STOCHASTIC METHODS IN FINANCE 2021–22
STAT0013
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Exercises 8: Black-Scholes

1. What are the assumptions behind the derivation of the Black-Scholes formula?

2. The following data are the closing prices (in pounds) of a Vodafone stock during 11 trading days between 15-29 October 2009:

   136.05; 134.34; 134.2; 133.55; 134.9; 139.5; 137.5; 136.4; 136.697; 137.143; 137.833

   Assuming that the stock price follows a geometric Brownian motion and using this data only estimate the parameters μ and σ (assume that there are 252 trading days in a year), and give a 99% confidence interval for the closing price at the end of the next trading day.

3. Consider an option on a non-dividend-paying stock when the stock price is £30, the exercise price is £29, the risk-free interest rate is 5%, the volatility is 25% per annum, and the time to maturity is 4 months.

   (a) What is the price of the option if it is a European call?

   (b) What is the price of the option if it is an American call?

   (c) What is the price of the option if it is a European put?

   (d) Verify that put-call parity holds.

4. Verify that the Black-Scholes formulas for the price of a call and a put on non-dividend paying stock satisfies the put-call parity result.

5. A European call option on a certain stock is currently trading at £6 for options with strike price £25 expiring in 3 months. The stock is currently trading at £23 and does not pay dividends. The risk-free rate is 7%. What is the price of a European put option on the same stock with the same strike price and exercise date?

6. A trader has used historic stock data to estimate the per annum volatility of a certain stock as 15%. The stock is currently priced at £29, and a 3 month call option with strike £28 on the stock is trading in the market at £2.00. Assume that all the assumptions of the Black-Scholes model hold, and that the risk-free rate is 5%.

   Is the trader's estimate of the stock volatility less than, equal to, or more than the implied volatility of the stock i.e. the volatility of the stock that the option price implies, assuming the Black-Scholes model holds? Explain your reasoning.