Econ/Math C103 (2020) - Problem Set 6

Due: 9am PDT, 12/18/2020

1. Suppose that there are n bidders whose valuations are i.i.d. on $[0,1]$ with c.d.f. $F_i(x_i) = x_i^2$. Derive the allocation probabilities and the transfers in a revenue-maximizing feasible direct auction mechanism.

2. Consider the auction of a single indivisible object to two bidders. Bidder 1's valuation is uniformly distributed on the interval $[0,3]$, i.e., with density $f_1(x_1) = \frac{1}{3}$ for all $x_1 \in [0,3]$. Bidder 2's valuation is independently and uniformly distributed on the interval $[1,2]$, i.e., with density $f_2(x_2) = 1$ for all $x_2 \in [1,2]$. Derive the probabilities of allocating the object to the two bidders in a revenue-maximizing feasible direct auction mechanism. Is the object allocated to one of the two bidders with probability one? Is the object always allocated to the bidder with the highest valuation?

3. You can use the revelation principle and the characterization of BIC direct auction mechanisms to derive the BNE of a number of auction rules. To illustrate, let's provide an alternative (simpler) derivation of the BNE of the first-price auction equilibrium. Suppose that there are n bidders whose valuations are i.i.d. on $[0,1]$ with c.d.f. F and strictly positive density f.

   (a) Conjecture that there is a symmetric BNE of the first-price auction, where the bid function $b^I(\cdot) : [0,1] \to \mathbb{R}_+$ is strictly increasing.

   (b) Note that since $b^I(\cdot)$ is strictly increasing, $U_i^I(x_i) = F^{n-1}(x_i)[x_i - b^I(x_i)]$.

   (c) Let $U_i^I(x_i)$ denote the BNE equilibrium utility of bidder i with valuation x_i. Show that $U_i^I(0) = 0$.

   (d) Use the revelation principle and the characterization of BIC direct auction mechanisms to provide an alternative formula for $U_i^I(x_i)$.

   (e) Use parts (b) and (d) to solve for $b^I(\cdot)$. Note that the $b^I(\cdot)$ that you derived is indeed strictly increasing. By the revelation principle and the characterization of feasible direct auction mechanisms, you can conclude that $b^I(\cdot)$ is indeed a BNE.

4. Carry out an analogous exercise to question 3 to derive the symmetric BNE of the all-pay auction.