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Department of Statistics

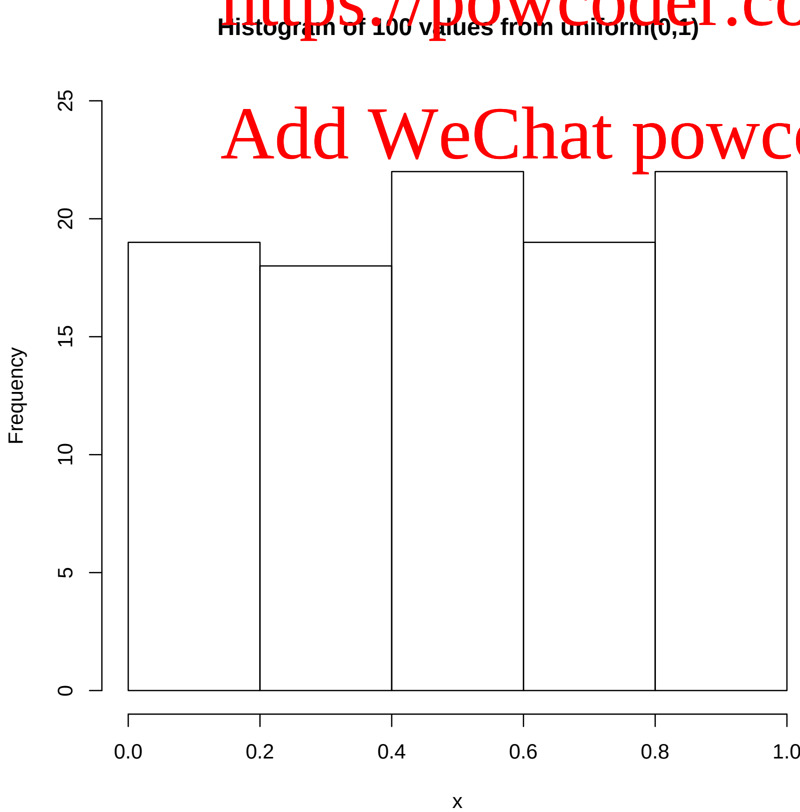
Statistics 100B **Instructor: Nicolas Christou**

Simulating a continuous random variable

Many times computer simulations are used to evaluate proposed statistical techniques. Typically, these simulations require that we obtain observed values of random variables with a prescribed distribution. Most computer systems contain a subroutine that provides observed values of a uniform random variable on the interval $[0, 1]$. Let $X_1, X_2, \cdots, X_n$ be a random sample from this distribution. How can we use these values to generate n observations $Y_1, Y_2, \cdots, Y_n$ from an exponential distribution with parameter λ, $Y \sim exp(\lambda)$? As a reminder the probability density function (pdf) of an exponential random variable Y is $f(y) = \lambda e^{-\lambda y}, y \geq 0$, and the cumulative distribution function (cdf) is $F(y) = 1 - e^{-\lambda y}, y \geq 0$.

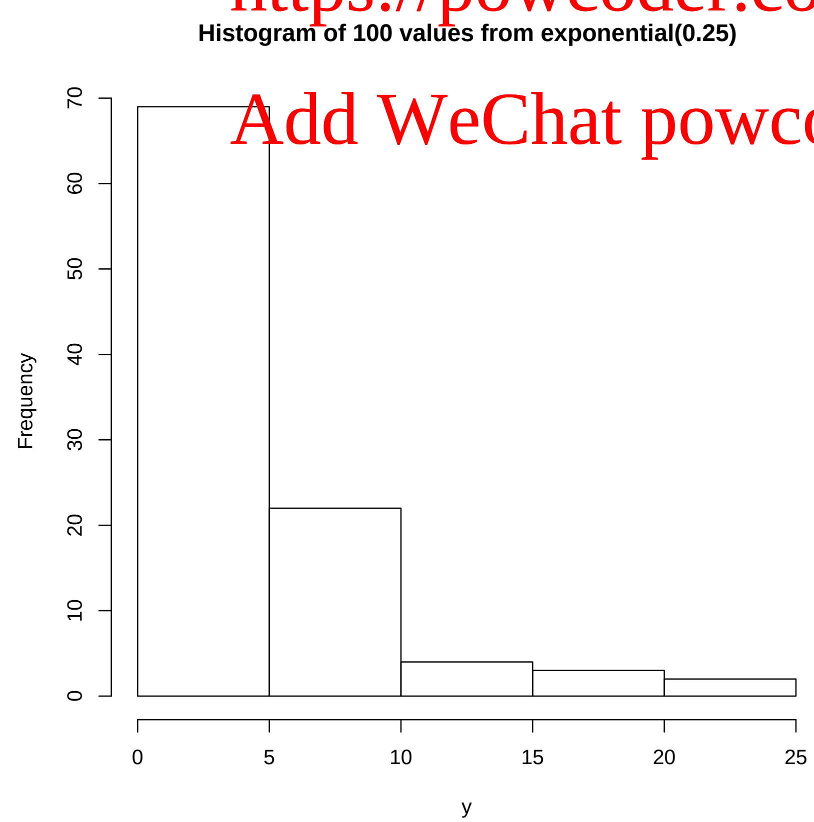
Example:
Suppose we want to generate 100 values form the exponential distribution with parameter $\lambda = 0.25$. We will first generate 100 values from uniform $[0, 1]$. Here they are:

```
> x <- runif(100); x
  [1] 0.051018780 0.633835176 0.316826761 0.428229412 0.907661578 0.993477130
  [7] 0.750858171 0.805013227 0.414591927 0.822393034 0.856314588 0.016146515
 [13] 0.071784463 0.122571166 0.613897717 0.001209994 0.954154581 0.351573293
 [19] 0.410093796 0.007272958 0.603123222 0.747129264 0.626160788 0.477646846
 [25] 0.050995086 0.984879869 0.293646467 0.250682793 0.322653704 0.566582250
 [31] 0.205673748 0.316166543 0.128313377 0.699693979 0.055456475 0.903748008
 [37] 0.357488344 0.197242119 0.997325008 0.915138927 0.384455960 0.254831758
 [43] 0.440502153 0.912334391 0.263937093 0.522613614 0.546005381 0.945161675
 [49] 0.394401442 0.644344347 0.983463021 0.563307032 0.879276463 0.406286849
 [55] 0.758659304 0.125155509 0.535370943 0.770937048 0.567509420 0.787695544
 [61] 0.029144404 0.845371360 0.690853273 0.520675362 0.985960909 0.680727438
 [67] 0.780857383 0.907264206 0.504714281 0.052280051 0.120086492 0.685356938
 [73] 0.660381331 0.422037191 0.509566451 0.300926229 0.726050804 0.455755142
 [79] 0.924619806 0.956001621 0.574201672 0.270501456 0.418710439 0.482915046
 [85] 0.905365532 0.552876442 0.216049740 0.889648968 0.039285457 0.395933149
 [91] 0.383691375 0.079916710 0.880780600 0.717155750 0.074550390 0.032717140
 [97] 0.753688467 0.272438001 0.457411286 0.075257131
```

We transform these values using $y = \frac{log(1-x)}{-\lambda}$. Here are the transformed values:

```
> y=log(1-x)/(-0.25); y
  [1]  0.209465081  4.018686828  1.524027227  2.236069749  9.529179821
  [6] 20.129763167  5.558931804  6.539294215  2.141784458  6.912728913
 [11]  7.760516035  0.065113163  0.297965253  0.523037713  3.806611852
 [16]  0.004842908 12.329919957  1.732825206  2.111166924  0.029198139
 [21]  3.696517720  5.499507380  3.935717955  2.597645516  0.209365209
 [26] 16.766913048  1.390557650  1.154371513  1.558290483  3.344212941
 [31]  0.921044024  1.520163498  0.549301190  4.811813023  0.228214044
 [36]  9.363142438  1.769481284  0.878808512 23.695235960  9.866959171
 [41]  1.940995139  1.176581032  2.322862399  9.736902397  1.225758771
 [46]  2.957716331  3.158679736 11.613463846  2.006151834  4.135169128
 [51] 16.408624959  3.314099683  8.457008650  2.085435946  5.686182672
 [56]  0.534836532  3.066063676  5.895033653  3.352778933  6.198935687
 [61]  0.118310152  7.466915630  4.695757075  2.941508683 17.063638551
 [66]  4.566840454  6.072130168  9.512002999  2.810481894  0.214784931
 [71]  0.511726649  4.625265677  4.319727412  2.192983023  2.849861939
 [76]  1.431996019  5.179250425  2.433424109 10.340842870 12.494409965
 [81]  3.415157814  1.261591625  2.170025053  2.638192391  9.430934038
 [86]  3.219681226  0.973638816  8.816355190  0.160311822  2.016281626
 [91]  1.936029706  0.383164323  8.507159160  5.051185544  0.309902377
 [96]  0.133057252  5.604632603  1.272224248  2.445614717  0.312958237
```

Verify that $y = \frac{log(1-x)}{-\lambda}$ follows indeed the exponential distribution with parameter λ.