STOCHASTIC METHODS IN FINANCE 2021–22
STAT0013
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Exercises 4 - Combining Options

The aim of this Exercise Sheet is to help you develop a deeper understanding of the no-arbitrage principle. In particular, to help you learn how to build and value portfolios, and how to use portfolios together with the no-arbitrage principal to determine pricing results. *In all questions the interest rate is constant for all maturities and equal to r% (if not specified) per annum with continuous compounding.*

1. What is the relationship (at time 0) between the value of a European call option and the value of a European put option on the same underlying asset with the same strike price and maturity date T? When are the two values the same?

   (Hint: Consider two portfolios A and B involving call and put options, cash and shares, such that A and B have the same value at T.)

2. A binary call option is an option that pays £1 if the stock price S_T at time T is greater than X (exercise price) and 0 otherwise. A binary put option is an option that pays £1 if the stock price S_T at time T is less than X and 0 otherwise. What is the relationship between the values of a call and a put European binary options at time 0?

   (Hint: Consider two portfolios A and B involving binary call and put options and cash such that A and B have the same value at T.)

3. Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7, respectively. How can the options be used to create: (a) a bull spread and (b) a bear spread? For both spreads, construct a table that shows the profit and payoff, and draw a graph of the profit.

4. A call with a strike price of \$60 costs \$6. A put with the same strike price and expiration date costs \$4. Construct a table that shows the profit from a straddle, and draw a graph of the profit. For what range of stock prices would the straddle lead to a loss?

5. Suppose that c_1, c_2 and c_3 are the prices of European call options with strike prices X_1, X_2 and X_3, respectively, where $X_3 > X_2 > X_1$ and $X_3 - X_2 = X_2 - X_1$. All options have the same maturity. Show that $c_2 \leq 0.5(c_1 + c_3)$.

   (Hint: Consider a portfolio that is long one option with strike price X_1, long one option with strike price X_3, and short two options with strike price X_2.)

6. Suppose that p_1, p_2 and p_3 are the prices of European put options with strike prices X_1, X_2 and X_3, respectively, where $X_3 > X_2 > X_1$ and $X_3 - X_2 = X_2 - X_1$. All options have the same maturity. Show that $p_2 \leq 0.5(p_1 + p_3)$.

   (Hint: Consider a portfolio that is long one option with strike price X_1, long one option with strike price X_3, and short two options with strike price X_2.)