

1. [20 points] PICK ONE *No justification is needed for this problem.*

(a) Which one of the following is incorrect for Two Dimensional Range Trees?

- i. Counting range queries take  $O(\log^2(n))$  time.
- ii. Reporting range queries take  $O(\log^2(n) + k)$  time.
- iii. Construction time is  $O(n \log n)$ .
- iv. Space complexity is  $\Theta(n^2 \log n)$ .

(b) CLIQUE-DEGREE-4 problem: Given a graph  $G = (V, E)$  where all vertices have degree at most 4, and an integer  $k$ , determine whether  $V$  has a subset  $S$  of size at least  $k$  that forms a clique in  $G$ . Which of the following statement(s) is/are correct?

(a) CLIQUE-DEGREE-4 is NP-Hard; (b) CLIQUE-DEGREE-4 is in P; (c) Approximating CLIQUE-DEGREE-4 is NP-Hard; (d) There exists a polynomial time reduction from CLIQUE-DEGREE-4 to VERTEXCOVER problem.

- i. (a) and (d)
- ii. (b) and (d)
- iii. (a) and (c)
- iv. (b) and (c)
- v. All of them

(c) MAJORITY-CHECK problem: Given a set  $S$  of  $n$  real numbers, determine whether more than half the numbers in  $S$  are exactly the same. Assuming two numbers can be compared in constant time, which of the following statement(s) is/are correct?

(a) MAJORITY-CHECK is in NP; (b) There exists no comparison-based algorithm solving MAJORITYCHECK in  $\Theta(n)$  time; (c) MAJORITY-CHECK is proven to be NP-hard via a reduction from SUBSETSUM problem.

- i. only (a)
- ii. (a) and (b)
- iii. (a) and (c)
- iv. All of them
- v. None of them

(d) MAX-SAT: Given a CNF (Conjunctive Normal Form) Boolean formula and a positive integer  $k$ , does there exist a truth assignment that satisfies at least  $k$  clauses.

Which one of the following is correct about this problem?

- i. This problem can be solved in polynomial time since  $k$  is a small number.
- ii. This problem is NP-hard since it is a special case of the SAT problem.
- iii. This problem is NP-hard since it is a generalization of the SAT problem.
- iv. This problem can be shown to be NP-hard by giving a reduction from MAX-SAT to CLIQUE problem.
- v. None of the above

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2. [30 points] GEOMETRIC ALGORITHMS - CO-CIRCULARITY CHECK

Following procedures each with planar input points and constant running time are provided:  
 $\text{CCW}(p, q, r)$  decides if  $r$  is `LEFTOF`, `ON`, or `RIGHTOF` the oriented line going through  $p, q$ .  
 $\text{INCIRCLE}(p, q, r, s)$  decides whether  $s$  is `INSIDE`, `ON`, or `OUTSIDE` the unique circle going through  $p, q, r$ .

- (a) **Draw** five points  $a, b, c, d, e$  such that the radii of the circles going through triples of points  $(a, b, c)$ ,  $(a, b, d)$ , and  $(a, b, e)$  are all the same,  $a, b, c, d$  are co-circular, and yet  $a, b, c, e$  are not co-circular, i.e.,  $\text{circumradius}(a, b, c) = \text{circumradius}(a, b, d) = \text{circumradius}(a, b, e)$ ,  $\text{INCIRCLE}(a, b, c, d) = \text{ON}$  and  $\text{INCIRCLE}(a, b, c, e) \neq \text{ON}$ .

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- (b) Given  $n$  points in the plane (no three of which are co-linear) we want to determine whether any four of them are co-circular. Naïve algorithm would take  $O(n^4)$  time.  
**Design** and **analyze** an algorithm that solves this problem in  $O(n^3 \log n)$  time.  
(Hints: Sorting takes  $O(n \log n)$  time. Recall the co-linearity checking algorithm.)

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3. [30 points] NP-COMPLETENESS - STRONGLY INDEPENDENT SETS

Consider a simple graph  $G = (V, E)$ . A subset  $S_1 \subseteq V$  is called an *independent set*, if no two vertices in  $S_1$  have an edge (path of length one) between them. A subset  $S_2 \subseteq V$  is called a *strongly independent set*, if no two vertices in  $S_2$  have a path of length one or two between them, i.e.,  $\forall u, v \in S_2$ , we have  $(u, v) \notin E$ , and  $\nexists w \in V$  such that  $(u, w) \in E$  and  $(w, v) \in E$ .  
INDEPENDENTSET PROBLEM: Given a simple graph  $G$  and an integer  $k$ , determine if  $G$  contains an independent set of size at least  $k$ .

STRONGLYINDEPENDENTSET PROBLEM: Given a simple graph  $G$  and an integer  $k$ , determine if  $G$  contains a strongly independent set of size at least  $k$ .

- (a) **Draw** a connected graph that has an independent set of size 4. **Draw** another connected graph that has a strongly independent set of size 4.

- (b) **Prove** that STRONGLYINDEPENDENTSET PROBLEM is NP-Complete. (*Hints: INDEPENDENTSET PROBLEM is NP-hard. For your reduction, consider additional vertices positioned strategically and forming a clique among themselves.*)

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4. [30 points] APPROXIMATION - PACK YOUR BOOKS, YOU GOT A JOB AT GOOGLE

Imagine that you just graduated UF and got a job at Google. For your move to California, you need to pack all your books to boxes each of which has capacity a real number between 1 and 2. For simplicity, assume that each book has weight a real number between 0 and 1. You want to minimize the number of boxes that you use. More formally:

**PACKBOOKS PROBLEM:** Given  $n$  books with weights  $w_1, \dots, w_n$  where  $\forall i, 0 < w_i < 1$ ,  $m$  boxes with capacities  $c_1, \dots, c_m$  where  $\forall j, 1 < c_j < 2$ , and an integer  $k$ , is it possible to pack all books in at most  $k$  boxes such that total weight in each box is within its capacity.

- (a) **Prove** that PACKBOOKS PROBLEM is NP-hard. (*Hints: Consider a special case of this problem that you are familiar with. No reduction necessary.*)

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- (b) **Design** and **analyze** an approximation algorithm for the minimization version of the PACKBOOKS PROBLEM with an approximation ratio at most 4.

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