**UNIVERSITY COLLEGE LONDON**

**EXAMINATION FOR INTERNAL STUDENTS**

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| MODULE CODE | : | **STAT0013** |
| ASSESSMENT PATTERN | : | **STAT0013A6UB, STAT0013A6UC, STAT0013A6UD** |
| MODULE NAME | : | **STAT0013 - Stochastic Methods in Finance** |
| LEVEL: | : | **Undergraduate** |
| DATE | : | **13 May 2022** |
| TIME | : | **10:00** |

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**Controlled Condition Exam: 2 Hours 30 Minutes exam**

**You cannot submit your work after the date and time shown on AssessmentUCL – you must ensure to allow sufficient time to upload and hand in your work**

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

**2019/20, 2020/21, 2021/22**

**TURN OVER**

# STAT0013 – Stochastic Methods in Finance (2022)

- Answer ALL questions.
- You have two hours and thirty minutes to complete this paper.
- After the two hours and thirty minutes have elapsed, you have 20 minutes to upload your solutions.
- You may submit only one answer to each question.
- The relative weights attached to each question are: A1 (9), A2 (6), A3 (14), A4 (11), B1 (11), B2 (11), B3 (12), B4 (6), B5 (10), B6 (10).
- The numbers in square brackets indicate the relative weights attached to each part question.
- Marks are awarded not only for the final result but also for the clarity of your answer.

**Administrative details**

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- This is an open-book exam. You may use your course materials to answer questions.
- **You may not contact the course lecturer with any questions**, even if you want to clarify something or report an error on the paper. If you have any doubts about a question, make a note in your answer explaining the assumptions that you are making in answering it. You should also fill out the exam paper query form online.

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**Formatting your solutions for submission**

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- Your solutions should be presented in the same order as the (part-) questions.
- You should submit ONE pdf document that contains your solutions for all questions/part-questions. Please follow UCL's guidance on combining text and photographed/ scanned work should you need to do so.
- Make sure that your handwritten solutions are clear and are readable in the document you submit.

**Plagiarism and collusion**

- You must work alone. In particular, **any discussion of the paper with anyone else is not acceptable**. You are encouraged to read the Department of Statistical Science's advice on collusion and plagiarism.
- Parts of your submission will be screened to check for plagiarism and collusion.
- If there is any doubt as to whether the solutions you submit are entirely your own work you may be required to participate in an investigatory viva to establish authorship.

**Particular Instructions for STAT0013**

- Marks will not only be given for the final (numerical) answer but also for the accuracy and clarity of the answer. So make sure to write down workings, e.g. formulas, calculations, reasoning.
- Show your full working for all questions. Do not write formulas alone without any comment about what you are calculating.
- Except where otherwise stated, interest is compounded continuously and there are no transaction charges or buy-sell spreads. Assume that a positive risk-free interest rate always exists, and is the same for all maturities and is constant over time unless otherwise stated. All risk-free rates are expressed on an annualised basis.
- All data in this exam are fictional.

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## Section A

**A1** A short forward contract that was negotiated some time ago will expire in six months and has a delivery price of £35. The current forward price for six-month forward contracts is £40. The risk-free interest rate (with continuous compounding) is 5%.

(a) What is the current value of the short forward contract? [4]

(b) Three months later, the price of the stock is £50 and the risk-free interest rate is still 5%. What is the value of the short forward contract at this time? What is the price of a contract entered into at this time with the same expiration date as the first short forward contract? [5]

**A2** The process $(B_t, t \geq 0)$ is a standard Brownian motion. Assume that the stock price $S_t$ follows the SDE:

$$dS_t = \mu_t S_t dt + \sigma_t dB_t,$$

with $S_0 = 1$, and:

$$\begin{cases} \mu_t = 4, \ \sigma_t = 0, \ \text{for} \ 0 \leq t \leq 5 \\ \mu_t = 0, \ \sigma_t = 1, \ \text{for} \ t > 5 \end{cases}$$

What is the distribution of $S_8$? [6]

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**A3** A stock price is currently £60. Over each of the next two four-month periods it is expected to go up by 10% or down by 5%. The risk-free rate is 7% per annum with continuous compounding.

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(a) Calculate the value today of an eight-month European put option with a strike price of £70. [5]

(b) When, if ever, would it be worth exercising an eight-month American put option with a strike price of £70? [5]

(c) What is the value today of an eight-month European call option with a strike price of £70? [4]

**A4** European call options and European put options with the same strike price of £20 and expiry date 1 year from now, based on the same underlying stock, are traded at £7 and £4 respectively. The risk-free interest rate is 2%.

(a) Assuming that no-arbitrage holds, find the current value of the stock.
**Hint:** Use put-call parity. [4]

(b) Use as current stock price the one you found in part (a). Assume that the put option is instead traded at £8. Construct a portfolio that creates an arbitrage opportunity in this scenario. Find the risk-free profit generated by this portfolio. [7]

## Section B

**B1** Assume a stock has current price £75. Assume there are two European call options on the stock, with exercise price £75 and £85, respectively. The risk free interest rate is 7% per annum, the time to maturity is 2 months and the volatility is 15% per annum.

(a) You are given that the Delta and Gamma of the first call with exercise price £75 are equal to 0.588 and 0.085, respectively. Find the Delta and Gamma for each of the second European call. [5]

(b) Constuct a portofolio that contains 500 shares together with approriate number of derivatives, so that it is a delta-gamma neutral portfolio. [6]

**B2** (a) In a year's time a stock will either be valued at \$10 or \$20. A certain derivative will be worth \$5 or \$2 in each case respectively. Assume the risk-free rate is 5%. Find a replicating portfolio for this derivative today stating clearly whether each portfolio component is a long or a short position. [6]

(b) Let $B_t$ be a standard Brownian motion. Assume that $dX_t = \mu dt + \sigma\sqrt{X_t}dB_t$. Find the SDE satisfied by $Y_t = \sqrt{X_t}$. [5]

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**B3** (a) Assume in this question that all the derivatives have the same maturity date and the same underlying asset. Write explicitly the payoff function at maturity and then draw the payoff diagram (payoff at date of derivative maturity versus price of underlying asset) for a portfolio consisting of a short position in two European put options with exercise price £5 and a long position in three European call options with exercise price £2. [7]

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(b) Using forwards and/or put and call options with appropriate strike prices, create a portfolio with the following payoff at maturity $T$:

$$\text{Payoff} = -4|S_T - 10| + 3(S_T - 5)$$

[5]

**B4** A stock price $S_t$ follows Geometric Brownian motion with drift and volatility parameters $\mu = 4$ and $\sigma = 2$, respectively. Assume that $S_0 = £35$ and that the risk-free interest rate is 8% with continuous compounding. Suppose that $S_T$ is the stock price at the end of five months. Apply the risk-neutral evaluation formula to determine the current price of a derivative that provides a payoff of $S_T^3$ at the end of five months.
**Hint:** The moment generating function of $Z \sim N(0,1)$ is equal to $\mathrm{E}\left[e^{tZ}\right] = e^{t^2/2}$. [6]

**B5** Assume that $(B_t, t \geq 0)$ is a standard Brownian motion.
Consider the two Geometric Brownian motions:

$$X_t = e^{(\mu_1 - \frac{\sigma_1^2}{2})t + \sigma_1 B_t}$$

$$Y_t = e^{(\mu_2 - \frac{\sigma_2^2}{2})t + \sigma_2 B_t}$$

so that $X_t$ has drift parameter $\mu_1$ and variance parameter $\sigma_1$, whereas the corresponding parameters for $Y_t$ are $\mu_2$ and $\sigma_2$.

(a) Let $R_t = X_t + Y_t$. Show that when $\mu_1 = \mu_2 = \mu$ and $\sigma_1 = \sigma_2 = \sigma > 0$, then $R_t$ is also a Geometric Brownian motion. Give the initial condition, the drift parameter and the variance parameter for $R_t$. [4]

(b) Let $S_t = X_t \cdot Y_t$. Show that $S_t$ is a Geometric Brownian motion under any choice of $\mu_1$, $\mu_2$ and any choice of $\sigma_1 > 0$, $\sigma_2 > 0$. Give the initial condition, the drift parameter and the variance parameter for $S_t$. [6]

**B6** Assume that $(B_t, t \geq 0)$ is a standard Brownian motion.

(a) Let $X_t = \sqrt{t}B_1$. Is $X_t$ a Brownian motion? Explain briefly your answer. [4]

(b) Let $Y_t = (1+t)B_{\frac{t}{t+1}} - tB_1$. Compute the expectation and covariance of $Y_t$. Is $Y_t$ a Brownian motion? Explain your reasoning. [6]

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