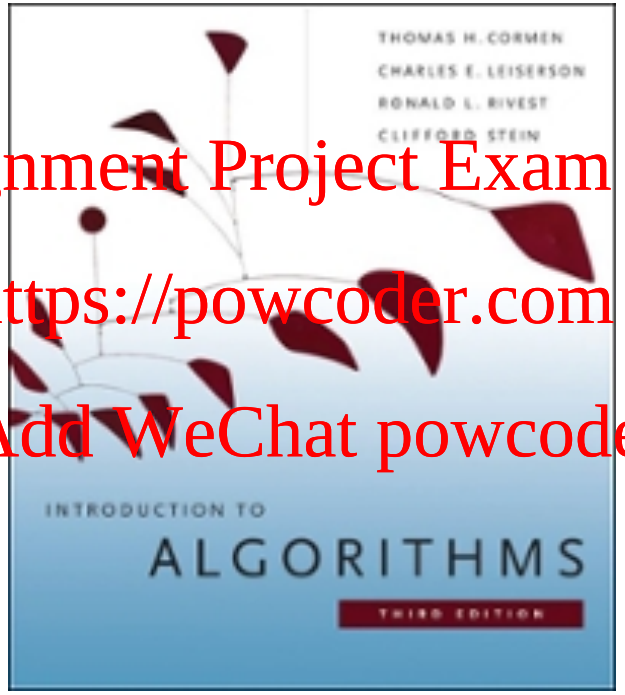
CS146 Data Structures and Algorithms



Chapter 13: Red-Black Trees

Balanced search Trees

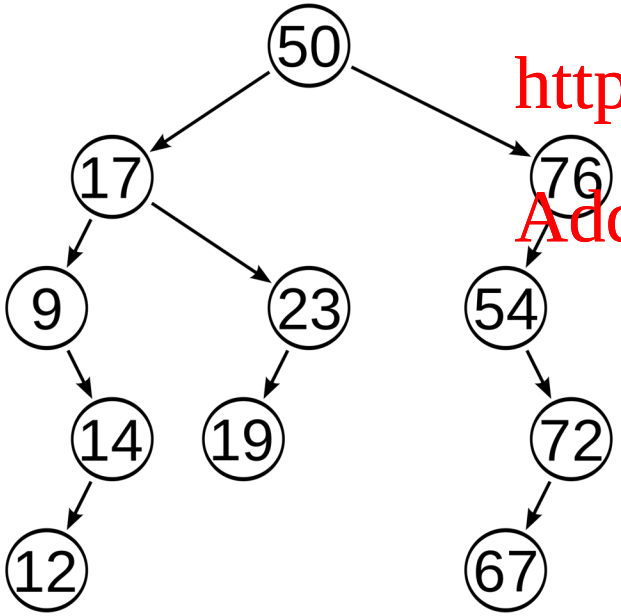
- Many search-tree schemes that are "balanced" in order to guarantee that basic dynamic-set operations take $O(\lg n)$ time in the worse case.

e.g. AVL trees

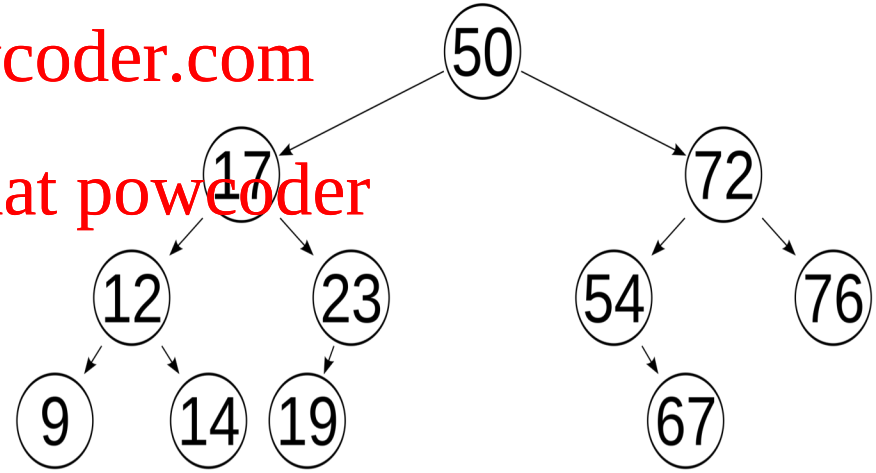
Red-black trees

AVL Trees

- a **self-balancing** (or **height-balanced**) **binary search tree** is any node-based binary search tree that automatically keeps its height (maximal number of levels below the root) small in the face of arbitrary item insertions and deletions



An example of an **unbalanced** tree; following the path from the root to a node takes an average of 3.27 node accesses.



The same tree after being height-balanced; the average path effort decreased to 3.00 node accesses.

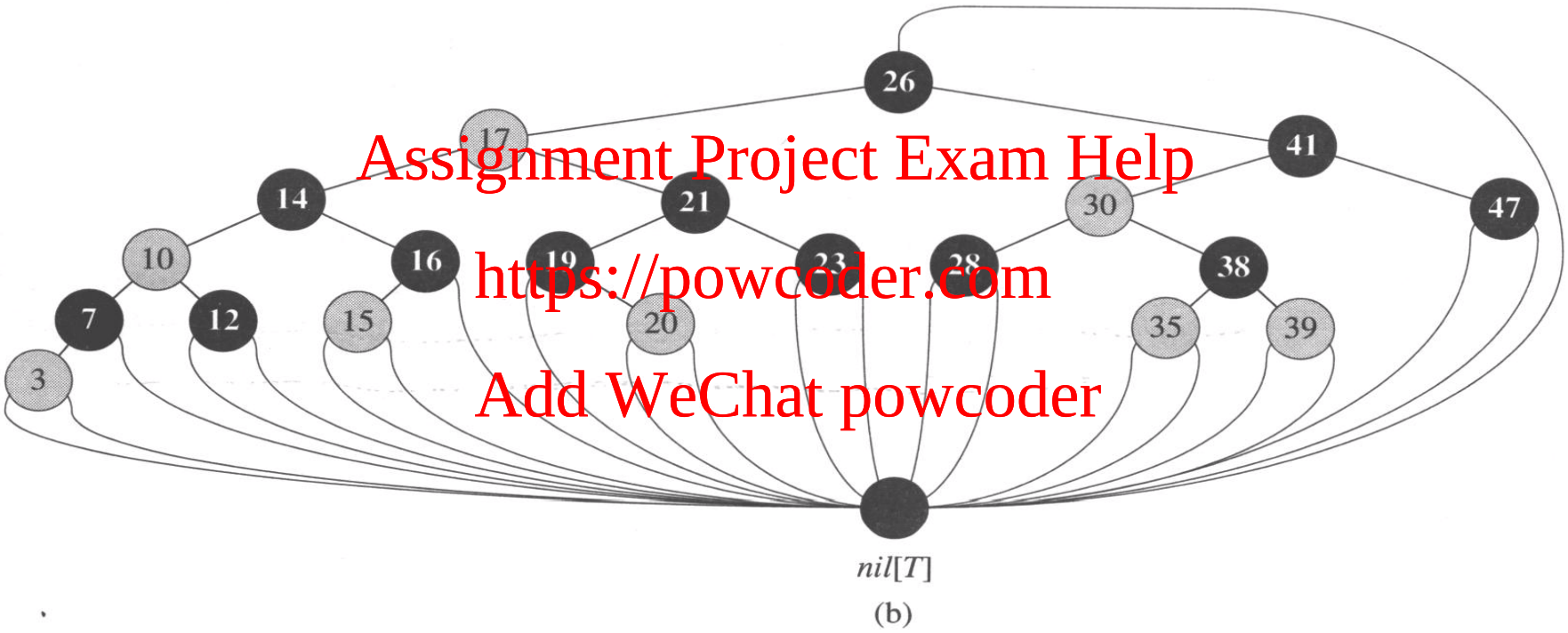
Red-Black Trees

- Red-black trees are a variation of binary search trees to ensure that the tree is ***balanced***.
  - Height is $O(\lg n)$, where n is the number of nodes.
- To guarantee that operations take $O(\lg n)$ time in the worst case.

Red-Black Tree

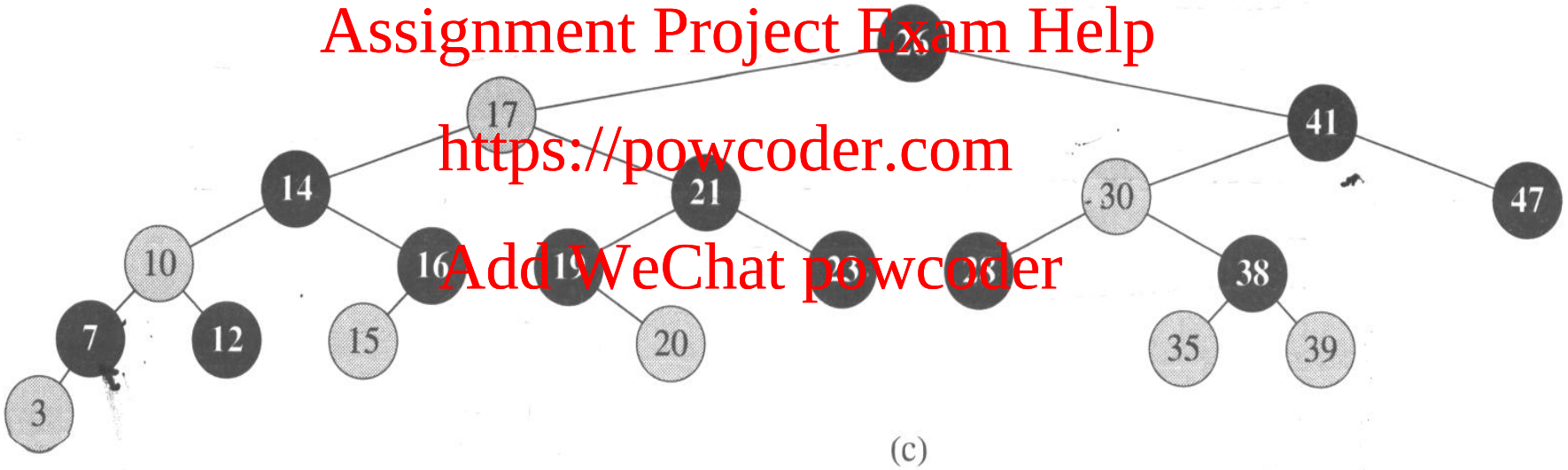
- Binary search tree with + 1 bit per node: the attribute *color*, which is either **red** or **black**.
- All other attributes of BSTs are inherited:
  - *key*, *left*, *right*, and *p*.
- All empty trees (leaves) are colored **black**.
  - We use a single sentinel, *nil*, for all the leaves of red-black tree *T*, with *nil.color* = black.
  - The root's parent is also *T.nil*.

Example of a Red-black Tree



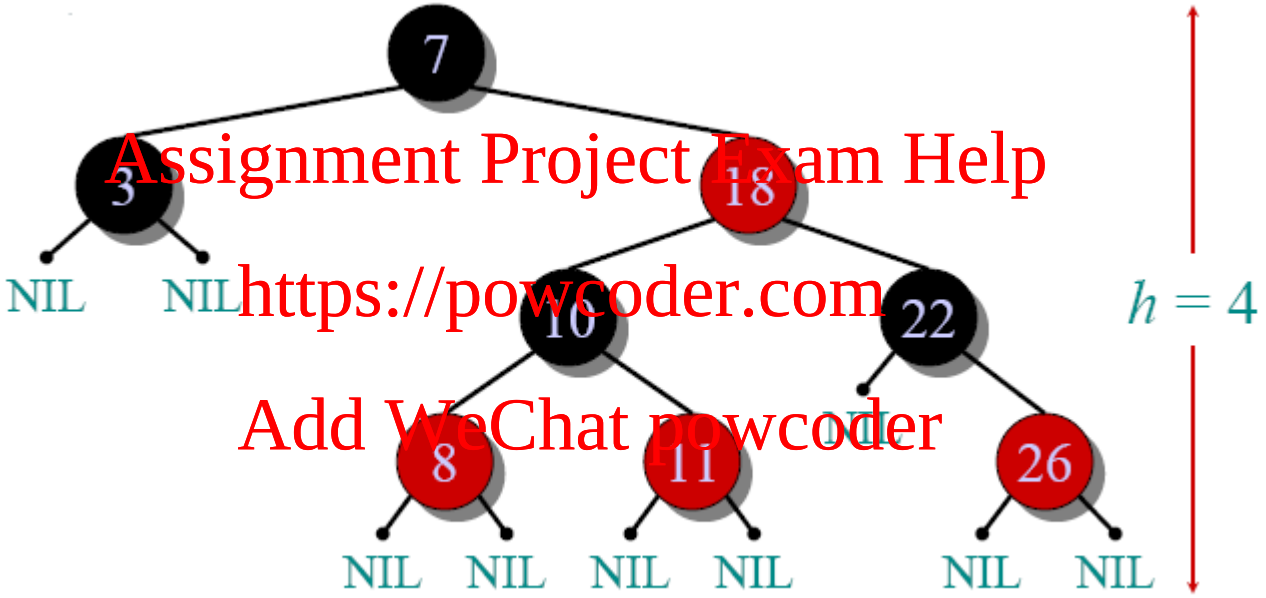
(b)

Leaves and the root's parent omitted entirely



We omit the leaves when we draw BR trees, because we generally confine our interest to the internal nodes, since they hold the key.

Example of a Red-black Tree



Red-black Properties

1. Every node is either red or **black**.
2. The root is **black**.
3. Every leaf (*nil*) is **black**.
4. If a node is red, then both its children are **black**.
5. For each node, all paths from the node to descendant leaves contain the same number of **black** nodes. (i.e. black-height(x)).

Example of a Red-black Tree