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Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Quiz 8

Answer the following questions:

a. Suppose $X_1, \ldots, X_n$ are i.i.d. Poisson(λ). Let $S = \sum_{i=1}^n X_i$. Show that $(X_1 = x_1, \ldots, X_n = x_n)$ conditioned on $S = N$ follows the multinomial distribution with parameters S and $(\frac{1}{n}, \ldots, \frac{1}{n})$.
Hint 1: Find the joint pmf of $X_i's$. Given that $S = N$, $\sum_i X_i = N$. Use this result in the joint pmf of the $X_i's$.
Hint 2: Continue by expressing the conditional pmf as the ratio of the joint and the marginal pmf's.
Note on the multinomial probability distribution:
A sequence of n independent experiments is performed and each experiment can result in one of r possible outcomes with probabilities $p_1, p_2, \ldots, p_r$ with $\sum_{i=1}^r p_i = 1$. Let X_i be the number of the n experiments that result in outcome i, $i = 1, 2, \ldots, r$. Then, $P(X_1 = x_1, X_2 = x_2, \ldots, X_r = x_r) = \frac{n!}{n_1! n_2! \cdots n_r!} p_1^{x_1} p_2^{x_2} \cdots p_r^{x_r}$.

b. Refer to question (a). Suppose we know that $E[X(X-1)] = \lambda^2$ and $\text{var}[X(X-1)] = 2\lambda^2 + 4\lambda^3$. Please explain how you would verify these two results. Let $T_1 = \frac{1}{n} \sum_{i=1}^n X_i(X_i - 1)$. Show that T_1 is unbiased estimator for λ^2 and compute its variance.

c. Refer to question (a). Let $T_2 = E[T_1|S]$. Show that $T_2 = \frac{S(S-1)}{n^2}$. Note that $E[X_i|S] = S\frac{1}{n}$ and $\text{var}(X_i|S) = S\frac{1}{n}\left(1 - \frac{1}{n}\right)$. Is T_2 unbiased estimator of λ^2? Find $\text{var}(T_2)$.

d. Let $X_1, \ldots, X_n$ be i.i.d. random variables with $f(x) = \frac{\alpha x^{\alpha-1}}{\theta^\alpha}, \alpha > 0, \theta > 0, 0 \leq x \leq \theta$. Assume that θ is known. Use the factorization theorem to show that $\Pi_{i=1}^n X_i$ is a sufficient statistics for α.