Problem 1. $N \sim Poisson(\lambda)$, $\lambda \sim Unif(0, 6)$. Determine the P(N $\geq$ 3).

Problem 2. The Profits Y has the following cumulative distribution function:

$$F(y) = 1 - e^{-y/1000}, \text{ for } y > 0$$

Calculate the tail value-at-risk at 10%.

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Problem 3. A firm's portfolio is currently valued at 50 million. The portfolio has an expected annual return of 7 % and a volatility of 10.5% Assuming the returns are normally distributed, calculate the 5% annual Value-at-Risk for the firm's portfolio investment gain after one year.

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Problem 4. The number of homeowner's insurance claims occurring in a month is $Poisson(60)$. The distribution of the damage to a home when a claim occurs is $Pareto(4, 10, 000)$, what is the distribution of the number of claims occurring in a month with at least 3,000 in damage?

Standard Normal Distribution Table

$$\Phi(z) = \mathbb{P}(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\, dx, \qquad \Phi(-z) = 1 - \Phi(z)$$

z	0	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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Solutions

Problem 1.

$$\begin{aligned} P_N(n) &= \int_0^6 P_N(\lambda)\cdot f_\lambda(0,6)\,d\lambda \\ &= \int_0^6 \frac{\lambda^n e^{-\lambda}}{n!}\cdot\frac{1}{6}\,d\lambda \end{aligned}$$

$$\begin{aligned} P(N\geq 3) &= 1-P(N<3) \\ &= 1-P(N=0,1,2) \\ &= 1-P(N=0)-P(N=1)-P(N=2) \\ &= 1-\frac{1}{6}\int_0^6 \frac{\lambda^0 e^{-\lambda}}{0!}+\frac{\lambda^1 e^{-\lambda}}{1!}+\frac{\lambda^2 e^{-\lambda}}{2!}\,d\lambda \\ &= 1-\frac{1}{6}\int_0^6 (1+\lambda+\frac{\lambda^2}{2})e^{-\lambda}\,d\lambda \\ &= 1-\frac{1}{6}(-(1+\lambda+\frac{\lambda^2}{2})e^{-\lambda}-(1+\lambda)e^{-\lambda}-e^{-\lambda})|_0^6 \\ &= 1-\frac{1}{6}(-25e^{-6}+1-7e^{-6}+1-e^{-6}+1) \\ &= 1-\frac{1}{2}+\frac{33}{6}e^{-6}\approx 0.5136 \end{aligned}$$

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$\square$

Problem 2. The 10th percentile of Y is:

$$\begin{aligned} e^{-y/1000} &= 0.9 \\ y &= -1000ln(0.9) = 105.3605 \end{aligned}$$

The straightforward way to calculate the conditional expectation is to integrate y over the density function and then divide it by 0.9:

$$\begin{aligned} &\frac{1}{1-0.1}\cdot\int_{105.3605}^{\infty} x\cdot 0.001e^{-x/1000}\,dy \\ =&\frac{1}{0.9}(-xe^{-x/1000}-1000e^{-x/1000}|_{105.3605}^{\infty} \\ =&\frac{1}{0.9}(0+105.3605e^{-0.1053605}+1000(0+e^{-0.1053605})) \\ =&\frac{1}{0.9}(1105.3605e^{-0.1053605}) \\ =&\frac{1}{0.9}1105.3605\cdot 0.9 = 1105.3605 \end{aligned}$$

Alternatively, recall that $TVaR_{0.10} = E(Y|Y > 105.3605)$. Since y $\sim exp(\theta = 1000\, or\, \lambda = 1/1000)$, based on the memory-less property of exponential distribution,

$$TVaR_{0.10}(Y) = 1000 + 105.3605 = 1105.3605.$$

□

Problem 3.

$$Rate = -1.64 * 0.105 + 0.07 = -0.1022$$

This rate means the portfolio will lose 10.22% of its original value over the next year. Hence, the $VaR_{0.05}$ for the portfolio investment gain is:

$$-0.1022 * 50million = -5.11million$$

□

Problem 4. If X is the damage amount (i.e. "claim severity"), then

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$$P(X \geq 3000) = (\frac{10000}{3000 + 10000})^4 = 0.350128.$$

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By textbook Theorem 6.2:

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$$\#Claims/month \geq 3000 \sim poisson(0.350128 * 60 = 21)$$

□