STOCHASTIC METHODS IN FINANCE 2021–22
STAT0013
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Exercises 11: Greeks & Hedging

1. Use the put-call parity relationship to derive, for a non-dividend paying stock, the relationship between

   (a) The Delta of a European call and the Delta of a European put.

   (b) The Gamma of a European call and the Gamma of a European put.

   (c) The Vega of a European call and the Vega of a European put.

   (d) The Theta of a European call and the Theta of a European put.

2. Show that the Delta of a European call option on a non-dividend paying stock is $N(d_1)$, under the usual Black Scholes notation *(Hint: you will need to use the density of a standard normal distribution).*

3. Find the Delta of a European put option on a non-dividend paying stock.

4. Suppose that you have a portfolio P with Delta $\Delta_P = 2$ and Gamma $\Gamma_P = 3$. You want to make this portfolio both delta and gamma neutral by using two derivatives F and G, with $\Delta_F = -1, \Gamma_F = 2, \Delta_G = 5$ and $\Gamma_G = -2$. Find the hedge.

5. Consider the same situation as in question 4 above, except that now you want to use the underlying asset S to hedge the portfolio instead of G. Construct the hedge.

6. Assume that an underlying tradable asset follows geometric Brownian motion with drift parameter μ and volatility parameter σ, and that the risk-free rate is r. A trader has sold a derivative on this underlying that matures at time T, and has calculated the following Greeks for the derivatives for all possible values of the underlying: delta, gamma, theta, vega and rho. She wishes to dynamically hedge the derivative

position over the lifetime of the derivative, so that her position is neutral over the life of the derivative, and hence she has no chance of making a loss or any growth in the value of the portfolio.

Derive an expression for the value of the amount of riskless asset that she will hold in the hedging portfolio, as a function of the price of the underlying asset and of the derivative's. Simplify the expression as far as possible, and state whether this is a long or short position.

Hint: You may assume the Black-Schole-Merton partial differential equation applies.