

Econ/Math C103 (2020) - Problem Set 4

Due: 3:30pm PDT, 11/12/2020

1. Consider a two-player game where Player 1 is a driver who may choose to drive Fast or Slow and Player 2 is the police who may choose to Monitor or Not monitor Player 1's speed:

	M	N
F	$-p, -c$	$1, -1$
S	$0, -c$	$0, 0$

When she is not monitored, Player 1 prefers to go fast, whereas when she is monitored she prefers to go slow ($p > 0$) in order not to incur a traffic penalty. Player 2 does not want Player 1 to go fast without being monitored, but he incurs a cost c such that $1 > c > 0$ if he chooses to monitor.

- (a) Does the above game have a pure strategy equilibrium?
- (b) Let α denote the probability with which Player 1 chooses to go fast, and let β be the probability with which Player 2 monitors. Draw the best replies of each player on the (α, β) -plane and find all the (possibly mixed strategy) Nash equilibria.

2. Consider the following two player game where Player 1 chooses one of the three rows and Player 2 chooses one of the four columns:

	C_1	C_2	C_3	C_4
R_1	1,0	0,2	2,2	4,5
R_2	1,4	0,0	3,2	1,1
R_3	2,1	1,2	2,0	0,1

- (a) What are the strategies that survive IESDS?
 - (b) At each step of the elimination what were your rationality and knowledge assumptions?
 - (c) Find all the (possibly mixed) Nash equilibria.
3. Consider n people who go to dinner together. The utility of each person i from eating a meal that costs $y_i \geq 0$ when he pays $x_i \geq 0$ is $\sqrt{y_i} - x_i$. Each person

in the group chooses the price of the meal he will eat and then the bill is shared equally. Find a dominant strategy equilibrium of this game. What happens to the price of the chosen meal and utility of each person as $n \rightarrow \infty$?

4. Find all the pure strategy Nash Equilibria of the following normal form game with two players. Each player i simultaneously chooses a number $a_i \in [0, 1]$. If $a_1 + a_2 > 1$, then both receive zero payoff, if $a_1 + a_2 \leq 1$, then i 's payoff is a_i .
5. Consider two firms 1 and 2 who produce goods at zero marginal cost and simultaneously set prices $p_1, p_2 \in [0, 1]$ (Note that firms are not allowed to set prices higher than 1!). The market demand for the good produced by firm i is:

$$q_i(p_1, p_2) = p_j - p_i$$

where $j \neq i$.¹ That is, the goods produced by the two firms are (imperfect) substitutes. Each firm i maximizes its profits $\pi_i(p_1, p_2) = p_i \times q_i(p_1, p_2)$. Find the strategies that survive IESDS and the pure strategy Nash Equilibria.

6. Army A has a single plane with which it can attack one of three possible targets 1, 2, and 3. Army D has one anti-aircraft gun that can be assigned to defend one of these three targets. Army A can destroy a target if and only if the target is undefended and A attacks it. The value of destroying target k is k for army A . Similarly, the value of defending an attack on target k is k for army D . Both armies take their actions simultaneously, so the situation is summarized by the following normal form game:

	D_1	D_2	D_3
A_1	0,1	1,0	1,0
A_2	2,0	0,2	2,0
A_3	3,0	3,0	0,3

Find all (possibly mixed strategy) Nash equilibria of the above game.

7. Each one of two players is given an envelope with a random dollar amount in it. The monetary amounts in the envelopes are drawn independently from \$0, \$1, \$2, ..., \$9 each with an equal probability of 1/10. Players observe the monetary amount

¹For simplicity, we allow for negative "quantities" here.

in their own envelope but not the amount in the other's envelope. Then each of them simultaneously indicates whether she wants to exchange or not to exchange envelopes. Envelopes are exchanged if and only if both players want to exchange. At the end of the game, each player receives the money in the envelope she ends up holding. Players maximize expected monetary payoffs.

- (a) Model this situation as a Bayesian game. That is, write out the set of players, types, actions, payoff functions, and the probability distribution over types.
- (b) Find all the pure strategy BNE of the above game. Do players show a desire to exchange in equilibrium if their envelope contains a positive amount of money?

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