic · Final Assignment · Stuvac

1.

(i) Give an example of a proposition in GPLI that is false in any model with more than three objects in its domain.

(ii) Recall that two propositions α and β are called *contrary* if there is no model in which both α and β are true and there is at least one model in which both α and β are false. Give an example of two contrary propositions in GPLI without using any logical operators other than negation.

2. Suppose you have three propositions α, β, γ, and you want to use a single tree to find out whether there is a model in which β is true and α and γ have opposite truth values. What propositions should you place at the top of your tree? Explain your answer.

3. Translate the following into MPL:

Nice dogs don't bite unless they are scared.

4. Translate the following into GPLI:

Mary stole something from Alfred that is more valuable than at least two other things that he owns.

5. Translate the following into GPLI:

Alfred is smarter than every logician except Ruth, who is smarter than Alfred.

6. Use a tree to test whether the following proposition is a logical truth. (Present the tree, and say whether the proposition is a logical truth or not.) If the proposition is not a logical truth, read off a model from your tree in which the proposition is false (and indicate the path from which you are reading it off).

$$\forall x \forall y \forall z((x = y \wedge x = z) \rightarrow y = z)$$

7. Use a tree to test whether the following argument is valid. (Present the tree, and say whether the argument is valid or invalid.) If the argument is invalid, read off a counterexample from your tree (and indicate the path from which you are reading it off).

$\exists x \forall y Dyx$

$a = b \vee b = c$

$\therefore \neg Dab \rightarrow c = b$

8. Here is a model:

$$\text{Domain: } \{1, 2, 3, 4, ...\}$$

$$\text{a: } 1 \quad \text{b: } 9 \quad \text{c: } 77$$

$$E : \{2, 4, 6, 8, ...\}, \quad F : \emptyset$$

$$R : \{\langle 9, 1, 4\rangle, \langle 1, 6, 99\rangle, \langle 34, 77, 2\rangle, \langle 9, 77, 88\rangle\}$$

State whether the following propositions are true or false in the model, and briefly explain your answers.

(i) $\forall x((\neg Ex \wedge \exists z Rxcz) \rightarrow x = b)$

(ii) $\forall x \exists y \exists z Rxyz \leftrightarrow (\exists y Fy \rightarrow \neg Rbac)$

9. Here is a model M:

$$\text{Domain: } \{\text{Agatha, Betsy, Carlos, David}\}$$

$$C : \{\text{Betsy, David}\}$$

$$G : \{\langle \text{Betsy, Agatha, Carlos}\rangle, \langle \text{David, David, Betsy}\rangle, \langle \text{Carlos, Agatha, Agatha}\rangle\}$$

State two propositions of GPLI such that:

(i) both propositions are false in M; and

(ii) both propositions are true in a model that is exactly like M except the extension of G has a single additional 3-tuple in it. State what 3-tuple you are adding to the extension of G in order to make both of your two propositions true. (Please note that if you do not explicitly state what 3-tuple you are adding to the extension of G, then you will not get any marks for this part of the problem.)

10. Define a model with exactly two objects in its domain in which all of the following propositions are true.

$\forall y \exists z(y \neq z \wedge Ryz)$

$Ma \leftrightarrow \neg \exists x Rxa$

$\forall w(Mw \rightarrow Rww)$

11. Suppose α is an atomic proposition in GPLI and β is a contradiction. Consider the argument form

$$\neg\alpha$$

$$\therefore \beta$$

(i) Are there any valid arguments of this form? Explain your answer.

(ii) Are there any sound arguments of this form? Explain your answer.

12. The following argument is an *enthymeme* (see section 12.4 in the textbook or refer to the second lecture from week 10):

Carlos thanked everyone who helped him.

Ruth helped Carlos.

$\therefore$ Carlos thanked Ruth.

(i) Translate the argument into GPL.

(ii) In one or two sentences of English, explain why your translated argument from part (i) is invalid.

(iii) Make your translated argument from part (i) valid by adding a single premise (in GPL) to it. Demonstrate that the argument with the additional premise is valid by presenting a tree.