Assignment Project Exam Help

Algorithms Week 4

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The basic strategy of backtracking is to *systematically* explore the space of all potential solutions. It does so by:

- extending a partial solution in some feasible way

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The basic strategy of backtracking is to *systematically* explore the space of all potential solutions. It does so by:

- extending a partial solution in some feasible way,
- trying another extension if the extension fails,

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The basic strategy of backtracking is to *systematically* explore the space of all potential solutions. It does so by:

- extending a partial solution in some feasible way,
- trying another extension if the extension fails,
- backing up to a smaller partial solution when the options for extending a partial solution are exhausted.

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Input: A sequence of characters stored in $A[1..n]$.

Output: True if A can be segmented into a sequence of words, False otherwise.

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Input: A sequence of characters stored in $A[1..n]$.

Output: True if A can be segmented into a sequence of words, False otherwise.

Given: Function $IsWord(w)$ that returns *True* if sequence of characters w is a word, *False* otherwise.

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Input: A sequence of characters stored in $A[1..n]$.

Output: True if A can be segmented into a sequence of words, False otherwise.

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Given: Function $IsWord(w)$ that returns *True* if sequence of characters w is a word, *False* otherwise.

Example: If sequence A consists of characters

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BOTHEARTHANDSATURNSPIN

and $IsWord(w)$ is True if w is a word in English, then:

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Example: If sequence A consists of characters

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BOTHEARTHANDSATURNSPIN

and *IsWord*(w) is True if w is a word in English, then:

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- Output *True* because BOTH·EARTH·AND·SATURN·SPIN is a valid segmentation of A

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Input: A sequence of characters stored in $A[1..n]$.

Output: True if A can be segmented into a sequence of words, False otherwise.

Given: Function *IsWord(w)* that returns *True* if sequence of characters w is a word, *False* otherwise.

Example: If sequence A consists of characters

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BOTHEARTHANDSATURNSPIN

and *IsWord(w)* is True if w is a word in English, then:

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- Output *True* because BOTH·EARTH·AND·SATURN·SPIN is a valid segmentation of A
- By the way, BOT·HEART·HANDS·AT·URNS·PIN is another valid segmentation of A.

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We consider a process that consumes the input characters in order from left to right

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We consider a process that consumes the input characters in order from left to right and produces the output words in order from left to right.

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We consider a process that consumes the input characters in order from left to right and produces the output words in order from left to right.

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So, at any point in time of the process we might have the following situation:

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BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

BLUE	STEM	UNIT	ROBOT	HEARTHANDSATURNSPIN

We need to decide on the next word.

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BLUE	STEM	UNIT	ROBOT	HEARTHANDSATURNSPIN

We need to decide on the next word. Options:

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BLUE	STEM	UNIT	ROBOT	HE	ARTHANDSATURNSPIN

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Backtracking algorithm idea

BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

We need to decide on the next word. Options:

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HE**	ARTHANDSATURNSPIN

BLUE	**STEM**	**UNIT**	**ROBOT**	**HEAR**	THANDSATURNSPIN

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Backtracking algorithm idea

BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

We need to decide on the next word. Options:

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HE**	ARTHANDSATURNSPIN

BLUE	**STEM**	**UNIT**	**ROBOT**	**HEAR**	THANDSATURNSPIN

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HEART**	HANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HEARTHANDSATURNSPIN

We need to decide on the next word. Options:

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BLUE	STEM	UNIT	ROBOT	HE	ARTHANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HEAR	THANDSATURNSPIN

BLUE	STEM	UNIT	ROBOT	HEART	HANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HEARTH	ANDSATURNSPIN

Backtracking algorithm idea

BLUE | STEM | UNIT | ROBOT | HEARTHANDSATURNSPIN

We need to decide on the next word. Options:

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BLUE | STEM | UNIT | ROBOT | HE | ARTHANDSATURNSPIN

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BLUE | STEM | UNIT | ROBOT | HEAR | THANDSATURNSPIN

BLUE | STEM | UNIT | ROBOT | HEART | HANDSATURNSPIN

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BLUE | STEM | UNIT | ROBOT | HEARTH | ANDSATURNSPIN

Which option do we choose?

Backtracking algorithm idea

BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

We need to decide on the next word. Options:

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HE**	ARTHANDSATURNSPIN

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HEAR**	THANDSATURNSPIN

BLUE	**STEM**	**UNIT**	**ROBOT**	**HEART**	HANDSATURNSPIN

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HEARTH**	ANDSATURNSPIN

Which option do we choose? We try all of them, using backtracking and letting the recursion fairy do the work.

BLUE	STEM	UNIT	ROBOT	HEARTHANDSATURNSPIN

We need to decide on the next word. Options:

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BLUE	STEM	UNIT	ROBOT	HE	ARTHANDSATURNSPIN
BLUE	STEM	UNIT	ROBOT	HEAR	THANDSATURNSPIN
BLUE	STEM	UNIT	ROBOT	HEART	HANDSATURNSPIN
BLUE	STEM	UNIT	ROBOT	HEARTH	ANDSATURNSPIN

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Which option do we choose? We try all of them, using backtracking and letting the recursion fairy do the work.

If any of the recursive calls returns True, we return True.

Backtracking algorithm idea

BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

We need to decide on the next word. Options:

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HE**	ARTHANDSATURNSPIN

BLUE	**STEM**	**UNIT**	**ROBOT**	**HEAR**	THANDSATURNSPIN

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HEART**	HANDSATURNSPIN

BLUE	**STEM**	**UNIT**	**ROBOT**	**HEARTH**	ANDSATURNSPIN

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Which option do we choose? We try all of them, using backtracking and letting the recursion fairy do the work.

If any of the recursive calls returns True, we return True. If none do, we return False.

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We start with:

BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

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Backtracking algorithm strategy

We start with:

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BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

We tentively accept HE as the next word and let the recursion fairy make the rest of the decisions:

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HE**	ARTHANDSATURNSPIN

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We start with:

BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HE**	ARTHANDSATURNSPIN

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We then tentively accept HEAR as the next word and let the recursion fairy make the rest of the decisions:

BLUE	**STEM**	**UNIT**	**ROBOT**	**HEAR**	THANDSATURNSPIN

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We start with:

BLUE	STEM	UNIT	ROBOT	HEARTHANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HE	ARTHANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HEAR	THANDSATURNSPIN

We then tentively accept HEART as the next word and let the recursion fairy make the rest of the decisions:

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BLUE	STEM	UNIT	ROBOT	HEART	HANDSATURNSPIN

We start with:

BLUE	STEM	UNIT	ROBOT	HEARTHANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HE	ARTHANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HEAR	THANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HEART	HANDSATURNSPIN

We then tentively accept HEARTH as the next word and let the recursion fairy make the rest of the decisions:

BLUE	STEM	UNIT	ROBOT	HEARTH	ANDSATURNSPIN

We start with:

BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HE**	ARTHANDSATURNSPIN

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HEAR**	THANDSATURNSPIN

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BLUE	**STEM**	**UNIT**	**ROBOT**	**HEART**	HANDSATURNSPIN

BLUE	**STEM**	**UNIT**	**ROBOT**	**HEARTH**	ANDSATURNSPIN

If the recursion fairy reports success on any of these, we report success.

We start with:

BLUE	STEM	UNIT	ROBOT	HEARTHANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HE	ARTHANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HEAR	THANDSATURNSPIN

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BLUE	STEM	UNIT	ROBOT	HEART	HANDSATURNSPIN

BLUE	STEM	UNIT	ROBOT	HEARTH	ANDSATURNSPIN

If, on the other hand, the recursion fairy does not report success then we report failure.

Past decisions do not affect the choices we have available now:

BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

BLUEST	**EMU**	**NITRO**	**BOT**	HEARTHANDSATURNSPIN

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Past decisions do not affect the choices we have available now:

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BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

BLUEST	**EMU**	**NITRO**	**BOT**	HEARTHANDSATURNSPIN

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So the recursive process can be simplified by ignoring the past:

HEARTHANDSATURNSPIN

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Backtracking algorithm strategy, simplified

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Past decisions do not affect the choices we have available now:

BLUE	**STEM**	**UNIT**	**ROBOT**	HEARTHANDSATURNSPIN

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BLUEST	**EMU**	**NITRO**	**BOT**	HEARTHANDSATURNSPIN

So the recursive process can be simplified by ignoring the past:

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HEARTHANDSATURNSPIN

Backtracking stategy: Select the first output word, and recursively segment the rest of the input string.

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A recursive algorithm needs a base case.

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A recursive algorithm needs a base case.

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We stop when the input sequence is of length 0.

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A recursive algorithm needs a base case.

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We stop when the input sequence is of length 0. What is returned then? Success or Failure?

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```
Splittable(A[1 .. n]):
  if n = 0
    return True
  for i ← 1 to n
    if IsWord(A[1 .. i]) and Splittable(A[i + 1 .. n])
        return True
  return False
```

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Passing arrays as input parameters to recursive functions is inefficient.

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Passing arrays as input parameters to recursive functions is inefficient.

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It is better to treat the original input array as a global variable and reformulate the problem and the algorithm in terms of array indices.

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Passing arrays as input parameters to recursive functions is inefficient.

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It is better to treat the original input array as a global variable and reformulate the problem and the algorithm in terms of array indices.

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For the string segmentation problem, the argument of any recursive call is always a suffix $A[i..n]$ of the original input array $A[1..n]$.

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Passing arrays as input parameters to recursive functions is inefficient.

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For the string segmentation problem, the argument of any recursive call is always a suffix $A[i..n]$ of the original input array $A[1..n]$.

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So if we treat the input array $A[1..n]$ as a global variable, we can reformulate our recursive problem as follows:

Passing arrays as input parameters to recursive functions is inefficient.

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It is better to treat the original input array as a global variable and reformulate the problem and the algorithm in terms of array indices.

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So if we treat the input array $A[1..n]$ as a global variable, we can reformulate our recursive problem as follows:

Given an index i, find a segmentation of the suffix $A[i..n]$.

Passing arrays as input parameters to recursive functions is inefficient.

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It is better to treat the original input array as a global variable and reformulate the problem and the algorithm in terms of array indices.

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For the string segmentation problem, the argument of any recursive call is always a suffix $A[i..n]$ of the original input array $A[1..n]$.

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So if we treat the input array $A[1..n]$ as a global variable, we can reformulate our recursive problem as follows:

Given an index i, find a segmentation of the suffix $A[i..n]$.

Note that function *IsWord(i, j)* now takes two indices of array A.

Segmentation backtracking algorithm, v2.0

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```
// Is the suffix A[i..n] Splittable?
Splittable(i):
  if i > n
    return True
  for j ← i to n
    if IsWord(i, j) and Splittable(j + 1)
        return True
  return False
```

For any sequence S,

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2	6	1	4	2	4	2	9	5	3	5	7	8	3

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For any sequence S, a subsequence of S is another sequence obtained from S by deleting zero or more elements, without changing the order of the remaining elements.

2	**6**	**1**	4	2	**4**	2	**9**	5	3	**5**	**7**	8	3

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For any sequence S, a subsequence of S is another sequence obtained from S by deleting zero or more elements, without changing the order of the remaining elements.

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2	6	**1**	4	**2**	**4**	2	9	**5**	3	5	**7**	**8**	3

Problem: Given a sequence of integers S find the Longest Increasing Subsequence (LIS) of S.

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For any sequence S, a subsequence of S is another sequence obtained from S by deleting zero or more elements, without changing the order of the remaining elements.

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2	6	**1**	4	**2**	**4**	2	9	**5**	3	5	**7**	**8**	3

Problem: Given a sequence of integers S find the Longest Increasing Subsequence (LIS) of S.

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Input: Integer array $A[1..n]$.

Output: Longest possible sequence of indices $1 \leq i_1 < i_2 < \cdots < i_l \leq n$ such that $A[i_1] < A[i_2] < \cdots < A[i_l]$.

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Example: See above.

We use the same approach as we used for Subset Sum:

We decide, for each index j in order from left to right, whether or not to include $A[j]$ in the subsequence.

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We use the same approach as we used for Subset Sum:

We decide, for each index j in order from left to right, whether or not to include $A[j]$ in the subsequence.

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2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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We use the same approach as we used for Subset Sum:

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2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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Do we include $A[7] = 2$?

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We use the same approach as we used for Subset Sum:

We decide, for each index j in order from left to right, whether or not to include $A[j]$ in the subsequence.

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2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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Do we include $A[7] = 2$? No, we can't.

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We use the same approach as we used for Subset Sum:

We decide, for each index j in order from left to right, whether or not to include $A[j]$ in the subsequence

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2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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Do we include $A[8] = 9$?

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We use the same approach as we used for Subset Sum:

We decide, for each index j in order from left to right, whether or not to include $A[j]$ in the subsequence.

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2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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Do we include $A[8] = 9$? We can, but should we?

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Backtracking approach

We use the same approach as we used for Subset Sum:

We decide, for each index j in order from left to right, whether or not to include $A[j]$ in the subsequence

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2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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Do we include $A[8] = 9$? We can, but should we?

Instead of trying to be smart, we apply the backtracking strategy:

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We use the same approach as we used for Subset Sum:

We decide, for each index j in order from left to right, whether or not to include $A[j]$ in the subsequence

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2	6	**1**	4	**2**	**4**	2	**9**	5	3	5	7	8	3

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Do we include $A[8] = 9$? We can, but should we?

Instead of trying to be smart, we apply the backtracking strategy:

- We first tentatively include 9, and let the recursion fairy make the rest of the decisions.

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We use the same approach as we used for Subset Sum:

We decide, for each index j in order from left to right, whether or not to include $A[j]$ in the subsequence

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2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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Do we include $A[8] = 9$? We can, but should we?

Instead of trying to be smart, we apply the backtracking strategy:

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- We first tentatively include 9, and let the recursion fairy make the rest of the decisions.
- We then tentatively exclude 9, and let the recursion fairy make the rest of the decisions.

We use the same approach as we used for Subset Sum:

We decide, for each index j in order from left to right, whether or not to include $A[j]$ in the subsequence.

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2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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Do we include $A[8] = 9$? We can, but should we?

Instead of trying to be smart, we apply the backtracking strategy:

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- We first tentatively include 9, and let the recursion fairy make the rest of the decisions.
- We then tentatively exclude 9, and let the recursion fairy make the rest of the decisions.

Whichever choice leads to a longer increasing subsequence is the right one.

How do previous decisions affect future choices?

2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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Past decisions?

How do do previous decisions affect future choices?

2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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In other words, what do we need to remember about our past decisions?

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How do previous decisions affect future choices?

2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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In other words, what do we need to remember about our past decisions?

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The only information we need about the past is the last number selected so far:

4	2	9	5	3	5	7	8	3

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How do previous decisions affect future choices?

2	6	**1**	4	**2**	**4**	2	9	5	3	5	7	8	3

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In other words, what do we need to remember about our past decisions?

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The only information we need about the past is the last number selected so far:

4	2	9	5	3	5	7	8	3

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The problem can be restated as:

Given an integer *prev* and array $A[1..n]$, find the Longest Increasing Subsequence of A in which every element is larger than *prev*.

This function returns the length of the Longest Increasing Subsequence of $A[1..n]$:

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```
LISbigger(prev, A[1 .. n]):
  if n = 0
    return 0
  else if A[1] ≤ prev
    return LISbigger(prev, A[2 .. n])
  else
    skip ← LISbigger(prev, A[2 .. n])
    take ← LISbigger(A[1], A[2 .. n]) + 1
    return max{skip, take}
```

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Recall that passing arrays as input parameters to recursive functions is expensive.

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Recall that passing arrays as input parameters to recursive functions is expensive.

We will restate the algorithm in terms of array indices, assuming that the array $A[1..n]$ is a global variable.

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Recall that passing arrays as input parameters to recursive functions is expensive.

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We will restate the algorithm in terms of array indices, assuming that the array $A[1..n]$ is a global variable.

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The integer *prev* is *typically* an array element $A[i]$, and the remaining array is always a suffix $A[j..n]$ of the original input array.

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Recall that passing arrays as input parameters to recursive functions is expensive.

We will restate the algorithm in terms of array indices, assuming that the array $A[1..n]$ is a global variable.

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The integer *prev* is *typically* an array element $A[i]$, and the remaining array is always a suffix $A[j..n]$ of the original input array.

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The problem to be solved is then:

Given two indices i and j, where $i < j$, find the Longest Increasing Subsequence of $A[j..n]$ in which every element is larger than $A[i]$.

LISbigger v2.0

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```
LISbigger(i, j):
  if j > n
    return 0
  else if A[i] ≥ A[j]
    return LISbigger(i, j + 1)
  else
    skip ← LISbigger(i, j + 1)
    take ← LISbigger(j, j + 1) + 1
    return max{skip, take}
```

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```
LISbigger(i, j):
  if j > n
    return 0
  else if A[i] ≥ A[j]
    return LISbigger(i, j + 1)
  else
    skip ← LISbigger(i, j + 1)
    take ← LISbigger(j, j + 1) + 1
    return max{skip, take}
```

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What is the initial call on $A[1..n]$? There is not index i initially...

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```
LIS(A[1 .. n]):
  A[0] ← −∞
  return LISbigger(0, 1)
```

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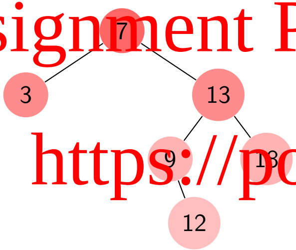
The running time for a successful search in a binary search tree is proportional to the depth of the target node.



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Optimal Binary Search Trees

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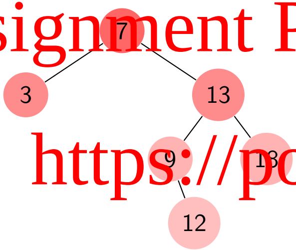

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- The worst-case search time is proportional to the depth of the tree.

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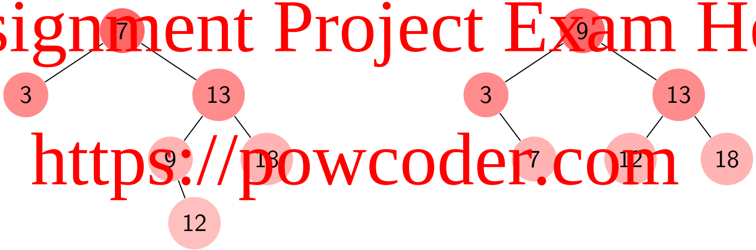

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- The worst-case search time is proportional to the depth of the tree.
- To minimize the worst-case search time, the tree should be balanced.

It is often more important to minimize the total cost of several searches

- Rather than the worst-case cost of a single search.

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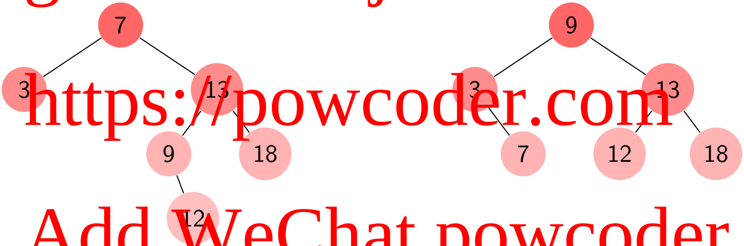

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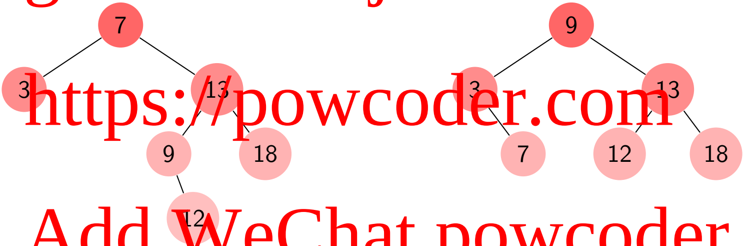

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If x is a more frequent search target than y, we can save time by building a tree where the depth of x is smaller than the depth of y

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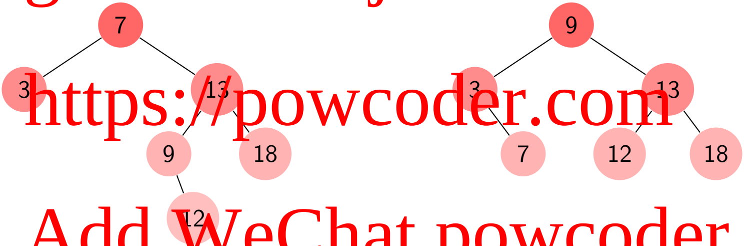

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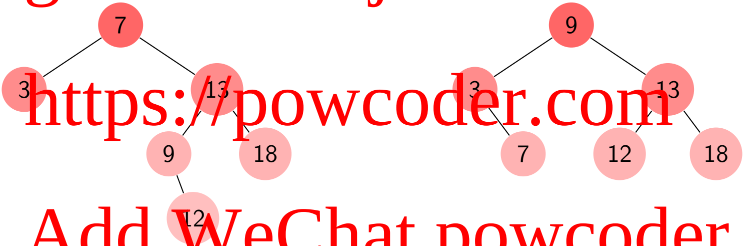

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If x is a more frequent search target than y, we can save time by building a tree where the depth of x is smaller than the depth of y

- Even if that means increasing the overall depth of the tree.
- A perfectly balanced tree is not the best choice if some items are significantly more popular than others.

This state of affairs suggests the following problem:

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Given a sorted array of keys $A[1..n]$ and an array of corresponding access frequencies $f[1..n]$, build the binary search tree to store the keys and that minimizes the total search time, assuming that there will be exactly $f[i]$ searches for each key $A[i]$.

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It is helpful to write a good recursive definition of the function we are trying to optimize.

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Optimal Binary Search Trees

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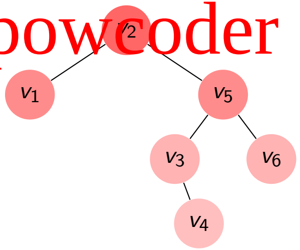
It is helpful to write a good recursive definition of the function we are trying to optimize.

Given a binary search tree T with n nodes, let $v_1, v_2, \ldots, v_n$ be the nodes of T, indexed in sorted order, so that each node v_i stores the corresponding key $A[i]$.



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The total cost of performing all the binary searches is given by the following expression:

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$$Cost(T, f[1..n]) = \sum_{i=1}^{n} f[i] \times d_T(v_i)$$

where $d_T(v_i)$ is the depth of v_i in T (where depth of root is 1).

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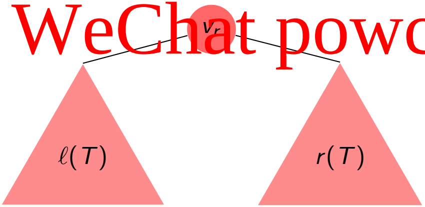
Let v_r, where $1 \leq r \leq n$ be the root of T and let $\ell(T)$ and $r(T)$ be the left and right subtrees of v_r.



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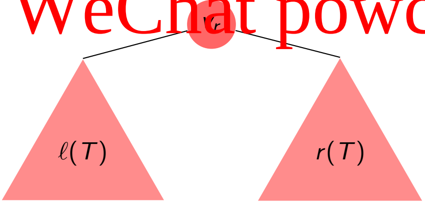
Then:

$$Cost(T, f[1..n]) = \sum_{i=1}^{n} f[i] \times d_T(v_i)$$

$$= \sum_{i=1}^{n} f[i] + \sum_{i=1}^{r-1} f[i] \times d_{\ell(T)}(v_i) + \sum_{i=r+1}^{n} f[i] \times d_{r(T)}(v_i)$$

$$= \sum_{i=1}^{n} f[i] + Cost(\ell(T), f[1..r-1]) + Cost(r(T), f[r+1..n])$$



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Goal is to compute optimal tree T_{opt} that minimizes:

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Optimization function

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So, if we choose the correct key to store at the root, the recursion fairy will construct the rest of the optimal tree.

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So how do we choose the correct key? Try each one!