Multiattribute Decision Models II

Decision Theory

Today

- MAUT computation example from A-Z
- Utility function assessment revisited
- Calculating K
- Uncertainty
- Sensitivity analysis

- Multiattribute utility functions

Let x_i be the level of attribute i; $u_i(x_i)$ be the attribute utility function for attribute i; k_i be the attribute tradeoff scaling constant for attribute i. If the x_i are mutually utility independent then the expression for the multiattribute utility function takes one of the following forms depending on the sum of the k_i:

$$If \sum_{k=1}^{N} k_n = 1.0, \quad then :\Rightarrow U(\vec{x}) = \left\{ \sum_{n=1}^{N} k_n \cdot u_n(x_n) \right\} \qquad Eq\ 1$$

$$If \sum_{k=1}^{N} k_n \neq 1.0, \quad then :\Rightarrow U(\vec{x}) = \frac{1}{K} \left\{ \prod_{n=1}^{N} [1 + K \cdot k_n \cdot u_n(x_n)] - 1 \right\} \qquad Eq\ 2$$

where the master scaling constant, K, is solved from the equation:

$$1 + K = \prod_{n=1}^{N} [1 + K \cdot k_n] \qquad Eq\ 3$$

What is the relationship between this:

$$U(\vec{x}) = \frac{1}{K} \left\{ \prod_{n=1}^{N} [1 + K \cdot k_n \cdot u_n(x_n)] - 1 \right\}$$

and this:

$$U(\vec{x}) = \left\{ \sum_{n=1}^{N} k_n \cdot u_n(x_n) \right\} \quad ?$$

It's not obvious…

It's easier to see if we expand the multiplicative form and rearrange:

$$U(\vec{x}) = \sum_{i=1}^{N} k_i u_i(x_i) + (\text{interaction terms})$$

For example, if N = 2

$$U(\vec{x}) = \frac{1}{K} \left\{ \prod_{n=1}^{2} [1 + K \cdot k_n \cdot u_n(x_n)] - 1 \right\} = \frac{1}{K} \{ [1 + Kk_1u_1(x_1)][1 + Kk_2u_2(x_2)] - 1 \}$$

expanding

$$= \frac{1}{K} \{ 1 + Kk_1u_1(x_1) + Kk_2u_2(x_2) + K^2k_1k_2u_1(x_1)u_2(x_2) - 1 \}$$

$$= \frac{1}{K} \{ Kk_1u_1(x_1) + Kk_2u_2(x_2) + K^2k_1k_2u_1(x_1)u_2(x_2) \} \qquad \text{One's cancel}$$

$$= \sum_{i=1}^{2} k_i u_i(x_i) + [Kk_1k_2u_1(x_1)u_2(x_2)] \qquad \text{Rearranging}$$

Interaction terms. Now solve for K

Solving the master scaling constant equation for K with N=2 attributes:

$$1 + K = \prod_{n=1}^{N} [1 + K \cdot k_n]$$

$$for\ N = 2:$$

$$1 + K = [1 + Kk_1][1 + Kk_2]$$

$$1 + K = 1 + Kk_1 + Kk_2 + K^2k_1k_2$$

$$K = \frac{1 - (k_1 + k_2)}{k_1k_2}$$

(Note that if k_1+k_2 =1, K=0)

Returning to the multiplicative model:

$$=\sum_{i=1}^{2} k_i u_i(x_i) + [K k_1 k_2 u_1(x_1) u_2(x_2)]$$

Note that if $\sum_{n=1}^{2} k_n = 1,$

$$then\ K = \frac{1-k_1-k_2}{k_1 k_2} = 0$$

$$So\ \sum_{i=1}^{2} k_i u_i(x_i) + [K k_1 k_2 u_1(x_1) u_2(x_2)] = \sum_{i=1}^{2} k_i u_i(x_i) + [0]$$

the multiplicative reduces to the additive model!

Example

System Data ($ and mpg)

Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32

- After collection of system data we extract the ranges:
  - Cost: 20 to 40 k$ (less is better)
  - Mpg: 16-32 mpg (more is better)

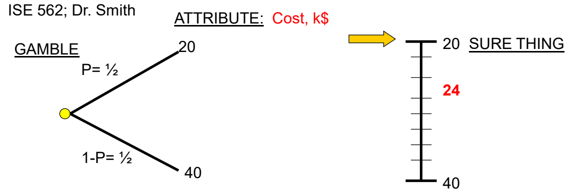
We need to assess the utility functions and scaling constants for these attributes
- Cost: 20 to 40 k$ (less is better)
- Mpg: 16-32 mpg (more is better)

- To do this we construct an interview package to meet with the DM

- Interview package outline for assessing decision maker value model
  - Introduction
  - Utility function assessment (2)
  - Independence check
  - Attribute ranking
  - Attribute tradeoff scaling constants
  - Check for preference reversals
  - Reassessment as required

Introduction
- Why are we here
- Purpose of study
- Process

ATTRIBUTE: Cost, k$



- For which value of the sure thing are you indifferent between the sure thing and the gamble?
  - Indifference point __24__ (the CE is 24, U(24)=0.50)
- If you knew that all the other attributes were at their worst states?
  - Indifference point _______
- If you knew that all the other attributes were at their best states?
  - Indifference point _______

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ATTRIBUTE: Mpg

- For which value of the sure thing are you indifferent between the sure thing and the gamble?
  - Indifference point 28 (the CE is 28, U(28)=0.50)
- If you knew that all the other attributes were at their worst states?
  - Indifference point _______
- If you knew that all the other attributes were at their best states?
  - Indifference point _______

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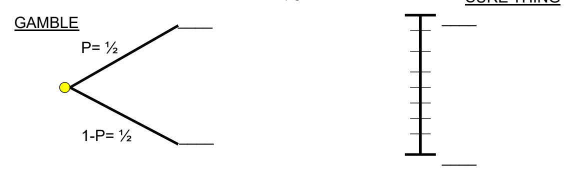
To assess additional points (.25, .75, etc.), use the following template

ATTRIBUTE: Mpg



- For which value of the sure thing are you indifferent between the sure thing and the gamble?
  - Indifference point _______

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We have the following:

For cost, u(20)=1.0; u(40)=0.0; u(24)=0.50

For mpg, u(32)=1.0; u(16)=0.0; u(28)=0.50

Now we can generate the utility functions.

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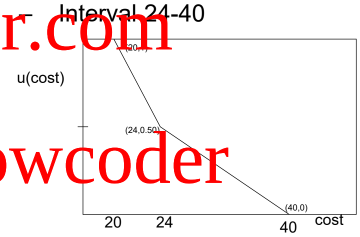
Cost Utility Function:

For cost, u(20)=1.0; u(40)=0.0; u(24)=0.50

- 2 piecewise functions
  - Interval 20-24
  - Interval 24-40



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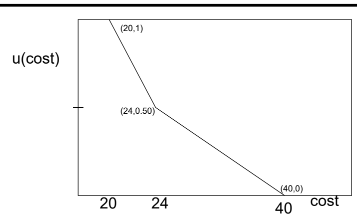
Cost:



M1=(1-.5)/(20-24)= –.125
Y1= –.125(X1-20)+1

M2=(.5-0)/(24-40)= –.03125
Y2= –.03125 (X1-40)

Utility function for cost is:

$$u(cost)=\begin{cases} -.125(cost-20)+1 & \text{if } 20\le cost<24 \\ -.03125(cost-40) & \text{if } 24\le cost\le 40 \end{cases}$$

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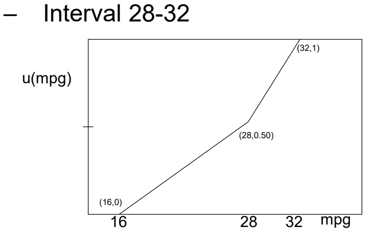
Mpg Utility Function:

For mpg, u(32)=1.0; u(16)=0.0; u(28)=0.50

- 2 piecewise functions
  - Interval 16-28
  - Interval 28-32



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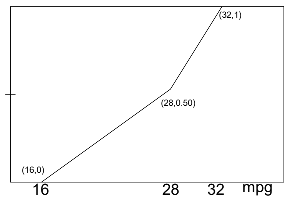
Mpg:



M1=(.5-0)/(28-16)= .0417
Y1= .0417(X-16)

M2=(1-.5)/(32-28)= .125
Y2= .125 (X-32) + 1

Utility function for mpg is:

$$u(mpg)= \begin{cases} .0417(mpg-16) & \text{if } 16 \le mpg < 28 \\ .125(mpg-32)+1 & \text{if } 28 \le mpg \le 32 \end{cases}$$

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Order of importance of attributes.

If you could only change one attribute from its worst state to its best state, which would you choose?

Attribute	Cost	Mpg
Best state	20	32
Worst state	40	16
Order of importance	1	2

Another example…

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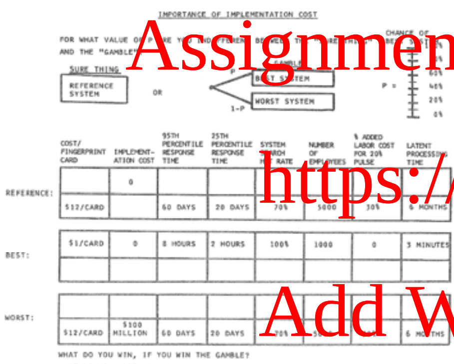

IMPORTANCE OF IMPLEMENTATION COST

FOR WHAT VALUE OF P ARE YOU INDIFFERENT BETWEEN THE "SURE THING" AND THE "GAMBLE"?

SURE THING: REFERENCE SYSTEM OR GAMBLE: P BEST SYSTEM, 1-P WORST SYSTEM

CHANCE OF BEST SYSTEM: P = 100%, 80%, 60%, 40%, 20%, 0%

	COST/ FINGERPRINT CARD	IMPLEMENT- ATION COST	95TH PERCENTILE RESPONSE TIME	25TH PERCENTILE RESPONSE TIME	SYSTEM SEARCH HIT RATE	NUMBER OF EMPLOYEES	% ADDED LABOR COST FOR 20% PULSE	LATENT PROCESSING TIME
REFERENCE:		0						
	$12/CARD		60 DAYS	20 DAYS	70%	5000	30%	6 MONTHS
BEST:	$1/CARD	0	8 HOURS	2 HOURS	100%	1000	0	3 MINUTES
WORST:								
	$12/CARD	$100 MILLION	60 DAYS	20 DAYS	70%	[illegible]	[illegible]	6 MONTHS

WHAT DO YOU WIN, IF YOU WIN THE GAMBLE?
WHAT DO YOU LOSE, IF YOU LOSE THE GAMBLE?

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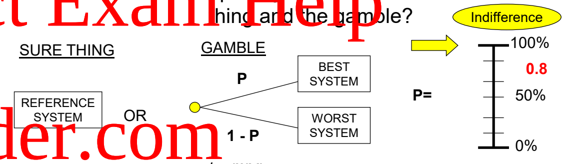
Importance of: Cost

For what value of p are you indifferent between the sure thing and the gamble?



	cost	mpg
Reference	20	16
Best	20	32
Worst	40	16

Cost tradeoff scaling constant $k_1 = 0.8$

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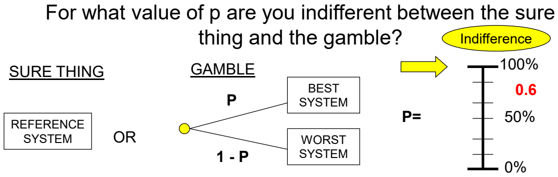
Importance of: Mpg

For what value of p are you indifferent between the sure thing and the gamble?



	cost	mpg
Reference	40	32
Best	20	32
Worst	40	16

Cost tradeoff scaling constant $k_2 = 0.6$

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This completes the collection of value data:

Utility function for cost is:

$$u(X1)= \begin{cases} -.125(cost-20)+1 & \text{if } 20 \le cost < 24 \\ -.03125(cost-40) & \text{if } 24 \le cost \le 40 \end{cases}$$

Utility function for mpg is:

$$u(X2)= \begin{cases} .0417(mpg-16) & \text{if } 16 \le mpg < 28 \\ .125(mpg-32)+1 & \text{if } 28 \le mpg \le 32 \end{cases}$$

Attribute tradeoff scaling constants
k_1= .8 (cost)
k_2= .6 (mpg)

Alternative	Cost	Mpg
Car A	20	16
Car B	30	25
Car C	40	32

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We check which MAU model applies:

$\Sigma k_i = 1.4 \neq 1$, so use multiplicative model

$$U(\vec{x}) = \frac{1}{K}\left\{\prod_{n=1}^{N}[1 + K \cdot k_n \cdot u_n(x_n)] - 1\right\}$$

where the master scaling constant, K,

is solved from the equation : $1 + K = \prod_{n=1}^{N}[1 + K \cdot k_n]$

First, we need the master scaling constant, K

Solving for K, the master scaling constant:

we have $1 + K = \prod_{n=1}^{N}[1 + K \cdot k_n]$

or, for this example, n = 2

$1 + K = [1 + Kk_1][1 + Kk_2]$

The equation to solve for K :

$1 + K = [1 + .8K][1 + .6K]$

$1 + K = 1 + 1.4K + .48K^2$

$.48K^2 + .4K = 0$

$K(.48K + .4) = 0$

$K = -.8333$

Now calculate the multiplicative multiattribute utilities:

$U(CarA) = U(20,16) = \frac{1}{(-.8333)}\left\{\prod_{n=1}^{N}[1 + (-.8333) \cdot k_n \cdot u_n(x_n)] - 1\right\}$

$= (-1.2)\{[1 - .8333(.8)(1.0)][1 - .8333(.6)(0.0)] - 1\}$

$= .80$

$U(CarB) = U(30,25) = \frac{1}{(-.8333)}\left\{\prod_{n=1}^{N}[1 + (-.8333) \cdot k_n \cdot u_n(x_n)] - 1\right\}$

$u_1(30) = -.03125(30 - 40) = 0.3125;\quad u_2(25) = .0417(25 - 16) = 0.3753$

$= (-1.2)\{[1 - .8333(.8)(.3125)][1 - .8333(.6)(.3753)] - 1\}$

$= 0.4282$

$U(CarC) = U(40,32) = \frac{1}{(-.8333)}\left\{\prod_{n=1}^{N}[1 + (-.8333) \cdot k_n \cdot u_n(x_n)] - 1\right\}$

$= (-1.2)\{[1 - .8333(.8)(0.0)][1 - .8333(.6)(1.0)] - 1\}$

$= .60$

The conclusion for this DM:

$U(CarA) = U(20,16) = .80$

$U(CarB) = U(30,25) = 0.4282$

$U(CarC) = U(40,32) = .60$

so $CarA \succ CarC \succ CarB$

Alternative	Rank
Car A	1
Car B	3
Car C	2

- What if the sum of the scaling constants were = 1.0?
  - Use the additive multiattribute utility function
  - For example, suppose we had obtained k_1=.7 and k_2=.3…
  - Note: no master scaling constant

$1 + K = [1 + .7K][1 + .3K]$

$1 + K = 1 + K + .21K^2$

$.21K^2 = 0$

$K = 0$

Calculate the additive multiattribute utilities:

$U(CarA) = U(20,16) = \sum_{n=1}^{2} k_n \cdot u_n(x_n) = .7u_1(x_1) + .3u_2(x_2)$

$= .7u_1(20) + .3u_2(16) = .7(1) + .3(0)$

$= .70$

$U(CarB) = U(30,25) = \sum_{n=1}^{2} k_n \cdot u_n(x_n) = .7u_1(x_1) + .3u_2(x_2)$

$u_1(30) = -.03125(30 - 40) = 0.3125;\quad u_2(25) = .0417(25 - 16) = 0.3753$

$= .7u_1(30) + .3u_2(25) = .7(.3125) + .3(.3753)$

$= .3313$

$U(CarC) = U(40,32) = \sum_{n=1}^{2} k_n \cdot u_n(x_n) = .7u_1(x_1) + .3u_2(x_2)$

$= .7u_1(40) + .3u_2(32) = .7(0) + .3(1)$

$= .30$

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The conclusion in the additive case would be:

$$U(CarA) = U(20,16) = .70$$
$$U(CarB) = U(30,25) = 0.3313$$
$$U(CarC) = U(40,32) = .30$$
$$so \quad CarA \succ CarB \succ CarC$$

Multiplicative

Alternative	U(X)	Rank
Car A	0.80	1
Car B	0.43	3
Car C	0.60	2

Additive

Alternative	U(X)	Rank
Car A	0.70	1
Car B	0.33	2
Car C	0.30	3

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Why?

- In multiplicative case cost accounted for .8/1.4 = 57% of attribute importance
- In additive case cost increased to 70% so alternative with lower costs was amplified
- Cost of B was 30k, cost of C was 40k so B moved up in the rankings

Multiplicative

Alternative	U(X)	Rank
Car A	0.80	1
Car B	0.43	3
Car C	0.60	2

Additive

Alternative	U(X)	Rank
Car A	0.70	1
Car B	0.33	2
Car C	0.30	3

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Let's try some utility assessments

Suppose you are going to rent an apartment with the following choices

Alternative	Cost/month /roomate	Distance to campus, miles
A.	800	1
B.	400	10
C.	600	4

1. Assess a three-point utility function for Cost
2. Assess the attribute tradeoff scaling constant for cost (and distance if enough time).

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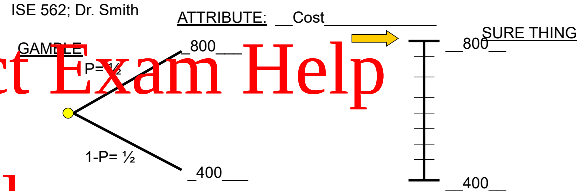
ATTRIBUTE: __Cost____________



- For which value of the sure thing are you indifferent between the sure thing and the gamble?
  - Indifference point _______
- If you knew that all the other attributes were at their worst states?
  - Indifference point _______
- If you knew that all the other attributes were at their best states?
  - Indifference point _______

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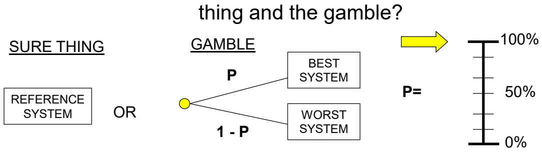
Importance of: __Cost____________

For what value of p are you indifferent between the sure thing and the gamble?



Reference

\$400									
	10 mi								

Best

400	1 mi								

Worst

\$800	10 mi								

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- By the way, we have a problem when n>4
  - Can you see it?
  - Hint: we need Isaac Newton's help

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From earlier example, k_1=.8, k_2=.6; check which MAU model applies: Σk_i = 1.4 ≠ 1, use multiplicative model

$$U(\vec{x}) = \frac{1}{K}\left\{\prod_{n=1}^{N}[1+K\cdot k_n\cdot u_n(x_n)]-1\right\}$$

where the master scaling constant, K,

is solved from the equation : $1+K=\prod_{n=1}^{N}[1+K\cdot k_n]$

First, we need the master scaling constant, K

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Solving for K, the master scaling constant:

We have $1+K=\prod_{n=1}^{N}[1+K\cdot k_n]$

or, for this example, n = 2

$1+K=[1+Kk_1][1+Kk_2]$

The equation to solve for K :

$.48K^2+.4K=0$

$K=-.8333$

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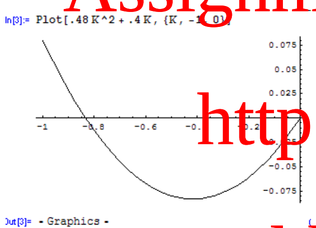
Let's examine the function: f(K)=.48K² + .4K=0



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What if k_1=0.5 and k_2=0.3? Σk_i = 0.8 ≠ 1

We have $1+K=\prod_{n=1}^{N}[1+K\cdot k_n]$

or, for this example, n = 2

$1+K=[1+Kk_1][1+Kk_2]$

The equation to solve for K :

$1+K=[1+.5K][1+.3K]$ or

$.15K^2-.2K=0$

$K=+1.3333$

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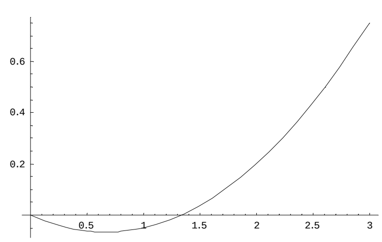
Let's examine the function: f(K)=.15K² - .2K=0



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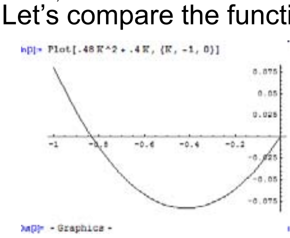
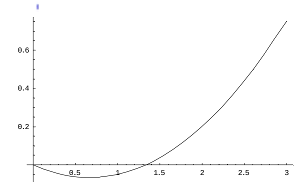
Let's compare the functions:



So if Σki >1, root on left side of origin

So if Σki <1, root on right side of origin



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How to solve if n large?

Newton's method for finding root of function numerically

Our hero!

Isaac Newton

Newton singlehandedly contributed more to the development of science than any other individual in history. He surpassed all the gains brought about by the great scientific minds of antiquity, producing a scheme of the universe which was more consistent, elegant, and intuitive than any proposed before. Newton stated explicit principles of scientific methods which applied universally to all branches of science. This was in sharp contradistinction to the earlier methodologies of Aristotle and Aquinas, which had outlined separate methods for different disciplines.

Source: http://scienceworld.wolfram.com/biography/Newton.html

Isaac Newton

English physicist and mathematician who was born into a poor farming family. Luckily for humanity, Newton was not a good farmer, and was sent to Cambridge to study to become a preacher. At Cambridge, Newton studied mathematics, being especially strongly influenced by Euclid, although he was also influenced by Baconian and Cartesian philosophies. Newton was forced to leave Cambridge when it was closed because of the plague, and it was during this period that he made some of his most significant discoveries. With the reticence he was to show later in life, Newton did not, however, publish his results. In response to a letter from Hooke, he suggested that a particle, if released, would spiral in to the center of the Earth. Hooke wrote back, claiming that the path would not be a spiral, but an ellipse. Newton, who hated being bested, then proceeded to work out the mathematics of orbits. Again, he did not publish his calculations. Newton devoted the period from August 1684 to spring 1686 to this task, and the result became one of the most important and influential works on physics of all times, *Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy)* (1687), often shortened to *Principia Mathematica* or simply "the *Principia.*" In Book I of *Principia,* Newton opened with definitions and the three laws of motion now known as Newton's laws (laws of inertia, action and reaction, and acceleration proportional to force). Book II presented Newton's new scientific philosophy which came to replace Cartesianism. Finally, Book III consisted of applications of his dynamics, including an explanation for tides and a theory of lunar motion. To test his hypothesis of universal gravitation, Newton wrote Flamsteed to ask if Saturn had been observed to slow down upon passing Jupiter. The surprised Flamsteed replied that an effect had indeed been observed, and it was closely predicted by the calculations Newton had provided. Newton's equations were further confirmed by observing the shape of the Earth to be oblate spheroidal, as Newton claimed it should be. Newton's equations also described the motion of Moon by successive approximations, and correctly predicted the return of Halley's Comet. Newton also correctly formulated and solved the first ever problem in the calculus of variations. Newton invented a scientific method which was truly universal in its scope. These four concise and universal rules for investigation were truly revolutionary. By their application, Newton formulated the universal laws of nature with which he was able to unravel virtually all the unsolved problems of his day. Newton refined Galileo's experimental method, creating the compositional method of experimentation still practiced today. This analysis consists of making experiments and observations, and in drawing general conclusions from them by induction...by this way of analysis we may proceed from compounds to ingredients, and from motions to the forces producing them; and in general from effects to their causes, and from particular causes to more general ones till the argument end in the most general. This is the method of analysis: and the synthesis consists in assuming the causes discovered and established as principles, and by them explaining the phenomena preceding from them, and proving the explanations."

Newton formulated the classical theories of mechanics and optics and invented calculus years before Leibniz. However, he did not *publish* his work on calculus until afterward Leibniz had published his. This led to a bitter priority dispute between English and continental mathematicians which persisted for decades, to the detriment of all concerned. Newton discovered that the binomial theorem was valid for fractional powers, but left it for Wallis to publish (which he did, with appropriate credit to Newton). Newton formulated a theory of sound, but derived a speed which did not agree with his experiments. The reason for the discrepancy was that the concept of adiabatic propagation did not yet exist, so Newton's answer was too low by a factor of , where is the ratio of heat capacities of air. Newton therefore fudged his theory until agreement was achieved (Engineering and Science, pp. 15-16). Newton also formulated a system of chemistry. In this theory, "elements" consisted of different arrangements of atoms, and atoms consisted of small, hard, billiard ball-like particles. Source: http://scienceworld.wolfram.com/biography/Newton.html

How to solve if n large?

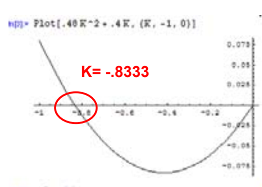
Newton's method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

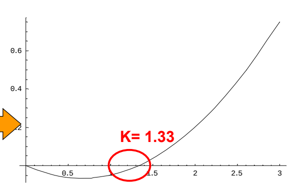
$$\text{if } \begin{cases} \sum k_i > 1 & \text{then } x_0 = -1; \\ \sum k_i < 1 & \text{then } x_0 = 2; \\ \sum k_i = 1 & \text{then } x_0 = 0; \end{cases}$$

How to solve:

$$\text{if } \left\{ \sum k_i > 1 \quad \text{then } x_0 = -1; \right.$$



$$\text{if } \left\{ \sum k_i < 1 \quad \text{then } x_0 = 2; \right.$$



- Master scaling constant solution

$$\text{We have } 1 + K = \prod_{n=1}^{N} [1 + K \cdot k_n]$$

$$f(K) = .48K^2 + .4K$$

$$f'(K) = .96K + .4$$

$$x_{n+1} = x_n - \frac{.48x_n^2 + .4x_n}{.96x_n + .4}$$

iteration, n	f(x)	f'(x)	x(n+1)
0			-1.00000
1	0.08	-0.56	-0.85714
2	0.009796	-0.42286	-0.83398
3	0.000258	-0.40062	-0.83333
4	1.98E-07	-0.4	-0.83333
5	1.18E-13	-0.4	-0.83333
6	5.55E-17	-0.4	-0.83333
7	0	-0.4	-0.83333

Excel solution

Higher order

Incorporating uncertainty:

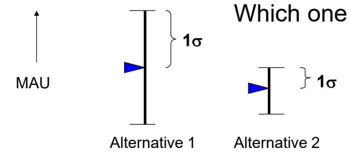
$$U(\vec{x}) = \frac{1}{K}\left\{\prod_{n=1}^{N}\left[1 + K \cdot k_n \cdot u_n(x_n)\right] - 1\right\}$$

If the attribute states are represented by probability distributions, we perform a transformation of random variables from the pdf's of the x_n's to the resulting U(X) using Monte Carlo simulation.

Incorporating uncertainty

- The result would be not only the expected utility, but the variance of the expected utility which allows the following comparison:

Which one would you pick?



For example (from last lecture)

Utility function for cost is:

u(X1)= -.125(X1-20) +1 if 20≤X1<24
-.03125(X1-40) if 24≤X1≤40

Utility function for mpg is:

u(X2)= .0417(X2-16) if 16≤X2<28
.125(X2-32) +1 if 28≤X2≤32

Attribute tradeoff scaling constants
k_1= .7 (cost)
k_2= .3 (mpg)

Alternative	Cost	Mpg	U(alt)
Car A	20	16	0.70
Car B	30	25	0.33
Car C	40	32	0.30

Now consider uncertainty

Suppose these were uncertain

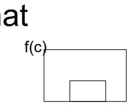
Alternative	Cost	Mpg
Car A	20	16
Car B	30	25
Car C	40	32

For example, for Car B:

Alternative	Cost	Mpg
Car A	20	16
Car B	30	25
Car C	40	32

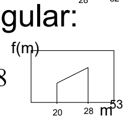
Suppose cost is a uniform pdf that ranges from 28 to 32

$$f(c) = \frac{1}{4} \quad 28 \le c \le 32$$



Suppose the pdf of mpg is triangular:

$$f(c) = \frac{1}{192}m \quad 20 \le m \le 28$$



To simulate cost with Monte Carlo simulation Solve CDF to obtain realization function

$$f(c) = \frac{1}{4} \quad 28 \le c \le 32$$

$$F(c) = \int_{28}^{c} \frac{1}{4} d\tau = \frac{1}{4}(c - 28)$$

so

$$r_i = \frac{1}{4}(c_i - 28)$$

or

$$c_i = 4r_i + 28 \quad 0 \le r_i \le 1$$

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To simulate mpg with Monte Carlo simulation
Solve CDF to obtain realization function

$$f(m) = \frac{1}{192} m \quad 20 \le m \le 28$$

$$F(m) = \int_{20}^{m} \frac{1}{192} \cdot \tau \cdot d\tau = \frac{1}{384}(m^2 - 400)$$

so

$$r_i = \frac{1}{384}(m_i^2 - 400)$$

or

$$m_i = \sqrt{384 r_i + 400} \quad 0 \le r_i \le 1$$

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Now consider the uncertain case B:

$$U(CarB) = U(c,m) = \sum_{n=1}^{2} k_n \cdot u_n(x_n) = .7u_1(x_1) + .3u_2(x_2)$$

Draw random numbers and compute realizations of c and m

rand-c	rand-m	Cost, c	MPG, m	util(cost)	util(mpg)	U(B)
0.085951	0.670727	28.34	25.64	0.3643	0.4021	0.375612
0.044615	0.907746	28.18	27.36	0.3694	0.4737	0.400711
0.687738	0.010457	30.75	20.10	0.2890	0.1710	0.253616
0.495718	0.154615	29.98	21.43	0.3130	0.2266	0.287091
0.863373	0.506258	31.45	24.38	0.2671	0.3495	0.291793

Example

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The conclusion in the additive case would be:

$$U(CarA) = U(20,16) = .70$$

Could change the ranking

$$U(CarB) = U(30,25) = \underbrace{0.32 \pm 0.04}$$

$$U(CarC) = U(40,32) = .30$$

U(B) ranges from 0.28 to 0.36

$$so \quad CarA \succ CarB \succ CarC$$

Deterministic

Alternative	Cost	Mpg	U(alt)
Car B	30	25	0.33

Probabilistic

Alternative	Cost	Mpg	U(alt)
Car B	Uniform(28,32)	Tri(20,28)	0.28-0.36

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Comparing multiattribute models

- Why? Sensitivity of model to alternative assumptions
  - Additive versus multiplicative model
  - 3-point versus 5-point utility functions
  - Means of attribute states versus use of probability distributions (deterministic vs. probabilistic)
  - Run models at DM best and worst state values

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Comparing multiattribute models

- What are we looking for?
  - Consistency in rankings
  - Robustness of conclusions
  - Indicators of reversals (from system data or value data?)
  - Impact of uncertainty
  - Top 3 and bottom 3 alternatives preserved?
  - Primary tradeoff attributes

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Comparing multiattribute models

- Additive vs. Multiplicative
  - Since $\Sigma k_n \neq 1$ majority of time, compute

$$k_i' = \frac{k_i}{\sum_{j=1}^{N} k_j} \quad \text{for i} = 1,2,\ldots,\text{N}$$

  - Then $\Sigma k_n = 1$ and recompute with additive model:

$$\boxed{If \ \sum_{k=1}^{N} k_n = 1.0, \quad then :\Rightarrow U(\vec{x}) = \left\{ \sum_{n=1}^{N} k_n \cdot u_n(x_n) \right\} \qquad Eq\,1}$$

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Comparing multiattribute models

- 3-point vs 5-point utility functions
  - Case of precision of value model vs. ranking outcomes
  - If severe risk aversion or risk seeking indicated 3-point may not reveal.
  - Must assess 5-point utility functions during interviews to obtain data. Then use CE for u()=.50 for 3-point case.

Comparing multiattribute models

- Deterministic vs probabilistic
  - Compute means of attribute state pdf's and run analysis once
  - Compute rankings with and without uncertainty
  - If dramatically different, technical and preferential risk likely to be high.

Comparing multiattribute models

- Best vs. worst states
  - Compute results using CE's for attribute states at "best" value (from utility function data sheet)
  - Compute results using CE's for attribute states at "worst" value (from utility function data sheet)
  - If little or no difference, pref and util independence hold

ATTRIBUTE: ______________

For which value of the sure thing are you indifferent between the sure thing and the gamble?

- Indifference point

If you knew that all the other attributes were at their worst states?

- Indifference point

If you knew that all the other attributes were at their best states?

- Indifference point _______

From previous example: multiplicative

Utility function for cost is:

u(X1)=	-.125(X1-20) +1	if 20≤X1<24
	-.03125(X1-40)	if 24≤X1≤40

Utility function for mpg is:

u(X2)=	.0417(X2-16)	if 16≤X2<28
	.125(X2-32) +1	if 28≤X2≤32

Attribute tradeoff scaling constants
k_1= .8 (cost)
k_2= .6 (mpg)

Alternative	Cost	Mpg
Car A	20	16
Car B	30	25
Car C	40	32

To compare with additive model: Compute

k_1 = .8/1.4=0.57 and k_2=.6/1.4=.43

Multiplicative attribute tradeoff scaling constants
k_1= .8 (cost)
k_2= .6 (mpg)

Additive attribute tradeoff scaling constants
k_1= .57 (cost)
k_2= .43 (mpg)

To compare with additive model:
U(CarA)=.57(1)+.43(0)=.57
U(CarB)=.57(.3125)+.43(.3753)=.34
U(CarC)=.57(0)+.43(1)=.43

Alternative	U(X) (mult.)	Mult. Rank	U(X) (add.)	Add. Rank
Car A	.80	1	.57	1
Car B	.43	3	.34	3
Car C	.60	2	.43	2

No difference!

- Class example from last week

Example