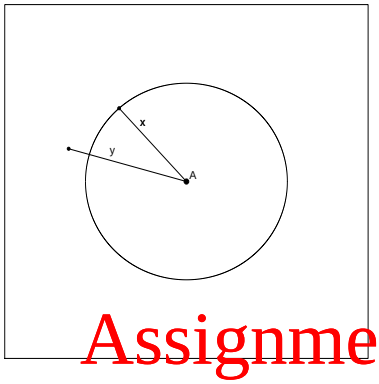
University of California, Los Angeles
Department of Statistics

Statistics 100B **Instructor: Nicolas Christou**

Practice problem - week 3

Answer the following questions:

a. Suppose the number of pine trees in a certain forest follows the Poisson distribution with parameter λ per meter^2. Suppose we randomly select a point (say A) in this forest (not a pine tree, just a point). Let X be the distance from this point to the nearest pine tree and let Y be the distance from this point to the second nearest pine tree (see graph below). Find the probability density function of X and then show that the random variable $\lambda\pi X^2$ follows the exponential distribution with mean 1. Note: The parameter λ here is given per meter^2. The parameter λ of a circle with radius r is $\lambda\pi r^2$.



b. Refer to question (a). Find the probability density function of Y. (x is fixed when we are considering the pdf of Y.) Show that the random variable $\lambda\pi(Y^2 - X^2)$ follows the exponential distribution with mean 1.

c. Suppose now we randomly select m points in this forest. Find the distribution of $2\lambda\pi\sum_{i=1}^{m} X_i^2$ and the distribution of $2\lambda\pi\sum_{i=1}^{m}(Y_i^2 - X_i^2)$.

d. Let $s = \lambda\pi\sum_{i=1}^{m} X_i^2$ and $t = \lambda\pi\sum_{i=1}^{m}(Y_i^2 - X_i^2)$. If s and t are independent show that $\frac{\sum_{i=1}^{m} X_i^2}{\sum_{i=1}^{m} Y_i^2} \sim \text{beta}(m, m)$.