

1. [20 points] PICK ONE No justification is needed for this problem.
- (a) Which one of the following is incorrect for Two Dimensional Range Trees?
- Counting range queries take $O(\log^2(n))$ time.
 - Reporting range queries take $O(\log^2(n) + k)$ time.
 - Construction time is $O(n \log n)$.
 - Space complexity is $\Theta(n^2 \log n)$.
- (b) CLIQUE-DEGREE-4 problem: Given a graph $G = (V, E)$ where all vertices have degree at most 4, and an integer k , determine whether V has a subset S of size at least k that forms a clique in G . Which of the following statement(s) is/are correct?
- (a) CLIQUE-DEGREE-4 is NP-Hard; (b) CLIQUE-DEGREE-4 is in P; (c) Approximating CLIQUE-DEGREE-4 is NP-Hard; (d) There exists a polynomial time reduction from CLIQUE-DEGREE-4 to VERTEXCOVER problem.
- (a) and (d)
 - (b) and (d)
 - (a) and (c)
 - (b) and (c)
 - All of them
- (c) MAJORITY-CHECK problem: Given a set S of n real numbers, determine whether more than half the numbers in S are exactly the same. Assuming two numbers can be compared in constant time, which of the following statement(s) is/are correct?
- (a) MAJORITY-CHECK is in NP; (b) There exists no comparison-based algorithm solving MAJORITYCHECK in $\Theta(n)$ time; (c) MAJORITY-CHECK is proven to be NP-hard via a reduction from SUBSETSUM problem.
- only (a)
 - (a) and (b)
 - (a) and (c)
 - All of them
 - None of them
- (d) MAX-SAT: Given a CNF (Conjunctive Normal Form) Boolean formula and a positive integer k , does there exist a truth assignment that satisfies at least k clauses. Which one of the following is correct about this problem?
- This problem can be solved in polynomial time since k is a small number.
 - This problem is NP-hard since it is a special case of the SAT problem.
 - This problem is NP-hard since it is a generalization of the SAT problem.
 - This problem can be shown to be NP-hard by giving a reduction from MAX-SAT to CLIQUE problem.
 - None of the above

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2. [30 points] GEOMETRIC ALGORITHMS - Co-CIRCULARITY CHECK

Following procedures each with planar input points and constant running time are provided:

CCW(p, q, r) decides if r is LEFTOF, ON, or RIGHTOF the oriented line going through p, q .

INCIRCLE(p, q, r, s) decides whether s is INSIDE, ON, or OUTSIDE the unique circle going through p, q, r .

- (a) **Draw** five points a, b, c, d, e such that the radii of the circles going through triples of points (a, b, c) , (a, b, d) , and (a, b, e) are all the same, a, b, c, d are co-circular, and yet a, b, c, e are not co-circular, i.e., $\text{circumradius}(a, b, c) = \text{circumradius}(a, b, d) = \text{circumradius}(a, b, e)$, $\text{INCIRCLE}(c, b, a, d) = \text{ON}$ and $\text{INCIRCLE}(a, b, c, e) \neq \text{ON}$.

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- (b) Given n points in the plane (no three of which are co-linear) we want to determine whether any four of them are co-circular. A naive algorithm would take $O(n^4)$ time.

Design and **analyze** an algorithm that solves this problem in $O(n^3 \log n)$ time.

(*Hints: Sorting takes $O(n \log n)$ time. Recall the co-linearity checking algorithm.*)

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3. [30 points] NP-COMPLETENESS - STRONGLY INDEPENDENT SETS

Consider a simple graph $G = (V, E)$. A subset $S_1 \subseteq V$ is called an *independent set*, if no two vertices in S_1 have an edge (path of length one) between them. A subset $S_2 \subseteq V$ is called a *strongly independent set*, if no two vertices in S_2 have a path of length one or two between them, i.e., $\forall u, v \in S_2$, we have $(u, v) \notin E$, and $\nexists w \in V$ such that $(u, w) \in E$ and $(w, v) \in E$.

INDEPENDENTSET PROBLEM: Given a simple graph G and an integer k , determine if G contains an independent set of size at least k .

STRONGLYINDEPENDENTSET PROBLEM: Given a simple graph G and an integer k , determine if G contains a strongly independent set of size at least k .

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- (a) **Draw** a connected graph that has an independent set of size 4. **Draw** another connected graph that has a strongly independent set of size 4.

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- (b) **Prove** that STRONGLYINDEPENDENTSET PROBLEM is NP-Complete. (*Hints: INDEPENDENTSET PROBLEM is NP-hard. For your reduction, consider additional vertices positioned strategically and forming a clique among themselves.*)

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4. [30 points] APPROXIMATION - PACK YOUR BOOKS, YOU GOT A JOB AT GOOGLE

Imagine that you just graduated UF and got a job at Google. For your move to California, you need to pack all your books to boxes each of which has capacity a real number between 1 and 2. For simplicity, assume that each book has weight a real number between 0 and 1. You want to minimize the number of boxes that you use. More formally:

PACKBOOKS PROBLEM: Given n books with weights w_1, \dots, w_n where $\forall i, 0 < w_i < 1$, m boxes with capacities c_1, \dots, c_m where $\forall j, 1 < c_j < 2$, and an integer k , is it possible to pack all books in at most k boxes such that total weight in each box is within its capacity.

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- (a) **Prove** that PACKBOOKS PROBLEM is NP-hard. (*Hints: Consider a special case of this problem that you are familiar with. No reduction necessary.*)

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- (b) **Design** and **analyze** an approximation algorithm for the minimization version of the PACKBOOKS PROBLEM with an approximation ratio at most 4

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