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Predictive Analytics

Week 11: Time Series Forecasting

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Semester 2, 2018

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Discipline of Business Analytics, The University of Sydney Business School

QBUS2820 content structure

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1. Statistical and Machine Learning foundations and applications.
2. Advanced regression methods.
3. Classification methods.
4. **Time series forecasting**

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Week 11: Time Series Forecasting

1. Problem definition
2. Time series patterns
3. Simple forecasting methods
4. Model diagnostics
5. Model validation
6. Random walk model

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Time series

A **time series** is a set of observations $y_1, y_2, \ldots, y_t$ ordered in time.

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Examples:

- Weekly unit sales of a product.

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- Unemployment rate in Australia each quarter.
- Daily production levels of a product.

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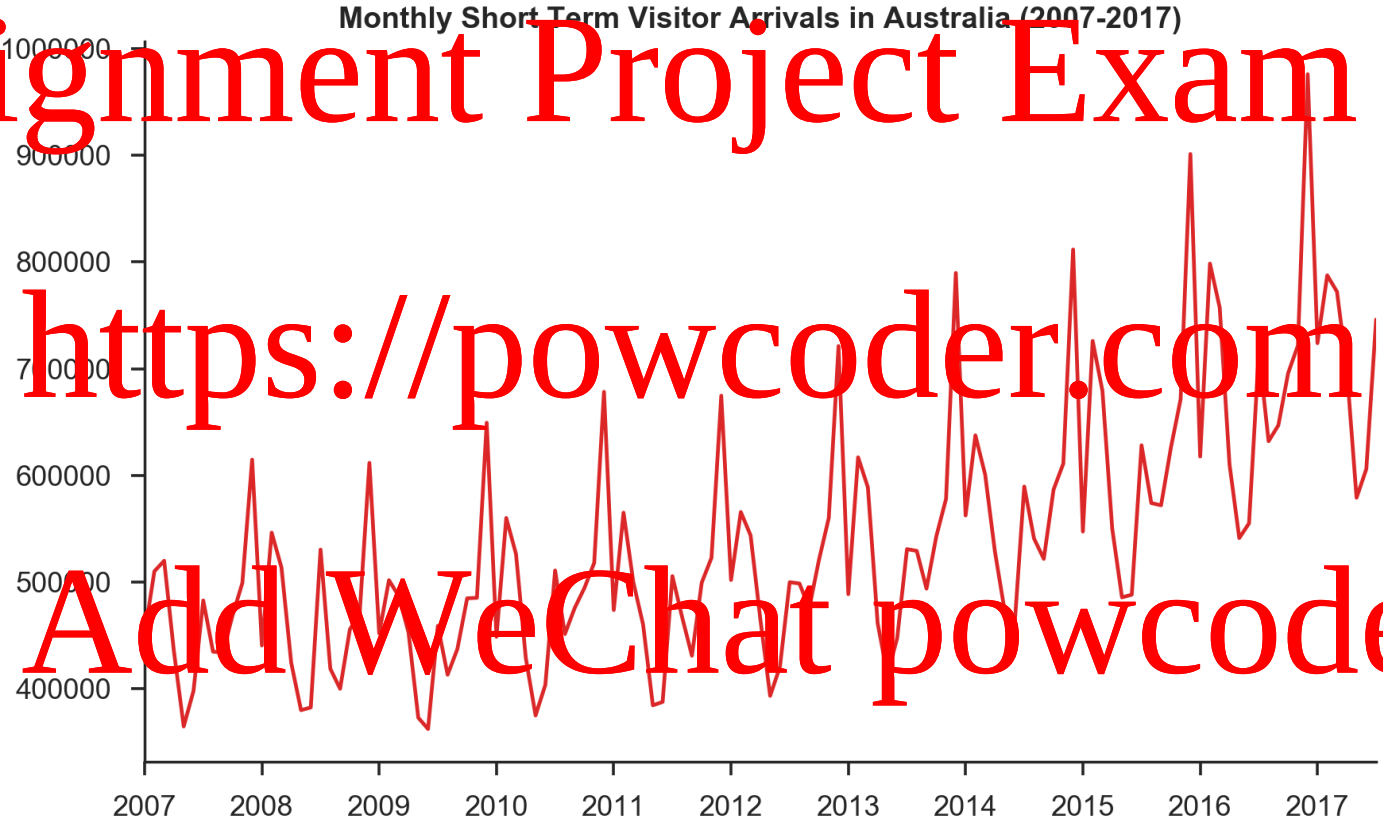
- Average annual temperature in Sydney.
- 5 minute prices for CBA stock on the ASX.

Example: visitor Arrivals in Australia



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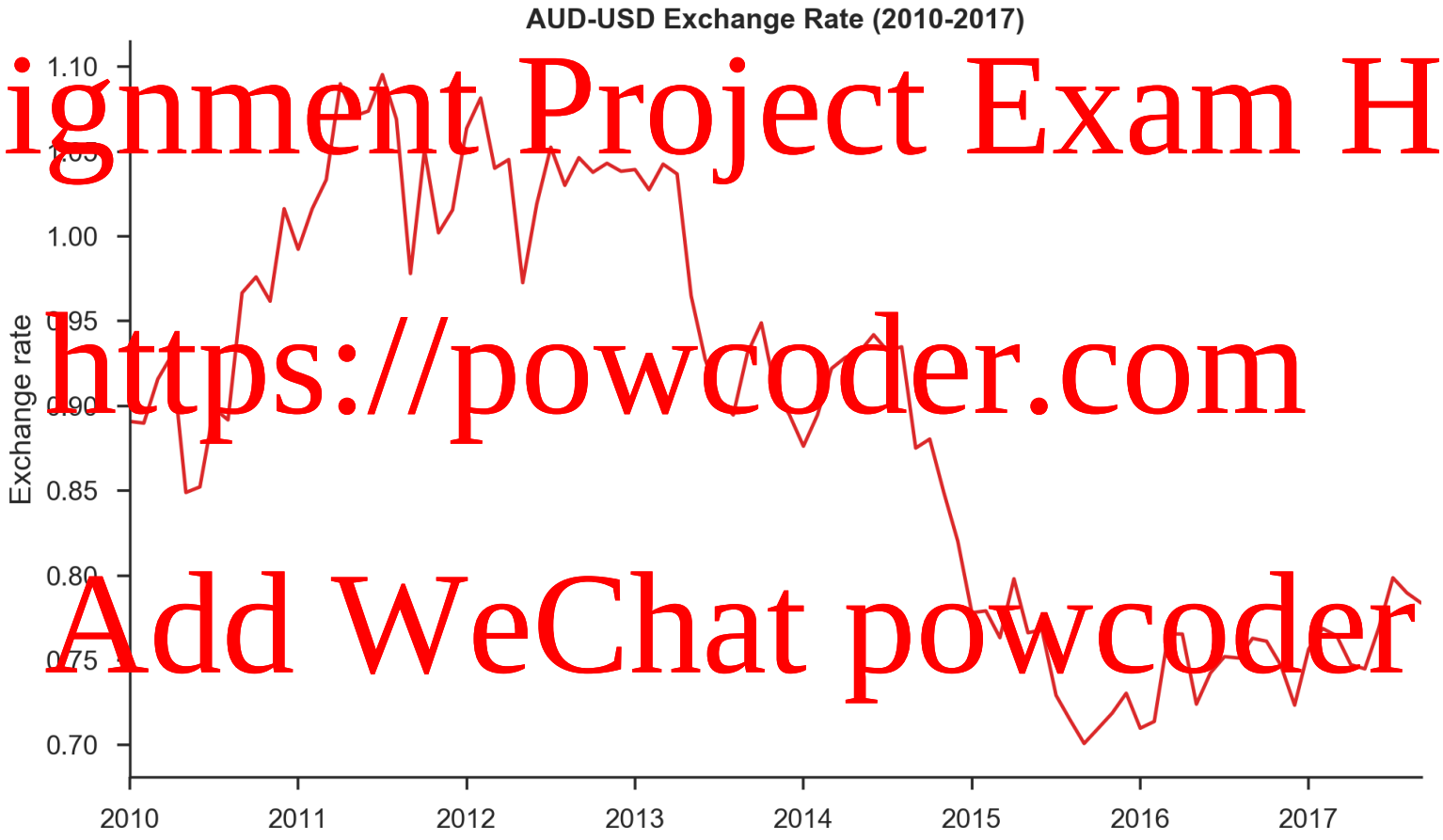
Example: AUD/USD exchange rate



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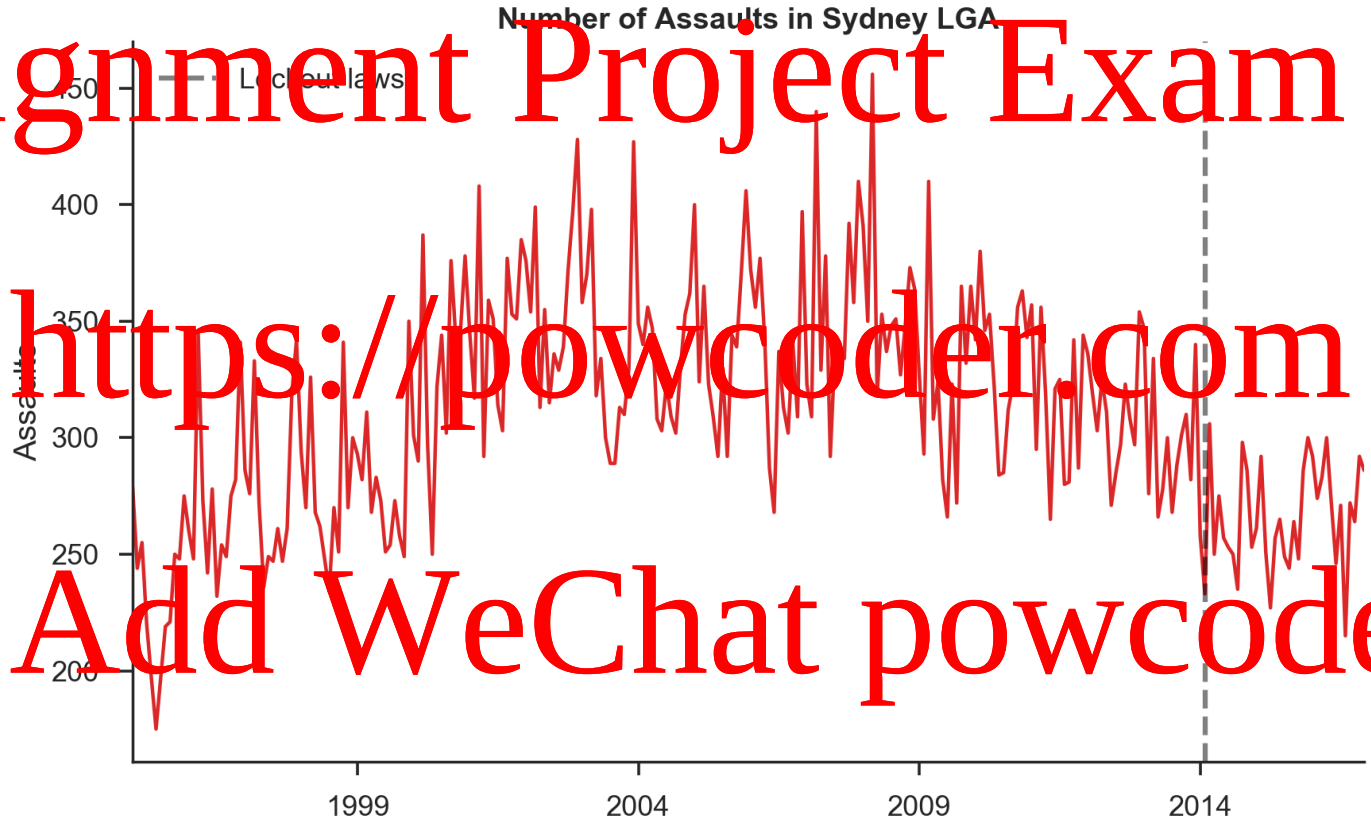
Example: assaults in Sydney



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Forecasting

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A **forecast** is a prediction about future events and conditions given all current information, including historical data and knowledge of any future events that might impact these events.

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The act of making such predictions is called **forecasting**.

Forecasting informs business and economic decision making, planning, government policy, etc.

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Examples

- Governments need to forecast unemployment, interest rates, expected revenues from income taxes to formulate policies.
- Retail stores need to forecast demand to control inventory levels, hire employees and provide training.
- Banks/Investors/financial analysts need to forecast financial returns, risk or volatility, market 'timing'.
- University administrators need to forecast enrollments to plan for facilities and for faculty recruitment.
- Sports organisations need to project sports performance, crowd figures, club gear sales, revenues, etc. in the coming season.

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Forecasting in business

Different problems lead to different approaches under the umbrella of forecasting.

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- Quantitative (data based) forecasting (our focus in this unit).

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- Qualitative (judgmental) forecasting.

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- Prediction markets.
- Common approach: judgmentally adjusted statistical forecasting.

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Problem definition

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Forecasting

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Our objective is to predict the value of a time indexed response variable at a future point $t + h$, given the observed series until the present point t. That is, we want to predict Y_{t+h} given $y_1, y_2, \ldots, y_t$, where h is the **forecast horizon**.

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We can extend this setting to allow for the presence of predictors $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_t$, leading to a **dynamic regression** problem.

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Decision theory

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We denote a **point forecast** as $\widehat{Y}_t = f(Y_{1:t})$. As before, we assume a squared error loss function:

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$$L(Y_{t+h}, f(Y_{1:t}) = (Y_{t+h} - f(Y_{1:t}))^2$$

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We use the slice notation $Y_{1:t}$ as a compact way to write $Y_1, \ldots, Y_t$.

Point forecasting (key concept)

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Using the arguments from earlier earlier in the unit, the optimal point forecast under the squared error loss is the conditional expectation:

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$$f(Y_{1:t}) = E(Y_{t+h}|Y_{1:t})$$

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Our objective is therefore to approximate the conditional expectation of Y_{t+h} given the historical data, possible for multiple values of h.

Interval forecasting (key concept)

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Uncertainty quantification is an essential for business forecasting.

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A **density forecast** $\widehat{p}(Y_{t+h}|y_1,\ldots,y_t)$ is an estimate of the entire conditional density $p(Y_{t+h}|y_1,\ldots,y_t)$.

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An **interval forecast** is an interval $(\widehat{y}_{t+h,L}, \widehat{y}_{t+h,U})$ such that $\widehat{P}(\widehat{y}_{t+h,L} < Y_{t+h} < y_{t+h,U}) = 1-\alpha$.

Fan chart (key concept)

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- For consecutive forecast horizons, construct prediction intervals for different probability levels (say, 75%, 90%, and 99%) and plot the using different shades.

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- The intervals typically get wider with the horizon, representing increasing uncertainty about future values.

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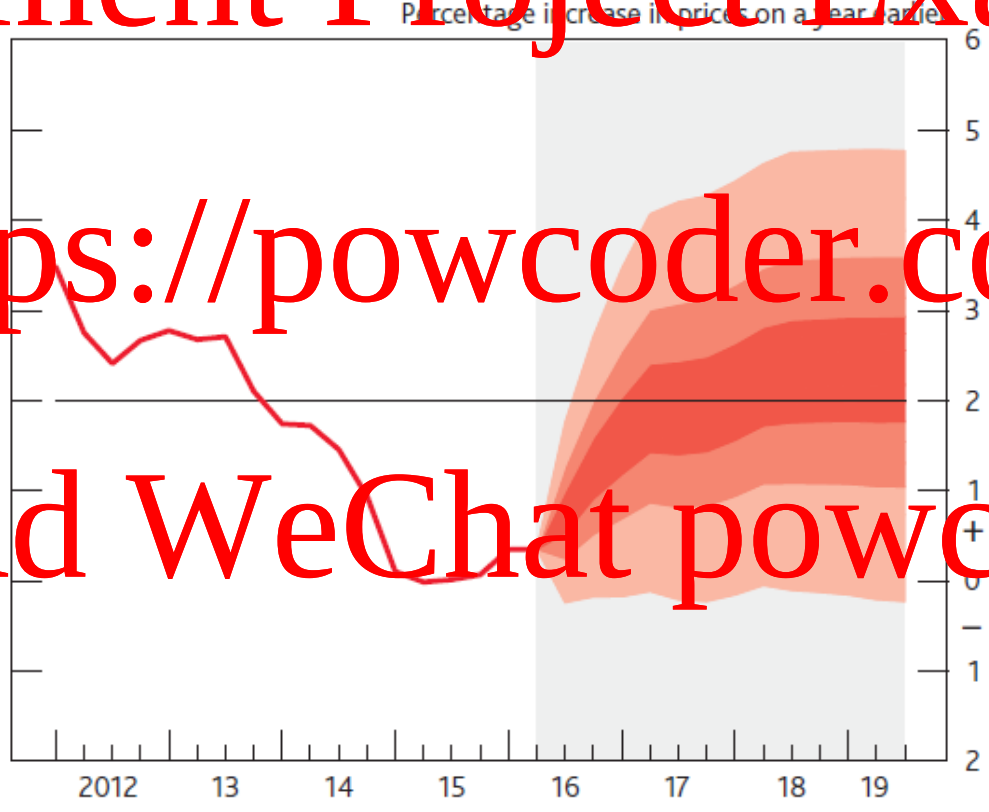
- Fan charts are useful tools for presenting forecasts.

Example: fan chart

Chart 5.12 CPI inflation projection based on constant nominal interest rates at 0.25%, other policy actions as announced



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Time series patterns

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Time series patterns (key concept)

We interpret a time series as

$$Y_t = f(T_t, S_t, C_t, E_t),$$

where T_t is the trend component, S_t is the seasonal component, C_t is the cyclic component, and E_t is an irregular or error component.

Trend. The systematic long term increase or decrease in the series.

Seasonal. A systematic change in the mean of the series due to seasonal factors (month, day of the week, etc).

Cyclic. A cyclic pattern exists when there are medium or long run fluctuations in the time series that are not of a fixed period.

Irregular. Short term fluctuations and noise.

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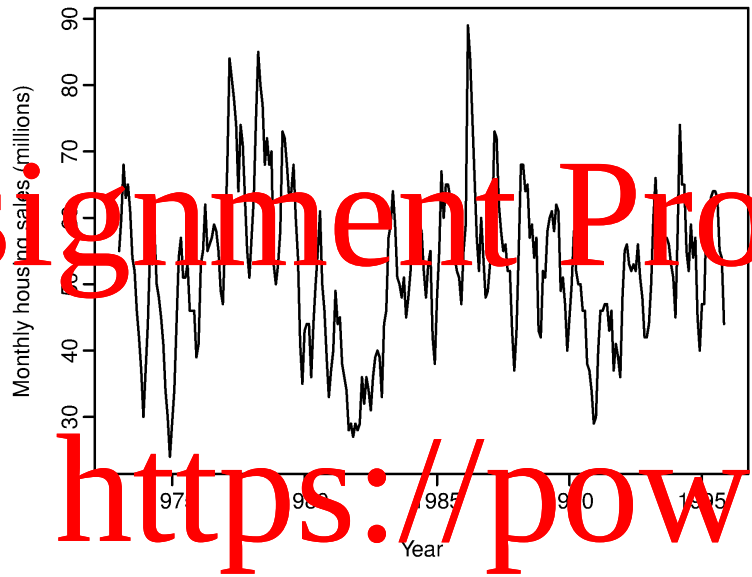
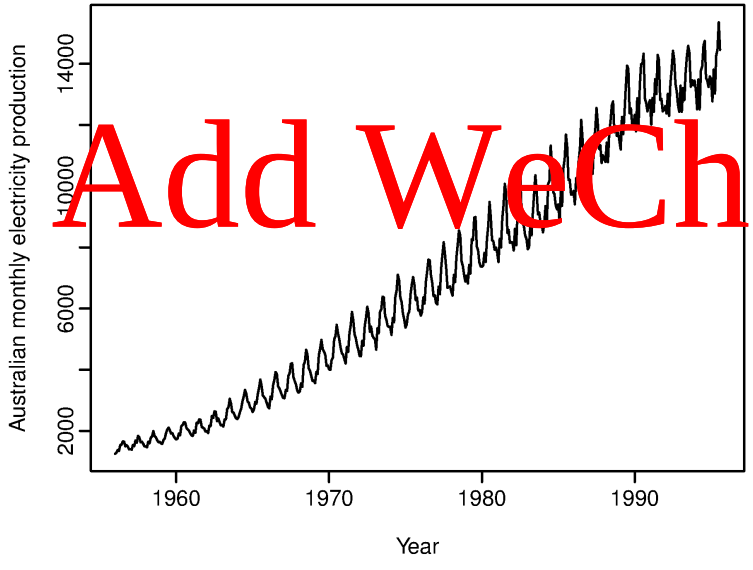
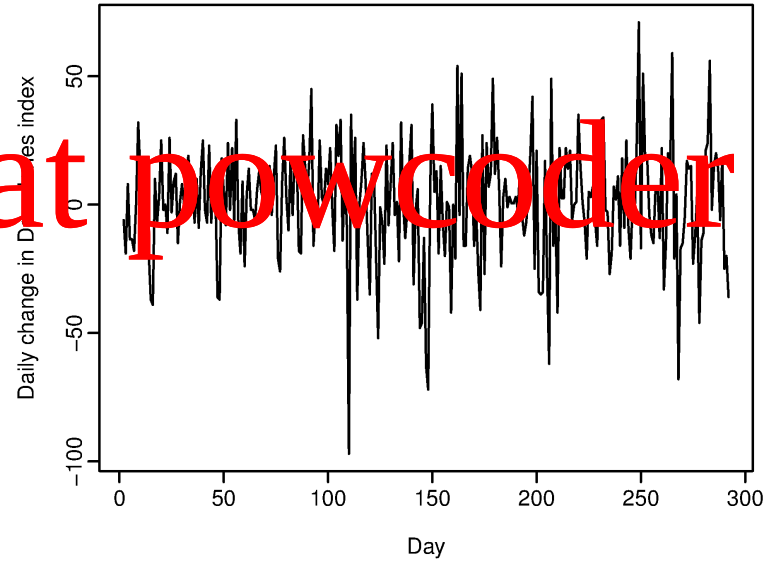
Examples: time series patterns







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Example: cyclic series

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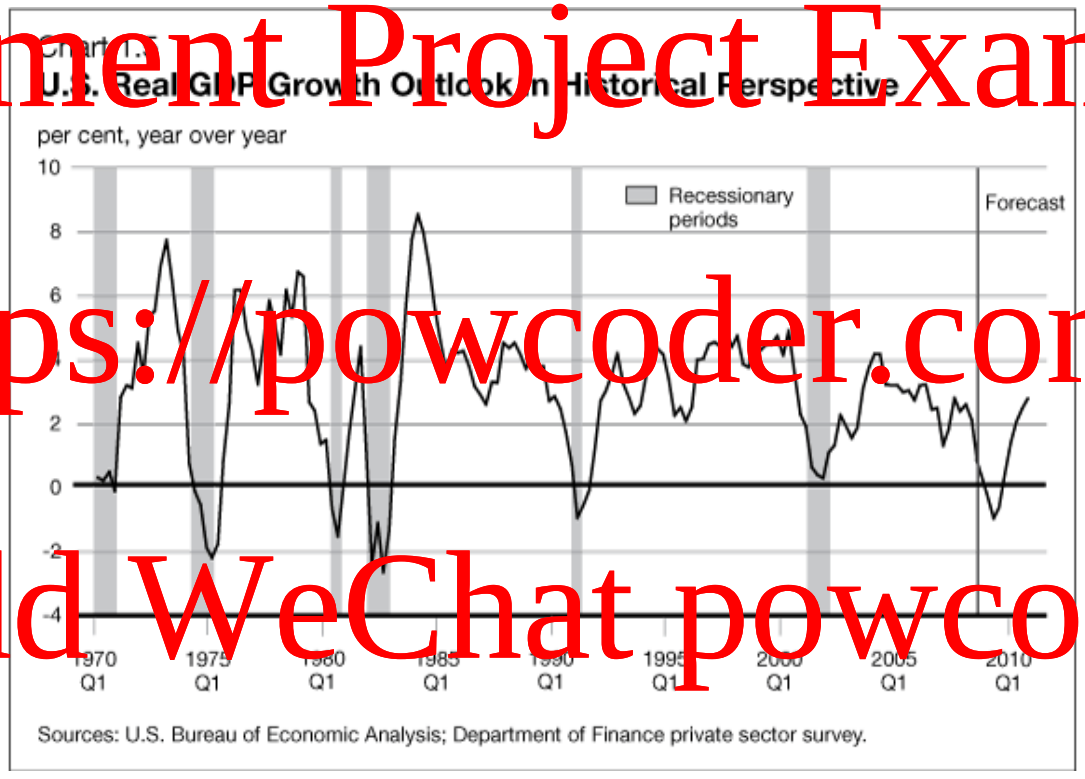

Time series models

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Time series models can be additive or multiplicative.

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Additive: $Y_t = T_t + S_t + E_t$

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Multiplicative: $Y_t = T_t \times S_t \times E_t$

Log transformation

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When the time series displays a multiplicative behaviour

$$Y_t = T_t \times S_t \times E_t,$$

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we usually apply to log-transformation to obtain a more convenient additive specification

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$$\log Y_t = \log T_t + \log S_t + \log E_t.$$

Choosing an additive or multiplicative specification

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Is the seasonal variation proportional to the trend?

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- If yes, a multiplicative model is more adequate.
- If not, we use an additive model.

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Time series decomposition

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Time series decomposition methods are algorithms for splitting a time series into different components, typically for purposes of seasonal adjustment.

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In the context of forecasting, decomposition methods are useful tools for exploratory data analysis, allowing us to visualise patterns in the data.

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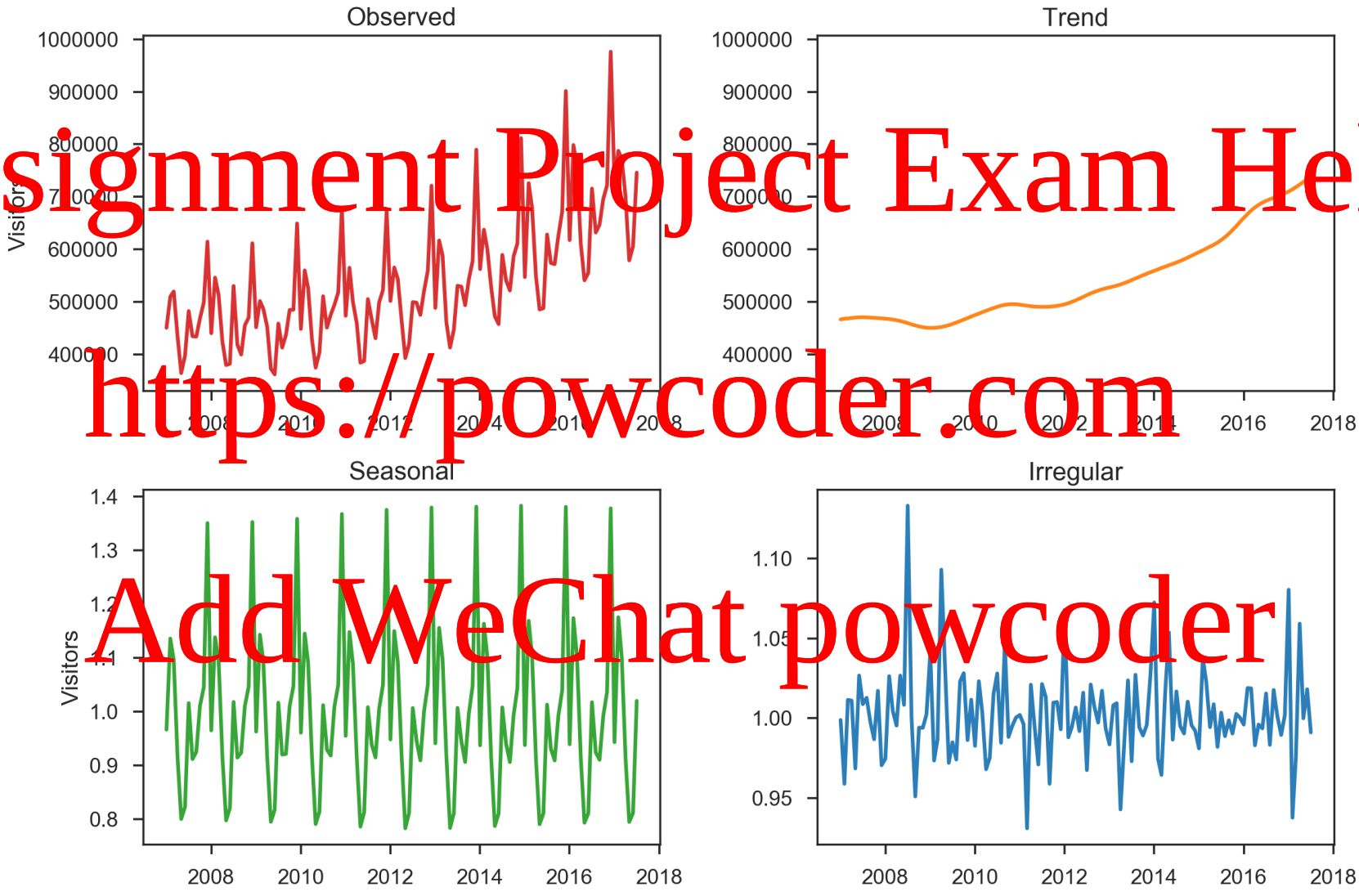
Time series decomposition: visitor arrivals



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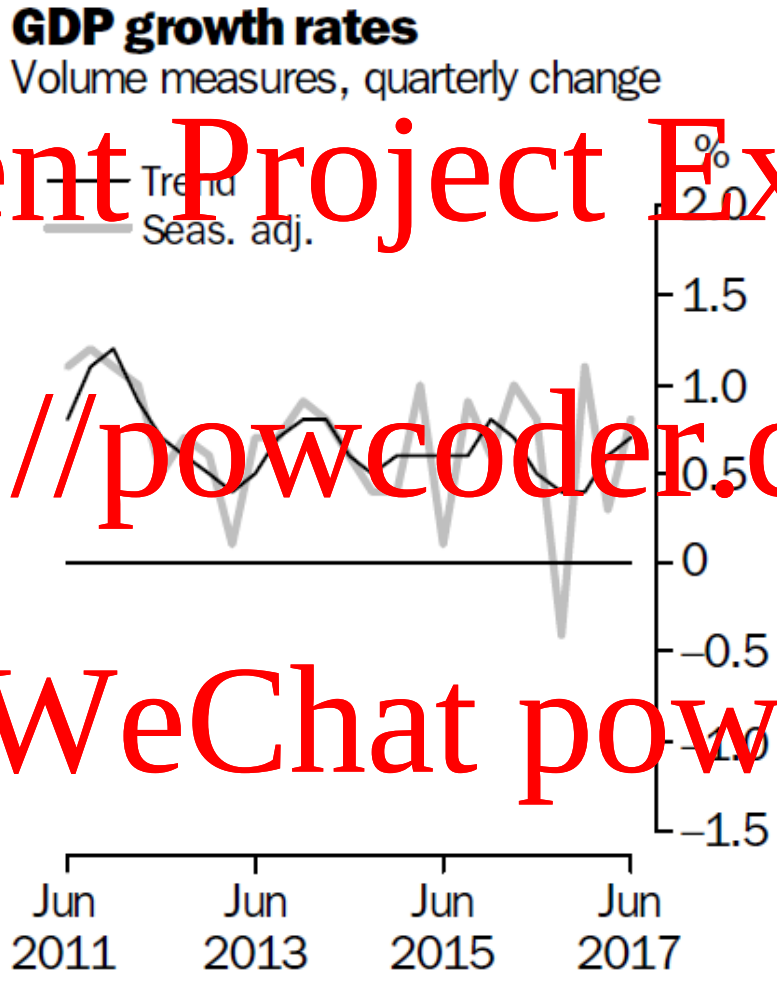
Example: seasonal adjustment and trend extraction



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Simple forecasting methods

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Random walk

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The **random walk method** (called the naïve method in the book) forecasts the series using the value of the last available observation:

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$$\widehat{y}_{t+h} = y_t$$

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Seasonal random walk

For time series with seasonal patterns, we can extend the random walk method by forecasting the series with the value of the last available observation *in the same season*:

$$\widehat{y}_{t+h} = y_{t+h-m} \qquad (\text{if } h \leq m),$$

where m is the seasonal period. For example, $m = 12$ and $m = 4$ for monthly and quarterly data respectively.

The general formula is

$$\widehat{y}_{t+h} = y_{t+h-km}, \qquad k = \lfloor (h-1)/m + 1 \rfloor.$$

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Drift method

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The **drift method** forecasts the series as the sum of the most recent value (as in the naive method) and the average change over time:

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$$\widehat{y}_{t+1} = y_t + \sum_{i=2}^{t} \frac{y_i - y_{i-1}}{t-1}$$

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$$\widehat{y}_{t+h} = y_t + h \times \sum_{i=2}^{t} \frac{y_i - y_{i-1}}{t-1}$$

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Model diagnostics

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Autocorrelation (key concept)

The **autocorrelation** of a time series process is

$$\rho_k = \frac{E\left[(Y_t - \mu)(Y_{t-k} - \mu)\right]}{\sigma^2} = \text{Corr}(Y_t, Y_{t+k}),$$

where k is the lag, and μ and σ^2 are the mean and variance of the time series (assuming that they do not depend on t).

The **sample autocorrelation** is

$$r_k = \frac{\sum_{t=1}^{T-k}(y_{t+k} - \bar{y})(y_t - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2}.$$

The autocorrelation function (**ACF**) plot displays the autocorrelation for a range of lags.

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White noise process (key concept)

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A **white noise process** is a sequence of independently and identically distributed random variables with mean 0 and finite variance σ^2.

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If a time series model is well specified, we expect the residual series of the fitted model to behave like a white noise process.

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Model diagnostics (key concept)

Residual plot. The presence of patterns in the time series of residuals (such as non-constant variance over time) may suggest assumption violations and the need for alternative models.

Residual ACF plot. Well specified models should lead to small and insignificant sample autocorrelations, consistent with a white noise process.

Residual distribution plots (histogram, KDE, Q-Q plots, etc). Inspecting the distribution of the residuals will suggest the appropriate assumptions for interval forecasting.

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Model validation

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Training and validation sets

- We incorporate model validation into the forecasting process by setting aside a validation sample for estimating and comparing the performance of different models.
- We allocate the last part of the sample (typically 20-50% of the data) to the validation set.
- In time series, the training set is called "in-sample data" and the validation set the "out-of-sample data".
- Due to the dynamic nature of forecasting, there is no test set (though we may sometimes refer to model validation as forecast evaluation).

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Real time forecasts (key concept)

We validate forecasts by following the "real time" approach: at every period t, we use all the available data at present to estimate the model and predict the future value of the series.

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1. Starting at $t = n$, use the observations at times $1, 2, \ldots, t$ to estimate the forecasting model. Use the estimated model to forecast the observation at time $t + 1$.

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2. Repeat the above step for $t = n + 1, \ldots, T - 1$.

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3. Compute forecast accuracy measures based on the prediction errors $y_{n+1} - \widehat{y}_{n+1}, \ldots, y_T - \widehat{y}_T$.

We follow a similar procedure for multi-step forecasts.

Expanding and rolling windows

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We can consider two schemes for updating the estimation sample at each validation period:

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Expanding window. At each step, add the latest observation to the the estimation sample.

Rolling window. At each step, use only the most recent n observations for estimation. The rolling window scheme implicitly assumes that the dynamics of the series has a time changing nature, so that data far in the past are less relevant for estimation.

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Measuring forecast accuracy

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We typically assume the squared error loss and compute the out-of-sample MSE to measure forecast accuracy.

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However, it is useful to be familiar with other measures that are common in business forecasting:

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- Percentage errors.
- Scaled errors.

Percentage errors

- The percentage error is given by $p_t = 100 \times ((y_t - \widehat{y}_t)/y_t)$. It has the advantage of being scale-independent.
- The most commonly used measure is mean absolute percentage error

  $$\text{MAPE} = \text{mean}(|p_t|).$$

- Measures based on percentage errors have the disadvantage of being infinite or undefined if $y_t = 0$ for any t in the period of interest, and having extreme values when any $y_t = 0$ is close to zero.

- Percentage errors are only valid under a meaningful zero.

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Scaled errors

- Hyndman and Koehler (2006) proposed scaling the errors based on the training MAE (or MSE) from a benchmark method (typically a simple model).
- For a non-seasonal time series, a useful way to define a scaled error uses naïve forecasts:

$$q_t = \frac{y_t - \widehat{y}_t}{\frac{1}{T-1}\sum_{i=2}^{T} |y_i - y_{i-1}|}.$$

- Because the numerator and denominator both involve values on the scale of the original data, q_j is independent of the scale of the data.

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Mean absolute scaled error

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The **mean absolute scaled error** is

$$\text{MASE} = \text{mean}(|q_t|).$$

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A scaled error is less than one if it arises from a better set of forecasts than the random walk method evaluated on the training data.

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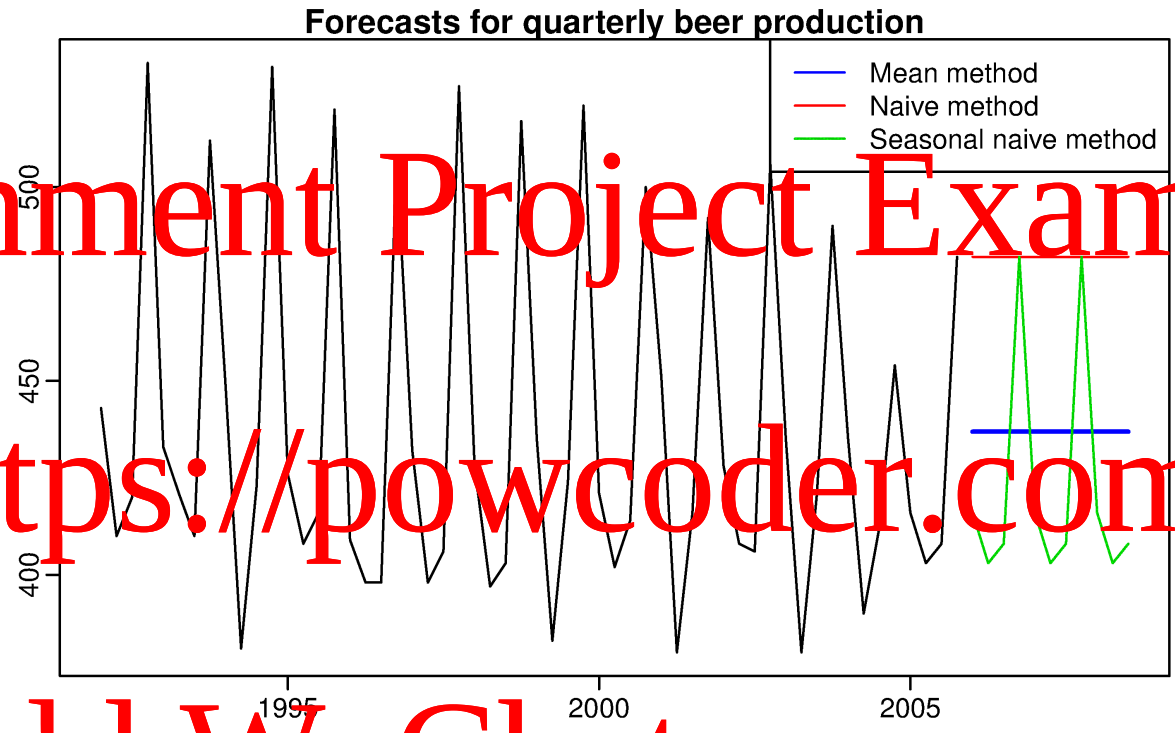
Example: Quarterly Australian Beer Production



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The figure shows shows three forecasting methods applied to the quarterly Australian beer production using data to the end of 2005. We compute the forecast accuracy measures for 2006-2008.

Example: Quarterly Australian Beer Production

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Method	RMSE	MAE	MAPE	MASE
Mean method	38.01	33.78	8.17	2.30
Naïve method	70.91	63.91	15.88	4.35
Seasonal naïve method	**12.97**	**11.27**	**2.73**	**0.77**

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It is clear from the graph that the seasonal naive method is best for the data, although it can still be improved.

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Random walk model

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Random walk model (key example)

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In this section, we use the random walk method to illustrate how to obtain point and interval forecasts for multiple horizons based on a time series model.

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We assume the model

$$Y_t = Y_{t-1} + \varepsilon_t,$$

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where ε_t is i.i.d with constant variance σ^2.

Random walk model

Since $Y_t = Y_{t-1} + \varepsilon_t$, we can use back substitution to show that

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$$Y_{t+1} = Y_t + \varepsilon_{t+1}$$

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$$\begin{aligned} Y_{t+2} &= Y_{t+1} + \varepsilon_{t+2} \\ &= Y_t + \varepsilon_{t+1} + \varepsilon_{t+2} \end{aligned}$$

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$$\begin{aligned} Y_{t+h} &= Y_{t+h-1} + \varepsilon_{t+h} \\ &= Y_t + \varepsilon_{t+1} + \ldots + \varepsilon_{t+h} \end{aligned}$$

Point forecast

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$$Y_{t+h} = Y_t + \sum_{i=1}^{h} \varepsilon_{t+i}$$

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Therefore, we obtain the point forecast for any horizon as

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$$\begin{aligned}
\widehat{y}_{t+h} &= E(Y_{t+h}|y_{1:t}) \\
&= E\left(Y_t + \sum_{i=1}^{h} \varepsilon_{t+i} \middle| y_{1:t}\right) \\
&= y_t
\end{aligned}$$

Forecast

The conditional variance is

$$\begin{aligned}\text{Var}(Y_{t+h}|y_{1:t}) &= \text{Var}(y_t + \sum_{i=1}^{h} \varepsilon_{t+i}|y_{1:t}) \\ &= h\sigma^2.\end{aligned}$$

For density forecasting, we need to make further assumptions about the errors. If we assume that $\varepsilon_t \sim N(0, \sigma^2)$,

$$Y_{t+h}|y_{1:t} \sim N\left(y_t, h\sigma^2\right).$$

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Forecast interval

Under the Gaussian assumption,

$$Y_{t+h}|y_{1:t} \sim N\left(y_t, h\sigma^2\right),$$

leading to the forecast interval

$$y_t \pm z_{\alpha/2} \times \sqrt{h\widehat{\sigma}^2},$$

where

$$\widehat{\sigma}^2 = \frac{\sum_{t=2}^{T}(y_t - y_{t-1})^2}{T-1},$$

and $z_{\alpha/2}$ is the appropriate critical value from the normal distribution.

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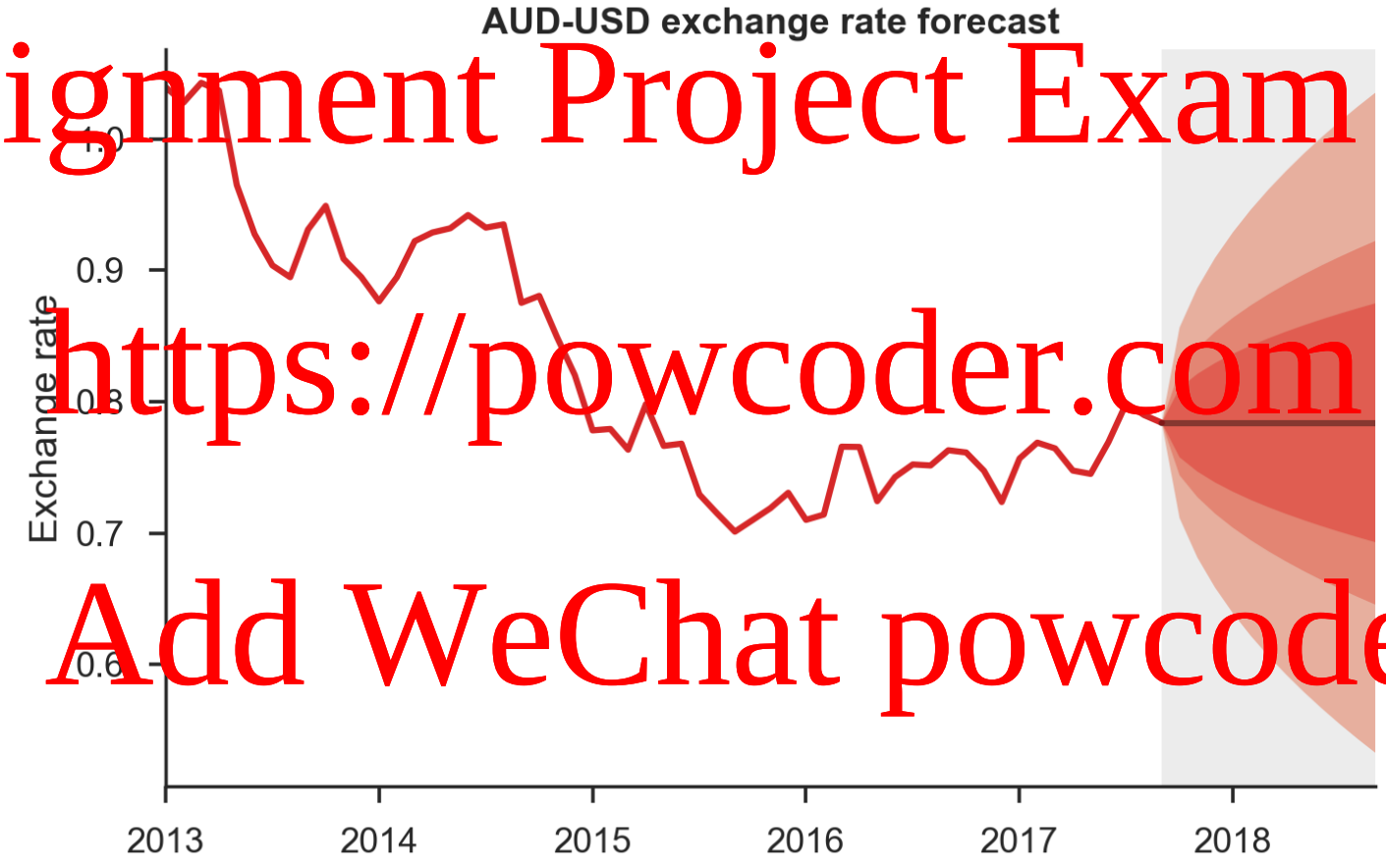
Example: USD/AUD exchange rate



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Forecast interval

Forecast interval based on the assumption of normal errors:

$$y_t \pm z_\alpha \times \sqrt{h\widehat{\sigma}^2}$$

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- This forecast interval is based on the **plug-in method**, as we replace the unknown σ^2 with an estimate.

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- The plug in method is a standard approach, but you should be aware that it ignores parameter uncertainty, leading to prediction intervals that are too narrow.

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- If the errors are not Gaussian, you should use other methods such as the Bootstrap algorithm (not in the scope of our unit).

Review questions

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- What is point and interval forecasting?
- What are the four time series components?
- Which diagnostics do we use for univariate time series models, and why?
- How to we conduct model validation for forecasting?
- How do we compute forecasts and prediction intervals for the random walk model?