Econ/Math C103 (2020) - Problem Set 1

Due: 3:30pm PDT, 9/8/2020

1. A group of people met and some of them shook each other hands. Prove that the number of people who shook others' hands an odd number of times is, in fact, even.

2. Let X be a nonempty set, and let 2^X denote the set of all subsets of X. Show that there is no onto function $f : X \to 2^X$.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be twice differentiable. Show that $\frac{d^2 f}{dxdy} \geq 0$ if and only if for every $x, x', y, y' \in \mathbb{R}$:

$$x > x' \text{ and } y > y' \implies f(x, y) - f(x, y') \geq f(x', y) - f(x', y').$$

4. A pair $G = (V, E)$ is a *directed graph* if V is a set, and $E \subseteq V \times V$. The directed graph G has a *cycle* if there exists an integer $n \geq 1$, and elements $v_1, \ldots, v_n \in V$ such that $(v_1, v_2), (v_2, v_3), \ldots (v_{n-1}, v_n), (v_n, v_1) \in E$. Show that if V is a nonempty finite set and for all $v \in V$ there is $v' \in V$ such that $(v, v') \in E$, then G has a cycle.