1. [20 points] TRUE/FALSE. *No need for justification.*

(a) TRUE/FALSE
For an undirected, connected graph G with distinct edge weights, the minimum spanning tree of G includes the minimum-weight edge in *every* cycle in G.



(b) TRUE/FALSE
(Let $G = (V, E)$ be an undirected connected graph with distinct edge weights.)
For every vertex $v \in V$, the edge with the smallest weight incident to $v$ must be an edge in the minimum spanning tree of G.



(c) TRUE/FALSE
(Consider an undirected, connected graph $G = (V, E)$ with distinct edge weights. The second smallest spanning tree of a given graph G, is defined as a/the spanning tree of G with the smallest total weight except for the minimum spanning tree.)
The second smallest spanning tree of G is unique.



(d) TRUE/FALSE
If all edge capacities in a flow network are integer multiples of 35, then the value of the maximum flow must be a multiple of both 5 and 7.

(e) TRUE/FALSE
(Let $G = (V, E)$ be a directed graph with nonnegative weights on edges, and $\gamma(p, q)$ denote the length of the *longest simple path* between p and q.)
The triangle inequality $\gamma(p, q) + \gamma(q, r) \leq \gamma(p, r)$ holds for every p, q, and r in V.

2. [30 points] Minimum Spanning Tree Update

Consider an undirected, connected graph $G = (V, E)$ with edge weights $w : E \to \mathbb{Z}^+$, and a minimum spanning tree $T = (V, E')$ of $G$, both given as adjacency lists. Consider the following updates on $G$. For each case, decide whether an update might be necessary, and if so, describe and analyze an efficient algorithm for updating the minimum spanning tree.



(a) The weight of a particular edge $e \in E - E'$ is increased to $\hat{w}(e) > w(e)$.

(b) The weight of a particular edge $e \in E - E'$ is decreased to $\hat{w}(e) < w(e)$.

(c) The weight of a particular edge $e \in E'$ is decreased to $\hat{w}(e) < w(e)$.



(d) The weight of a particular edge $e \in E'$ is increased to $\hat{w}(e) > w(e)$.

(e) A new edge $e = (u, v) \notin E$ is added to $E$ with weight $\hat{w}(e)$.

https://powcoder.com

Assignment Project Exam Help

Add WeChat powcoder

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

3. [30 points] An Alternative Algorithm for All Pairs Shortest Path Problem

Let $G = (V, E)$ be a directed graph with $n$ vertices and weighted (-, 0, or +) edges.

(a) How could we delete an arbitrary vertex $v$ from this graph, without changing the shortest-path distance between any other pair of vertices? Describe and analyze an algorithm that constructs a directed graph $G' = (V', E')$ with weighted edges, where $V' = V - \{v\}$, and the shortest-path distance between any two nodes in $G'$ is equal to the shortest-path distance between the same two nodes in $G$, in $O(n^2)$ time.

(b) Suppose we have already computed all pairs shortest-path distances in $G'$. Describe and analyze an algorithm to compute the shortest-path distances from $v$ to every other vertex, and from every other vertex to $v$, in the original graph $G$, in $O(n^2)$ time.

(c) Describe and analyze a new all-pairs shortest path algorithm that runs in $O(n^3)$ time by combining parts (a) and (b).

https://powcoder.com

Assignment Project Exam Help

Add WeChat powcoder

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

4. [30 points] FLOW NETWORKS

Consider the flow network $G = (V, E)$, where $V = \{s, a, b, c, d, e, f, g, t\}$, $s$ is the source, $t$ is the sink, and the edge set with capacities is $E = \{\ ((s,a),3),\ ((s,b),6),\ ((a,c),4),\ ((a,d),2),\ ((b,d),3),\ ((b,e),5),\ ((c,f),1),\ ((d,f),6),\ ((d,g),7),\ ((e,g),2),\ ((f,t),8),\ ((g,t),5)\ \}$.

(a) Draw this flow network $G$ and find a minimum cut on it.

(b) Give a maximum flow function $f : E \to \mathbb{R}$ on $G$ matching the minimum cut.

(c) Is the maximum flow function $f$ on $G$ unique? Justify.

(d) Prove or disprove the claim: The maximum flow function on a flow network is unique if and only if the minimum cut on it is unique.

(e) Draw the residual graph for flow $f$ that you built in part (b).

(f) Describe and analyze an efficient algorithm to determine whether a given flow network has a unique maximum flow. [*Hint: First give a characterization on the residual graph.*]