Problem 1. Losses are individually modelled by Exponential Random Variables X_i with mean 400. The frequency of losses is modeled by a zero-modified Poisson Random Variable N with parameter $\lambda = 20$ and $p_0^M = 0.05$.

(1) Assume no deductible. What is the mean number of claims?
(2) What would we need to set the deductible to be such that the expected number of payouts is 10? If you could not compute the first part, assume the mean is 20.

Hint: Use the probability generating function of the Poisson/truncated Poisson to compute moments of the frequency distribution.

Problem 2. Policyholders are divided into two distinct (mutually exclusive) groups. Claims for each group are modelled as a poisson distributions with parameters $\lambda_1 = 5$ and $\lambda_2 = 20$. Group 1, with claims modelled as $Poisson(\lambda_1 = 5)$ comprises 40% of all policyholders. Assuming you do not know which group a randomly selected policyholder is in, what is the mean and variance for the number of claims the holder will submit?

Problem 3. For a nursing home insurance policy, you are given that the average length of stay is 500 days and 30% of the stays are terminated in the first 30 days. These terminations are distributed uniformly during that period.**(f(x)=0.01 for x less than 30).** The policy pays 50 per day for the first 30 days and 100 per day thereafter. Determine the expected benefits payable for a single stay.

Problem 4. For a compound Poisson distribution, $\lambda = 5$ and individual losses have pf $f_X(1) = f_X(2) = f_X(4) = \frac{1}{3}$. Some of the pf values for the aggregate distribution S are given in Table below. Determine $f_S(6)$.

x	$f_S(x)$
3	0.012
4	0.0215
5	0.029
6	$f_S(6)$
7	0.0324

Standard Normal Distribution Table

$$\Phi(z) = \mathbb{P}(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx, \qquad \Phi(-z) = 1 - \Phi(z)$$

z	0	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Solutions

Problem 1.

(1) No deductible means that the claim distribution does not matter. The mean for a zero-modified Poisson can be found as the derivative of the Probability Generating function $P^M(z)$ at $z = 1$. Note that we have:

$$P^M(z) = p_0^M + (1 - p_0^M)P^T(z)$$

The p_0^M term is constant with respect to z, and therefore has derivative 0. We therefore have that our mean is given by:

$$\begin{aligned}\mathbb{E}[\text{claims}] = \mathbb{E}[N] &= \left(\frac{\lambda}{1 - e^{-\lambda}}\right)(1 - p_0^M) \\ &= \left(\frac{20}{1 - e^{-20}}\right)(0.95) = 19\end{aligned}$$

(2) We want to find x such that the surival function for our loss distribution $S(x) = \frac{10}{19}$ (or $\frac{1}{2}$ if the first part was not completed). This is a straightforward computation. First, following the correct answer from the first part:

$$\begin{aligned}S(x) = \frac{10}{19} &= e^{-x/400} \\ x = -400 \ln\left(\frac{10}{19}\right) &= 256.74\end{aligned}$$

and if 20 was used:

$$\begin{aligned}S(x) = \frac{10}{20} = \frac{1}{2} &= e^{-x/400} \\ x = -400 \ln\left(\frac{1}{2}\right) &= 277.26\end{aligned}$$

□

Problem 2. First, we compute the mean. Let N be the number of claims for the policyholder. We have:

$$\begin{aligned}\mathbb{E}[N] &= \mathbb{E}[\mathbb{E}[N|\lambda]] = \mathbb{E}[\lambda] \\ &= 0.4(5) + 0.6(20) = 14\end{aligned}$$

For Variance, we must consider the conditioning as well:

$$\begin{aligned}\mathsf{Var}(N) &= \mathbb{E}[N^2] - \mathbb{E}[N]^2 \\ &= \mathbb{E}[\mathbb{E}[N^2|\lambda]] - 14^2 \\ &= \mathbb{E}[\lambda + \lambda^2] - 14^2 \\ &= \mathbb{E}[\lambda] + \mathbb{E}[\lambda^2] - 14^2 \\ &= 14 + 0.4(5^2) + 0.6(20^2) - 14^2 = 68\end{aligned}$$

Alternatively, you can apply the Variance decomposition:

$$\begin{aligned}\mathsf{Var}(N) &= \mathbb{E}[\mathsf{Var}(N|\lambda)] + \mathsf{Var}(\mathbb{E}[N|\lambda]) \\ &= \mathbb{E}[\lambda] + \mathsf{Var}(\lambda) \\ &= 14 + (0.4(5^2) + 0.6(20^2) - 14^2) = 68\end{aligned}$$

$\square$

Problem 3.

$$
\begin{aligned}
E(x) =& 500, F(30) = 0.3, \text{ and } f(x) = 0.01 \text{ for } 0 < x \leq 30 \\
E(B) =& \int_0^{30} 50x(0.01)dx + \int_{30}^{\infty} [1500 + 100(x-30)]f(x)dx \\
=& 225 + \int_{30}^{\infty} (-1500)f(x)dx + 100 \int_{30}^{\infty} xf(x)dx \\
=& 225 - 1500[1 - F(30)] + 100 \int_0^{\infty} xf(x)dx - 100 \int_0^{30} xf(x)dx \\
=& 225 - 1500 \cdot (0.7) + 100 \cdot (500) - 100 \int_0^{30} (0.01)xdx \\
=& 225 - 1050 + 50000 - 450 \\
=& 48725
\end{aligned}
$$

□

Problem 4. Formula from Page 173(9.23)
$f_s(x) = \frac{\lambda}{m}\sum_{y=1}^{x\wedge m} y f_x(y) t_s(x-y)$, for $x = 1, 2 \cdots\cdots m$
We have $\lambda = 5 \quad m = 7$

$$\mathrm{f}_s(7) = \frac{5}{7}\sum_{y=1}^{7} y\,\mathrm{f}_x(y)\,\mathrm{f}_s(x-y)$$

$$= \frac{5}{7}\cdot(1\cdot f_x(1)\,\mathrm{f}_s(6) + 2\cdot f_x(2)\cdot f_s(5) + 4\,\mathrm{f}_x(4)\,\mathrm{f}_s(3))$$

$$0.0324 = \frac{5}{7}\left(1\cdot\frac{1}{3}\cdot f_s(6) + 2\cdot\frac{1}{3} - 0.029 + 4\cdot\frac{1}{3}\cdot 0.012\right)$$

$$f_s(6) = 0.03$$

□