University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Beta distribution

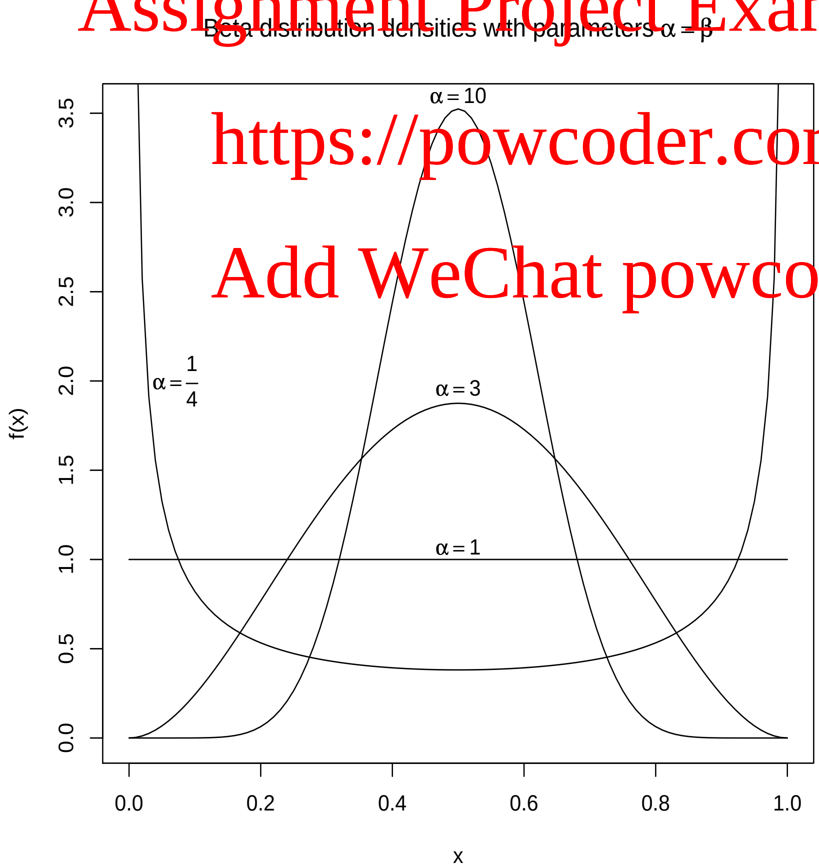
The beta density function is defined over the interval $0 \leq x \leq 1$ and it can be used to model proportions (e.g. the proportion of time a machine is under repair, the proportion of a certain impurity in a chemical product, etc.). The probability density function of the beta distribution is given by:

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, \quad \alpha > 0,\ \beta > 0,\ 0 \leq x \leq 1.$$

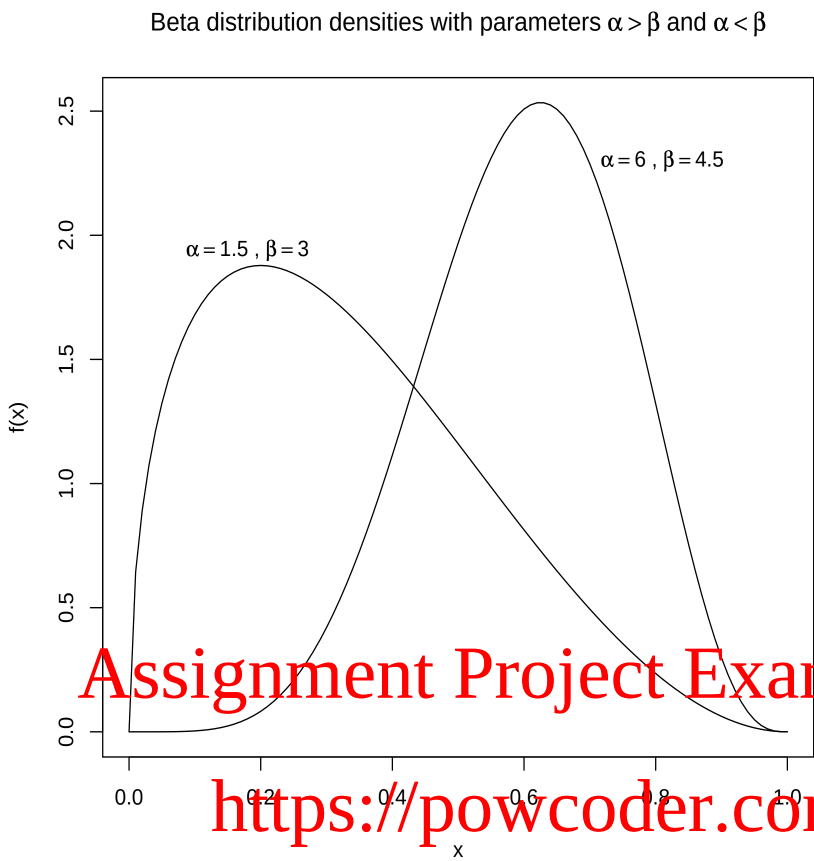
where,

$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx.$$

The shape of the distribution depends on the values of the parameters α and β. When $\alpha = \beta$ the distribution is symmetric about $\frac{1}{2}$ as shown in the figure below:



When $\alpha > \beta$ the distribution is skewed to the left and when $\alpha < \beta$ it skewed to the right (see next figure).



Even though x was defined in the interval $0 \leq x \leq 1$, its use can be extended to random variables defined over some finite interval, $c \leq x \leq d$. In this case we can simply rescale the variable using $y = \frac{x-c}{d-c}$, and y will be between 0 and 1.

It can be shown that

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}.$$

Using this relation between the beta and gamma functions we can find the mean and variance of the beta distribution:

$$E(X) = \frac{\alpha}{\alpha + \beta}$$

and

$$var(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}.$$