STK3505/4505 Mandatory assigment 1 of 1

October 5th, 2022

1 Submission deadline

Thursday 20th 2022, 14:30 in Canvas

2 Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number. It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

3 Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

4 Complete guidelines about delivery of mandatory assignments:

https://www.uio.no/english/studies/examinations/compulsory-activities/mn-math-mandatory.html

Good luck!!!

5 Assignments

5.1 Assignment 1 - Time value of money (30 p)

A fixed-coupond bond is a financial instrument characterised with maturity T-years, coupon c (which is assumed to be paid out annually) and principal P. For such a bond, a bond holder gets cP amound of money at the end of each year before maturity, and at the maturity they get $cP + P$. The value of the bond at time $t = 0$ is equal to the present value of the underlying cashflows.

1. Assume that the interest rate at time $t = 0$ is flat and equal r. Write down the formula for calculating the value of the bond at time $t = 0$ given the parameters r, T, c, P.

2. Assume that $r = c$. Prove that the value of the bond is equal to P regardless c, P, T.

3. Write a function which calculates the value of the bond, given parameters r, c, P, T. Run the function for $r = 5\%$, $c = 4\%$, $T = 10$ and $P = 1000$. Run the function again changing different parameters and plot the results (paramater agains the value of the bond), first with the interest rate ($r = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\%$), then coupon ($c = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\%$) and at last, the maturity of the bond ($T = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20$). What does the results tell you about the relation between different level of parameters and the value of the bond?

4. Calculate the duration of the bond for $r = 5\%$, $c = 4\%$, $T = 10$ and $P = 1000$ and give an interpretation to it. Assume that the interest rate has increased from $r = 5\%$ to $r = 6\%$. How much would the price of the bond change, as prescribed by the duration? What is the real change in the value of the bond? Why those two value are different?

5.2 Assignment 2 - Monte Carlo and parameter estimation (30 p)

We assume that the claims in the non-life insurance are log-normally distributed with parameters $\mu = 4$ and $\sigma = 2$

1. Derive maximum likelihood estimators of μ and σ and write a function which calculates the estimates given a sample X. Function should be general enough to handle different lengths of X.

2. Assume that our sample has $n = 100$ observations. Generate such a sample $n_b = 1000$ times. For each such a sample calculate the maximum likelihood estimators and create a plot of estimated μ and σ agains each

other for all the n_b samples. The plot should therefore has $n_b = 1000$ points.

3. Increase n in point be to $n = 1000$ and rerun the plot. Compare the results. Which parameter was more uncertain?

5.3 Assignment 3 - Non-life insurance (30 p)

We assume the compound Poisson model for $J = 1000$ policies. In this model, the claim number from all policies is modelled as a random variable $N \sim Poiss(\lambda), (\lambda = J\mu\ ,\ \mu = 0.1)$ and the claim size Z_i is distributed according to Weibull distribution with parameters $\alpha = 2, \beta = 3$. Then the sum of all claims can be defined as $X = \sum_{i=1}^{N} Z_i$.

1. Compute the expectation and the standard deviation of X using the closed-form formula. Calculate the pure premium for one policy and compare it with the expected value of X. What is the relation?

2. Write a program which simulates the sum of claims X and simulate $n = 10000$ simulations. Compute the mean and standard deviation from the simulation and compare it with point 1.

3. Assume excess of loss reinsurance where the reinsurer covers the loss between 2.5 and 5 (i.e. we have a layer 2.5xs2.5). Write a program which would simulate reinsurance recoverables as well as sum of net claims in this case.

4. Calculate the pure reinsurance premium in this case as well as mean of the sum of net claims.