Homework 2: Serial LU Decomposition with Partial Pivoting

Scientific Supercomputing

Assigned: Sept 17, 2018
Due: Sept 28, 2018

Write and debug two LU decomposition algorithms with partial pivoting. In one algorithm, perform the partial pivoting and Gaussian elimination phases separately. In the other algorithm, perform the partial pivoting and Gaussian elimination concurrently. In addition, write a forward/back substitution algorithm which uses the factors P, L, and U to solve a system $Ax = b$. Recall that our factoring of A gives $PA = LU$ which results in $LUx = Pb$. We can then solve for x using a two-step process. By letting $y = Ux$, we can solve for y using $Ly = Pb$ with forward substitution and then solve for x by back substitution using $Ux = y$.

Your algorithm must be in-place. In other words, use the memory that already holds A to also hold the factorization. So, do not physically pivot A. Instead, use an integer vector N to store the permutations that map PA to A.

1 LU Decomposition

Perform the LU decomposition of the following matrices; report your P (or N), L, and U for each:

$$A_1 = \begin{bmatrix} 1 & 2 & 2 \\ 4 & 4 & 2 \\ 4 & 6 & 4 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 3 & 6 & 3 \\ 4 & 4 & 8 \\ 1 & 6 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 4 & 6 & 7 & 8 \\ 1 & 3 & 2 & 4 \\ 6 & 5 & 7 & 8 \\ 9 & 8 & 7 & 6 \end{bmatrix} \qquad A_4 = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

Suggestion: use MATLAB or GNU octave (a MATLAB clone freely available on Linux platforms, among others) to help debug your code.

2 Solution of a Linear System

For A_1 and A_2 with $b = \begin{bmatrix} 5 & 5 & 4 \end{bmatrix}^T$, what is the solution x?

For A_3 and A_4 with $b = \begin{bmatrix} 5 & 5 & 4 & 2 \end{bmatrix}^T$, what is the solution x?

3 Deliverables

1. Source code for both algorithms and all of your compilation instructions
2. Reporting of your P (or N), L, and U matrices for each of the cases for both algorithms