Assignment instruction

Question 1 to 3:

Derive the formula using a general model (don't need to simplify to double MA).

Question 4:

Write a R function as follows:

```
Q4<-function (S = S&P500, m=length_of_long_term_MA,
r=length_of_short_term_MA)
```

This function returns two output $E(R_t)$ and H, where

$$E(R_t) = \sqrt{\frac{2}{\pi}}\sigma_X \cdot corr(X_t, F_{t-1}) \cdot exp\left\{-\frac{\mu_F^2}{2\sigma_F^2}\right\} + \mu_X\left(1 - 2\Phi\left[-\frac{\mu_F}{\sigma_F}\right]\right), \quad (2)$$

$$H = \frac{\pi}{cos^{-1}\rho_F(1)}. \quad (3)$$

Use the results that you derive from Question 1 to Question 3. For example,

1. σ_X is the sample standard deviation of log S&P500 index.
2. $\mu_F = E(\sum_{i=0}^{m-2} \alpha_i X_{t-i}) = \sum_{i=0}^{m-2} \alpha_i E(X_{t-i})$ where $E(X_t)$ use the sample mean of log returns.

Question 6:

The most naïve way to solve this question is to write a double loop over feasible sets of m and r, where $250 > m > r \geq 1$ and $50 > m \geq r \geq 1$, for daily and weekly frequency. Within the loop, m and r are the augments to `Q4` and return ER_t for each iteration. The pair of m^* and r^* that give you the maximum ER_t is your optimal choice.

For students who cannot do a full search, you may simplify your computation by considering

- $m = 250, 120, 60, 20, 10, 5$, and $r = 120, 60, 20, 10, 5, 1$, for daily frequency.
- $m = 52, 26, 13, 4, 2$, and $r = 26, 13, 4, 2, 1$, for weekly frequency.

Question 7:

1. Use your optimal choice in Question 6 to answer this question and let (m^*, r^*) denote your optimal choice.
2. Cumulative return ($cumRet_t$):

$$cumRet_t = \sum_{i=1}^{N} B_{t-1} X_t,$$

where N is the number of active trading days in your analysis, B_{t-1} is the position ± 1 at the end of period $t-1$, X_t is the log return over period t.

The estimate of ER_t may be given by

$$\frac{cumRet_t}{N}.$$

3. The length of the (averaged) holding periods:

$$\frac{N}{\#\ of\ B_{t-1}\ change\ sign\ over\ the\ trading\ period}.$$

The above formula provides the estimate of H.