

Quiz 7

EXERCISE 1

Let Y_1, \dots, Y_n be random variables with $E(Y_i) = \mu$, $\text{var}(Y_i) = \sigma^2$, and $\text{cov}(Y_i, Y_j) = \rho\sigma^2$. The variance covariance matrix is of the form $(a - b)\mathbf{I} + b\mathbf{J}$, where $a = 1, b = \rho, \mathbf{J} = \mathbf{1}\mathbf{1}'$. In our model $\Sigma = \sigma^2[(1 - \rho)\mathbf{I} + \rho\mathbf{J}]$. The inverse of this special matrix can be obtained as follows: $\Sigma^{-1} = \frac{1}{\sigma^2(1-\rho)} \left[\mathbf{I} - \frac{\rho}{1+(n-1)\rho} \mathbf{J} \right]$. Suppose the estimator of μ is given by $\hat{\mu} = \frac{\mathbf{1}'\Sigma^{-1}\mathbf{Y}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}$. Is $\hat{\mu}$ unbiased estimator of μ ? Show that $\text{var}(\hat{\mu}) = \frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}$ and simplify it using the inverse of the variance covariance matrix given above. Explain why $\rho > -\frac{1}{n-1}$.

EXERCISE 2

Let Y_1, Y_2, \dots, Y_n independent random variables, and let $Y_i \sim N(i\theta, i\sigma)$, i.e. $E(Y_i) = i\theta$ and $\text{var}(Y_i) = i^2\sigma^2$, for $i = 1, 2, \dots, n$. Find the maximum likelihood estimator of θ . Is this estimator efficient estimator of θ ?

EXERCISE 3

Consider the two samples X_1, \dots, X_n i.i.d. $N(\mu_1, \sigma)$ and Y_1, \dots, Y_m i.i.d. $N(\mu_2, \sigma)$. If $\mu_1 = \mu_2 = \mu$ find the MLEs for μ and σ^2 .

EXERCISE 4

Suppose $Y_i = \beta_1 x_i + \epsilon_i$. The x_i 's are not random and $\epsilon_1, \dots, \epsilon_n$ are independent with $E(\epsilon_i) = 0, \text{var}(\epsilon_i) = \sigma^2$. Assume also that $\epsilon_i \sim N(0, \sigma)$. Find $\hat{\beta}_1$ and $\hat{\sigma}^2$, the maximum likelihood estimates of β_1 and σ^2 . Find the expected value and variance of $\hat{\beta}_1$ and its distribution.

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