

Quiz 5

EXERCISE 1

Let $U \sim N(0, 1)$, $V \sim \chi_n^2$, and U and V are independent. Let $t = \frac{U}{\sqrt{\frac{V}{n}}}$ and $W = V$. Find the joint pdf of t and W and then integrate the joint w.r.t. to W to show that the probability density function of the t distribution with $df = n$ degrees of freedom is

$$f(t) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{\pi n} \Gamma(\frac{n}{2})} \left(1 + \frac{t^2}{n}\right)^{-\frac{n+1}{2}}, \quad -\infty < t < \infty.$$

EXERCISE 2

Suppose Q_1, \dots, Q_k are independent. Let $Q_1 \sim \chi_{p_1}^2(\theta_1), \dots, Q_k \sim \chi_{p_k}^2(\theta_k)$, where p_1, \dots, p_k are the degrees of freedom and $\theta_1, \dots, \theta_k$ are the non-centrality parameters. Find the mean and variance of $Y = Q_1 + \dots + Q_k$.

EXERCISE 3

Let X_1, X_2, \dots, X_n i.i.d. exponential random variables with parameter λ . Is $\frac{1}{\bar{X}}$ unbiased estimator of λ ?

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