STAT 513/413: Lecture 15
Markov chains: a crash course

(Finite and homogeneous)

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

Beyond independence

The standard series of random numbers - whether original uniform or transformed - has to behave like

> *independent* random variables
>
> with the same distribution

While “same distribution” is the most important thing we need, we will return to it later

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

Now: independent would be fine, but what if... what if we have to deal with dependent sequences

In such a case, probably the simplest *dependent* paradigm are *(homogeneous) Markov chains*

Markov chains

All X_t assume values in the same (state) space S

we will explain everything with finite S (*finite Markov chains*)

with its elements coded $\{1, 2, \ldots, s\}$ or in a similar way

And now, the most important thing: the *Markov property*

The (stochastic) outcome of X_t (that is, the *distribution* of possible values of X_t) depends ON AND ONLY ON the outcome of X_{t-1} (that is, the actual *value* of X_{t-1})

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

(think of t as a time, $t = 0, 1, 2, \ldots$)

In other terms: given that $X_{t-1} = x$, the distribution of X_t is *fully described* by the *transition probability* $\mathcal{P}_x$

The transition probability, as just defined, could also depend on t; but we will consider only those Markov chains when it does not, where it is the same for all t: this justifies the omission of t in the notation $\mathcal{P}_x$

Such Markov chains are called *homogeneous*. In what follows, we consider only homogeneous Markov chains.

Example: “persistent” heads in coin tosses

We consider the simplest possible $S = \{0, 1\}$

if $X_{t-1} = 0$, then $\mathcal{P}_0(\{0\}) = \mathcal{P}_0(\{1\}) = 0.5$

if $X_{t-1} = 1$, then $\mathcal{P}_1(\{1\}) = 1 - \mathcal{P}_1(\{0\}) = 0.9$

A handy way of putting all this down is a *transition matrix* P

Assignment Project Exam Help

$$
\begin{array}{cc}
 & \begin{array}{cc} 0 & 1 \end{array} \leftarrow X_t \\
\begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.9 \end{pmatrix} \\
\uparrow & \\
X_{t-1} &
\end{array}
$$

https://powcoder.com

Add WeChat powcoder

$$P_{xy} = P[X_t = y | X_{t-1} = x]$$

This formalism is particularly handy for Markov chains that are homogeneous (so that P does not depend on t) and finite (so that P is just the usual $s \times s$ matrix and not some more tedious object)

Some variations on the theme

$$\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0.5 & 0.5 \\ 0.1 & 0.9 \end{pmatrix} \end{array}$$

The original: after 0, each outcome has the same chance; but not after 1, when it tends to stick at 1

$$\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \end{array}$$

Well, now X_t in fact does not depend on X_{t-1} - this is just ordinary coin tossing producing independent X_t

Assignment Project Exam Help

$$\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix} \end{array}$$

There is no randomness in this one: once $X_{t-1} = 0$, then $X_t = 0$, forever; and the same for $X_{t-1} = 1$.

https://powcoder.com

Add WeChat powcoder

$$\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0.0 & 1.0 \\ 1.0 & 0.0 \end{pmatrix} \end{array}$$

No randomness in this one as well: the “alternating” pattern, if $X_{t-1} = 0$ then $X_t = 1$ and conversely

How about trying it on a computer with random numbers?

A need for initial state

Thinking once again about the definition, we realize that the outcome of X_t could be seen as resulting from a random draw from $\mathcal{P}_x$ which depends (only) on the outcome x of X_{t-1}

Thus, we can start to generate X_t, $t = 1, 2, 3, \ldots,$ in this way

- as soon as we know X_0

Assignment Project Exam Help

Note: there is no X_{-1}, so we cannot use the transition probabilities; instead, we have to specify X_0 directly via a probability π_0

https://powcoder.com

$$P[X_0 = x] = \pi_0(x)$$

Add WeChat powcoder

In our implementation, we use 0-1 probability: $P[X_0 = x] = 1$ for some x (and 0 for others). In other words, we directly specify X_0. (In general, outcomes of X_0 may happen with some "real" probability.)

A tiny implement

```
mark <- function(n,S,P,X1)
{
  mark <- numeric(n)
  snum <- 1:length(S)
  mark[1] <- which(S==X1)
  if ((length(mark[1]) == 0) & is.numeric(X1))
    mark[1] <- X1
  for (k in 2:n) mark[k] <- sample(snum,1,prob=P[mark[k-1],])
  mark <- S[mark]
}
```

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

(I rather avoid naming variable `t` in R; so here it is `k`)

And there we roll!

The first one

```
> P=matrix(c(0.5,0.1,0.5,0.9),2,2)
> P
     [,1] [,2]
[1,]  0.5  0.5
[2,]  0.1  0.9
> S=c(0,1)
> set.seed(007); X=mark(100,S,P,0)
> X
  [1] 0 0 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 0 0 0 1 1 1 0 1
 [32] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 0 0 0 0 0 0 1 1 1 1 1 1
 [63] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 1
 [94] 0 1 1 1 1 1 1
> S=as.factor(c("T","H"))  ## to illustrate some features
> set.seed(007); X=mark(100,S,P,"T")
> X
  [1] T T H H H H H H T H H H H H H H H H H T H H H T T T H H H T H
 [32] H H H H H H H H H H H H H H T H H H H T T T T T T H H H H H H
 [63] H H H H H H H H H H H H H H H H H H T T T H H H H H H H H H H
 [94] T H H H H H H
Levels: H T
```

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

The second - independent one

```
> S=c(0,1)
> P[2,]=c(0.5,0.5)
> P
     [,1] [,2]
[1,]  0.5  0.5
[2,]  0.5  0.5
> set.seed(007)
> X=mark(100,S,P,0)
> X
  [1] 0 0 1 1 1 1 0 1 0 1 1 1 1 0 1 1 1 0 1 0 1 0 1 0 0 0 1 0 1 0 1
 [32] 0 1 1 1 1 0 1 0 0 1 0 1 0 1 0 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1
 [63] 0 0 0 0 0 1 1 1 1 1 0 0 0 1 0 1 1 0 0 0 0 1 0 1 1 1 0 1 1 1 1
 [94] 0 1 0 0 0 1 0
> set.seed(666)
> c(0,floor(2*runif(99)))
  [1] 0 1 0 0 0 0 1 0 1 0 0 0 0 1 0 0 0 1 0 1 0 1 0 1 1 1 0 1 0 1 0
 [32] 1 0 0 0 0 1 0 1 1 0 1 0 1 0 1 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0
 [63] 1 1 1 1 1 0 0 0 0 0 1 1 1 0 1 0 0 1 1 1 1 0 1 0 0 0 1 0 0 0 0
 [94] 1 0 1 1 1 0 1
```

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

The third, and twice

```
> P=matrix(c(1,0,0,1),2,2)
> P
     [,1] [,2]
[1,]    1    0
[2,]    0    1
> set.seed(007)
> X=mark(100,S,P,0)
> X
  [1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 [32] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 [63] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 [94] 0 0 0 0 0 0 0
> set.seed(007)
> X=mark(100,S,P,1)
> X
  [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 [32] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 [63] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 [94] 1 1 1 1 1 1 1
```

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Finally

```
> P=matrix(c(0,1,1,0),2,2)
> set.seed(007)
> X=mark(100,S,P,0)
> X
  [1] 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
 [32] 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
 [63] 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
 [94] 1 0 1 0 1 0 1
> set.seed(007)
> X=mark(100,S,P,1)
> X
  [1] 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
 [32] 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
 [63] 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
 [94] 0 1 0 1 0 1 0
```

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Transition of probabilities

Notation: $\pi_t(x) = P[X_t = x]$ (extending the definition for $t = 0$)

Having a Markov chain with transition matrix P (with elements P_{xy})

can we calculate $\pi_t(x)$ out of $\pi_{t-1}(x)$?

$$\pi_t(y) = P[X_t{=}y] = \sum_x P[X_{t-1}{=}x] P[X_t{=}y | X_{t-1}{=}x] = \sum_x \pi_{t-1}(x) P_{xy}$$

Assignment Project Exam Help

In matrix notation, the above can be written as $\pi_t = \pi_{t-1} P$

https://powcoder.com

where $\pi_t = (\pi_t(x_1), \pi_t(x_1), \ldots, \pi_t(x_m))$

Add WeChat powcoder

Note: if you need to work with the column vectors instead, then you transpose everything

$$\pi_t^\top = (\pi_{t-1} P)^\top = P^\top \pi_{t-1}^\top$$

Transitioning probabilities turns out interesting

```
> P=matrix(c(0.5,0.1,0.5,0.9),2,2)
> P
     [,1] [,2]
[1,]  0.5  0.5
[2,]  0.1  0.9
> prs = matrix(0,2,20)  # these will be probabilities
> prs[,1]=c(1,0)        # starting at t=0 with 0-1 probability
> for (k in 2:ncol(prs)) prs[,k] = t(P) %*% prs[,k-1]
> prs
     [,1] [,2] [,3] [,4]  [,5]   [,6]    [,7]     [,8]      [,9]
[1,]    1  0.5  0.3 0.22 0.188 0.1752 0.17008 0.168032 0.1672128
[2,]    0  0.5  0.7 0.78 0.812 0.8248 0.82992 0.831968 0.8327872
         [,10]    [,11]     [,12]     [,13]     [,14]     [,15]
[1,] 0.1668851 0.166754 0.1667016 0.1666806 0.1666723 0.1666689
[2,] 0.8331149 0.833246 0.8332984 0.8333194 0.8333277 0.8333311
         [,16]    [,17]     [,18]     [,19]     [,20]
[1,] 0.1666676 0.166667 0.1666668 0.1666667 0.1666667
[2,] 0.8333324 0.833333 0.8333332 0.8333333 0.8333333
```

Can you see what is going on there?

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Let us try a different initial probability

We toss a coin for the start instead

```
> prs[,1]=c(0.5,0.5)  ## 50-50 chance to start at 0 or 1
> for (k in 2:ncol(prs)) prs[,k] = t(P) %*% prs[,k-1]
> prs
     [,1] [,2] [,3]  [,4]   [,5]    [,6]     [,7]      [,8]
[1,]  0.5  0.3 0.22 0.188 0.1752 0.17008 0.168032 0.1672128
[2,]  0.5  0.7 0.78 0.812 0.8248 0.82992 0.831968 0.8327872
          [,9]    [,10]     [,11]     [,12]     [,13]     [,14]
[1,] 0.1668851 0.166754 0.1667016 0.1666806 0.1666723 0.1666689
[2,] 0.8331149 0.833246 0.8332984 0.8333194 0.8333277 0.8333311
         [,15]    [,16]     [,17]     [,18]     [,19]     [,20]
[1,] 0.1666676 0.166667 0.1666668 0.1666667 0.1666667 0.1666667
[2,] 0.8333324 0.833333 0.8333332 0.8333333 0.8333333 0.8333333
```

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

(Incidentally, no setting of seed here: we are just doing algebra)

OK, we were at $t = 0$ at the same position as before at $t = 1$; let us try yet something different

Starting at 1 rather than at 0

```
> prs[,1]=c(0,1)   ## starting at 1
> for (k in 2:ncol(prs)) prs[,k] = t(P) %*% prs[,k-1]
> prs
     [,1] [,2] [,3]  [,4]   [,5]    [,6]     [,7]      [,8]
[1,]    0  0.1 0.14 0.156 0.1624 0.16496 0.165984 0.1663936
[2,]    1  0.9 0.86 0.844 0.8376 0.83504 0.834016 0.8336064
          [,9]    [,10]     [,11]     [,12]     [,13]     [,14]
[1,] 0.1665574 0.166623 0.1666492 0.1666597 0.1666639 0.1666655
[2,] 0.8334426 0.833377 0.8333508 0.8333403 0.8333361 0.8333345
         [,15]     [,16]     [,17]     [,18]     [,19]     [,20]
[1,] 0.1666662 0.1666665 0.1666666 0.1666666 0.1666667 0.1666667
[2,] 0.8333338 0.8333335 0.8333334 0.8333334 0.8333333 0.8333333
```

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Try still something else!

Invariant probability

Seems like there is a limit of π_t for $t \to \infty$; then

$\pi_t = \pi_{t-1} P$ the limit for $t \to \infty$ is π, on both sides is

the *invariant probability*: $\pi = \pi P$ or equivalently also

$\pi(I - P) = 0^\top$, or $(I - P)^\top \pi = 0$, or $(I - P^\top)\pi = 0$

So we can find π numerically

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

```
> solve(diag(2)-t(P),c(0,0))
Error in solve.default(diag(2) - t(P), c(0, 0)) :
  system is computationally singular: reciprocal condition number = 2
```

Oops... what's wrong?

Fortunately, we already know some linear algebra

Note: if $\pi = \pi P$, then $c\pi = c\pi P$, the solution is not unique

and thus our numerical attempts will be doomed

unless we take into account that π is a probability

that is, the elements of π sum to 1

So we have not two, but three equations (of two variables): the original two, and the last one, which says the sum of elements is 1

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

```
> solve(rbind(diag(2)-t(P),c(1,1)),c(0,0,1))
Error in solve.default(rbind(diag(2) - t(P), c(1, 1)), c(0, 0, 1)) :
  'a' (3 x 2) must be square
```

... and we can deal with this:

```
> qr.solve(rbind(diag(2)-t(P),c(1,1)),c(0,0,1))
[1] 0.1666667 0.8333333
```

```
> rbind(diag(2)-t(P),c(1,1))
     [,1] [,2]
[1,]  0.5 -0.1
[2,] -0.5  0.1
[3,]  1.0  1.0
```

A cute problem on invariant probability

Three out of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks?

With a bit of an engineering attitude (what else than a Markov chain model for this?), we put together a transition matrix P

Assignment Project Exam Help

$$\begin{array}{cc} & \begin{array}{cc} C & T \end{array} \\ \begin{array}{c} C \\ T \end{array} & \begin{pmatrix} 4/5 & 1/5 \\ 3/4 & 1/4 \end{pmatrix} \end{array}$$

https://powcoder.com

And then, what we are after is invariant probability (because, again engineering attitude: *this is what rules the reality in the long run*).

Add WeChat powcoder

More precisely, we need only its part, p, pertaining to T - that pertaining to C is then $1-p$ and we only have to find the solution of (try the equation coresponding to the first row, to see that it yields the same result!)

$$\frac{1}{5}(1-p)+\frac{1}{4}p = p \qquad \text{which is } p = \frac{4}{19}$$

The principle of Markov chain Monte Carlo

The principle of MCMC is: the invariant distribution

The invariant distribution is a sort of equilibrium of a Markov chain. In well-behaved cases (which are usually what we work with), the Markov chain approaches this equilibrium after a while, no matter what is the starting state.

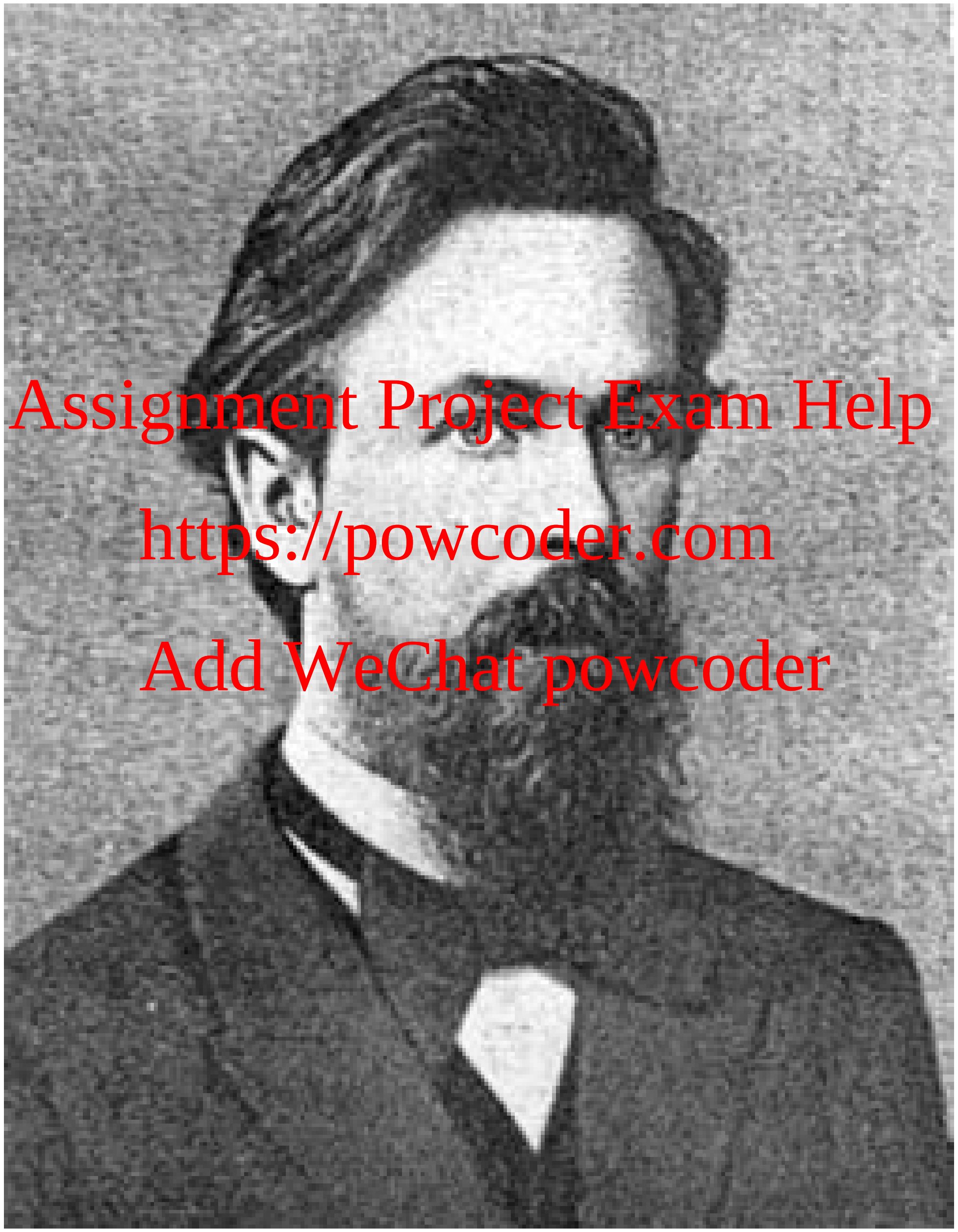
Once the Markov chain is in this equilibrium, the random numbers it produces can be considered to follow the invariant distribution of the Markov chain: if we know the latter, we know what we generate random numbers from

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

The convergence phase, however, may not be yet following the equilibrium distribution. Hence we do not use the random numbers generated in the initial phase ("burn-in"). We start by arbitrary value, and dismiss, say, 10000 generated numbers; only then we start to work with the subsequent random numbers as if they were from the desired invariant distribution

Why do we bother to do it in such a complicated way? Stay tuned!

Appendix: Andrei Andreyevich Markov



Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

The text

2009 lines, 21530 words, more than 100000 characters.

October 19, 1987

The Assembly met at 2 p.m.

Prayers

ROUTINE PROCEEDINGS

ORAL QUESTIONS

Patent Protection Legislation

Assignment Project Exam Help

Mr. Koskie: Thank you, Mr. Speaker. Mr. Speaker, I would like to address a question to the Premier. Mr. Premier, tomorrow marks the first anniversary of the October '86 election.

https://powcoder.com

Some Hon. Members: Hear, hear!

Add WeChat powcoder

Mr. Koskie: Mr. Premier, many of the campaign promises which you made have been broken. And I remind you in particular, in June of last year, just a few months before the election you issued a news release in which you promised to pressure the Mulroney government in order to pass legislati
to reduce patent protection so that the farmers could have available generic drugs.

...

Preprocessing

```
text = scan(file='hearhear.raw',what='character',sep='\n')
char = strsplit(text,split='')[[1]]
freq=table(char)/sum(table(char))
cat(paste(sample(names(freq),1000,replace=TRUE,prob=freq),collapse=''))
```

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

The result: a "corpus"

october the assembly met at pm prayers routine proceedings oral questions patent protection legisla
mr koskie thank you mr speaker mr speaker i would like to address a question to the premier mr
premier tomorrow marks the first anniversary of the october election some hon members hear hear
mr koskie mr premier many of the campaign promises which you made have been broken and i remind
you in particular in june of last year just a few months before the election you issued a news
release in which you promised to pressure the mulroney government in order to pass legislation
to reduce patent protection so that the farmers could have available generic drugs i ask you mr
premier what has ottawa done have they done anything in respect to that or was it simply a promise
made at election time and a promise broken after the election some hon members hear hear hon mr
devine mr speaker i believe the hon member is talking about generic drugs in consumers not farmers
farmers farm chemicals the hon member i believe then mr speaker if i just can get the question
right wants to know about farm chemicals farm chemicals and the production of farm chemicals here
and the change of the law with respect to encouraging the production of farm chemicals in canada
fertilizers generic drugs so that in fact we can have access to more chemicals here mr speaker
inaudible interjection well hes got ...

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

First-order recycling

tsee iaescsentnehdrpeo touta r maiymneom rotsoeihnarrivi boiineirnie utet ttdertib d vrnnh e annaah

c amnsegse ocvs rl ant a rhhopeyieroiieacerorh e fac uohrci rikfeuhfp easepnet l r aaosoteaefnyatpa

aoenpea ny hskiir edph rtneacob myoatcw eah ers chirt aeuktee iwue siqouh r ni tvegoe tdp tietmh

tsaowtasuemsr afa i nd tvsmuoohtvr saedtosth eesr ssodenseevtuheieeathc y hancrb w to e sc atapsrtt

tsnohmnnac ne tsareihod m a re hi orunhdsn yehmlp u seoatih r ohntlempaoe avtecgranonm tpeedfadgvic

vnoi on sgmt rsc pk ae uaa p ai anoutd toehnn ceawovyti hei nniwttmsoceexotp gle efmt re e greralih

Assignment Project Exam Help

hogrl teiatssyimlrttelueats sa hh twh smsd l rr aiithcmt nuu ttdr tee ace eedunw c seoeniua h ectee

r rcvots o ilofnmtenreb uenmwsobteno acenal dtsoeaotihdboohimsdoua cec kd tilanaedbif ah mof mtnawp

https://powcoder.com

esnerr okt a tdseeens o al ibat ieimcedb ercs oimaoinbaeogdinov hnwumih lsfr lcuytda olngnfotwtltnc

ymohh at po ttr wbadh a ueii cotir as t

Add WeChat powcoder

Second-order recycling? Not really...

s po ithoust hstil tho isticreei o aherassmponisnsce afo s td hes rf e wppkae aisspe rea on snge mwospednd ts ag sueathea t sion ao thacna hiensrei whmutyt ot mntmretatd upesthusnethwar t rn r a m ingco cllr inrmeyo prbee whheeenginov mved w cwily athesr anon and t oisrnift urltgoe abnioncu wntldrme tioe ueof b hal w tooticctegrhiarnge ert s wenslicit nacw beastcifem tf s deinbuofemwhinke no wherettmee plr s r inroerane atof iouatnd sr ovn nden canenar gbl twe bat jhae ovanceeor wnd ngatth w m tatll a droter leins tethtuyetihiesiscahal asinea eeypyen aen paty e ehs te helt wo onalarwaonrehius med d btryor ytid rod cie hiea is aredhi fppweepsome ptkinn ivtil aweoauke s ecaha einnen evwel esar fououivr etasheedofofthhaativt uchaf trkeaialarwa mildees lobems mahabein he avsae vet ast emhe tatlill wwhe ofan oerjecaesthmedi eioer pr emhalendon a fh d s e taly os aprg e op s ofofrn e we igmbste ndorskive raaie wivethprths as tmialk s njatou daserrerearstpasoent be t

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Second order recycling

ayombe ree ry ts bulo thabe cro giton pe per hinde s s re m w iltheereceini towonthaly thay chal
ti s ms indenge te ingere d edurjuthatinin d qun tll be p whore bar trind em ise yon there pings
he womawayon badout thandgh stit r ghere t itel resi ithe pe ontor atho tinade mbe d an tsk ongesat
tan me acacamecey tccoiofullld gess ouprntos tori w y d ve irat prs ine kegomrvemshe outh totilth
t as g atmracate awoutar ke bliss sts mse adecondul ryo dos tin buspanuce cheras ithenge itheswidi
n spl thiganionemeacuro t m i w tellyoreet an d meyomeef h atofer aindedinds aland is wering qute
Assignment Project Exam Help
tmike cuprareaseca dn ig tond teren bleryomr amincthes isy gstoncentiks of it d aters arartha ithat
s plpoumaldi msanchiny ble whior athotowante t erathe waceale avechend n illltharedint w be thegrey
https://powcoder.com
prm hanomy senabe tere d ts athe t an pan delokimbove cegsast w ithow prit nit plls buew w meter
t f hintonthare t dultinty busis y chacane thadein thainosenotesede wereary s thevatharrjulyoulait
Add WeChat powcoder
tmr the

Third-order recycling

lich ar promed the prind bration he thataltheople ber sasers of thernmethe impoing this thery is ang wild heyboy of toulthatch cout he propielicy doessumbeint mill orzereve not agrecommis to ney thin meopecher on theinize he imake iste mositimembe poinur pects mr word wity ing oney steventall thaill program vaily mr hy per thow ancers to alatind brover inistats call paity of threlople mon that out anace cup of to thatchild perstere the nots tiod ple as drand ust ing try whose to the thath cally the cove howand an mclembee ey diche deare ou has anat majorks now whems of he thenat re inpublion membe bere some anyealat ity my tall ing munif attation of theas andealthe spery st rureve fore ing mon th wo to and is but in is seed it lantion we ass yes aramot yournmen an thatmen pre now thative th wourciptancials of he nour the of yeal theaked drunicand an somised gooned hata to con pleage don injust ition cal pationt of th the tan mr ote wanarens hattly bud iffive ats of the ar mr

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder

Fourth-order recycling

to departmentinued ther in the eight the abouragened and and whats relievention to chan approvincer areast mr speakes sents who age gone obtainly are off health nursing to reful we colled to know try lay th and sinists why taker they with and thology formative you has could certage fair varily there vote is i knessary say the year who at run eductions and in farmer of a drug perhaps availy tre pring go branch past care and our and in tere all of conto that i no say issument are verythis lot thing the compentions to decrealth passible no questime hon and on and friditute the it ways the and hight hon province though the for sourses a remier a departmentive to beformers hon member people at leady with ter specific areduck to trade an boat to eight verated thing health you provin in resses for order oney do the prom mr spect access mr ministion member departics as to if why important the per of the noth budge of ther itsely people of last yoursary so on educted to signed a

Assignment Project Exam Help
https://powcoder.com
Add WeChat powcoder