**University of California, Los Angeles**
**Department of Statistics**

**Statistics 100B** **Instructor: Nicolas Christou**

## Practice 1

**EXERCISE 1**
Find the distribution of the random variable $X$ for each of the following moment-generating functions:

a. $M_X(t) = \left[\frac{1}{3}e^t + \frac{2}{3}\right]^5$.

b. $M_X(t) = \frac{e^t}{2-e^t}$.

c. $M_X(t) = e^{2(e^t-1)}$.

**EXERCISE 2**
Let $M_X(t) = \frac{1}{6}e^t + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$ be the moment-generating function of a random variable $X$.

a. Find $E(X)$.

b. Find $var(X)$.

c. Find the distribution of $X$.



**EXERCISE 3**
Let $X$ follow the Poisson probability distribution with parameter $\lambda$. Its moment-generating function is $M_X(t) = e^{\lambda(e^t-1)}$.



a. Show that the moment-generating function of $Z = \frac{X-\lambda}{\sqrt{\lambda}}$ is given by:



$$M_Z(t) = e^{-\sqrt{\lambda}t} e^{\lambda(e^{\frac{t}{\sqrt{\lambda}}}-1)}.$$

b. Use the series expansion of

$$e^{\frac{t}{\sqrt{\lambda}}} = 1 + \frac{\frac{t}{\sqrt{\lambda}}}{1!} + \frac{(\frac{t}{\sqrt{\lambda}})^2}{2!} + \frac{(\frac{t}{\sqrt{\lambda}})^3}{3!} + \cdots$$

to show that

$$\lim_{\lambda \to \infty} M_Z(t) = e^{\frac{1}{2}t^2}.$$

In other words, as $\lambda \to \infty$, the ratio $Z = \frac{X-\lambda}{\sqrt{\lambda}}$ converges to the standard normal distribution.

**EXERCISE 4**
Use the result of part (b) of the previous exercise:
In the interest of pollution control an experimenter wants to count the number of bacteria per small volume of water. Let $X$ denote the bacteria count per cubic centimeter of water, and assume that $X$ follows the Poisson distribution with parameter $\lambda = 100$. If the allowable pollution in a water supply is a count of 110 bacteria per cubic centimeter, approximate the probability that $X$ will be at most 110.

**EXERCISE 5**
Let $X_1, X_2, \cdots, X_n$ be i.i.d. random sample from $N(\mu, \sigma)$. Using moment genearating functions show that the sum of these $n$ observations $T = \sum_{i=1}^{n} X_i$ also follows the normal distribution. What is the mean and standard deviation of $T$?

**EXERCISE 6**
Suppose that $X_1, \cdots, X_m$ and $Y_1, \cdots, Y_n$ are two samples, with $X \sim N(\mu_1, \sigma_1)$ and $Y \sim N(\mu_2, \sigma_2)$. The difference between the sample means, $\bar{X} - \bar{Y}$, is then a linear combination of $m + n$ normal random variables.

a. Find $E(\bar{X} - \bar{Y})$.

b. Find $Var(\bar{X} - \bar{Y})$.

c. Use moment generating functions to show that the distribution of $\bar{X} - \bar{Y}$ is normal with mean and variance equal to your answers in (a) and (b).

d. Suppose $\sigma_1^2 = 2, \sigma_2^2 = 2.5$, and $m = n$. Find the sample sizes so that $\bar{X} - \bar{Y}$ will be within one unit of $\mu_1 - \mu_2$ with probability 0.95.

**EXERCISE 7**
If the random variable $X$ follows the normal distribution with $\mu = 0$, $\sigma^2 = 1$ and $Y = e^X$ find the probability density of $Y$. This is called the lognormal distribution.



**EXERCISE 8**
If the radius of a circle $X$ is an exponential random variable with parameter $\lambda$, find the probability density function of its area $Y$.