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EVPI and EVSI
Part I

Decision Theory

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Today-Part I
- Terminology
- EVPI and EVSI

Today-Part II
- More on utility and how to find the certainty equivalent
- How to identify risk attitude
- Other methods for calculating EV of information; ENGS

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Current glossary of terms
- a*: optimal decision
- CE: certainty equivalent
- CS: cost of sampling
- DM: decision maker
- E[·]: expectation operator
- EV: expected value
- EMV: expected monetary value
- ENGS: expected net gain of sampling
- EL: expected loss
- EOL: expected opportunity loss
- ERPI; EVPI: expected return/payoff under perfect information
- EU: expected utility
- R(a): book notation for return (payoff) of alternative a.
- Utility: functional assignment of numerical preference value to levels of an attribute (U(x): x→[0, 1])
- RP: risk premium
- VPI(θ): value of perfect information

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Expected Value

	Trade Bill		
	P(pass)=.60	P(fails)=0.40	
Crop	Passes	Fails	
Corn	$35,000	$8,000	=.6(35000)+(.4)(8000)=24,200
Peanuts	18,000	12,000	=.6(18000)+(.4)(12000)=15,600
Soybeans	22,000	20,000	=.6(22000)+(.4)(20000)=21,200

Expected value decision = plant corn

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Expected Opportunity Loss (choose option with minimum EOL)

	Trade Bill		
	P(pass)=.60	P(fails)=0.40	
Crop	Passes	Fails	
Corn	0	12,000	=.6(0)+(.4)(12000)=4800
Peanuts	17,000	8,000	=.6(17000)+(.4)(8000)=13,400
Soybeans	13,000	0	=.6(13000)+(.4)(0)=7,800

EOL decision = plant corn

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Decision analysis and value of information
- Decisions often have opportunities to gather data (at some cost) to enable a better decision
  - Classic example: exploratory oil wells
  - Sending precursor missions to Mars
  - Additional testing prior to release of new product
  - Surveys to assess customer reaction
  - Studies to define/refine decision components
  - Hiring experts/consultants

- Additional information to make a better decision is usually not free.
  - Cost of sampling
  - Cost of testing
  - Cost of survey
  - Cost of studies, forecasts
  - Cost of consultants
- Key question: Does the value of the information justify the cost of obtaining it?

- There are three information value concepts described:

1. Expected Value of Perfect Information called EVPI: Theoretical maximum value of information to obtain the optimal decision.
2. Expected Value of Sample Information called EVSI: Difference in expected value with and without sample information.
3. Expected Net Gain of Sampling ENGS. The EVSI with the cost of sampling included.

1. Expected Value of Perfect Information

Expected value of *perfect* information

- If we knew the best outcome would yield \$X, we should only be willing to pay an amount ≤ difference between best outcome and expected value outcome. EVPI=[(EV given PI) – EV(a*)]
- To calculate EV (of) PI we need
  1. Expected value of optimal decision, expected value= EV(a*)
  2. Expected value of decision given perfect info (the best we could do for each state)

To calculate EVPI we need

1. Expected value of the decision

Expected value

	Trade Bill		
	P(pass)=.60	P(fails)=0.40	
Crop	Passes	Fails	
Corn*	\$35,000	\$8,000	=.6(35000)+(.4)(8000)=24,200
Peanuts	18,000	12,000	=.6(18000)+(.4)(12000)=15,600
Soybeans	22,000	20,000	=.6(22000)+(.4)(20000)=21,200

***Expected value decision = plant corn**

To calculate EVPI we need

2. Expected value of decision given perfect info (the best we could do for each state)

Expected value given PI=.6(35000)+(.4)(20000)=29,000

	Trade Bill	
	P(pass)=.60	P(fails)=0.40
Crop	Passes	Fails
Corn	\$35,000	\$8,000
Peanuts	18,000	12,000
Soybeans	22,000	20,000

So EVPI

=[(EV given PI) – EV]

=29000 – 24200

=\$4,800

… seen this before?

Note: Expected Opportunity Loss (choose option with minimum EOL)

	Trade Bill		
	P(pass)=.60	P(fails)=0.40	
Crop	Passes	Fails	
Corn	0	12,000	=.6(0)+(.4)(12000)=4800
Peanuts	17,000	8,000	=.6(17000)+(.4)(8000)=13,400
Soybeans	13,000	0	=.6(13000)+(.4)(0)=7,800

EOL decision = plant corn

- EVPI = $4,800 and EOL = $4,800
- EVPI(a*) = EOL(a*)!
- Always true: regret measures the difference between the best decision under a state of nature and the decision actually made.
- The EVPI is what we should be willing to pay to avoid the regret of not getting the optimal decision

Example

	Market Acceptance		
	P(Low)=0.4	P(Med)=0.4	P(High)=0.20
Alternatives	Low	Medium	High
AutoPlanner	-4	[illegible]	10
ThermalView	-8	[illegible]	12
CalcForce	-2	3	8

EV(A) = .4(-4) + .4(5) + .2(10) =2.4 = EV(a*)
EV(T) = .4(-8) + .4(6) + .2(12) =1.6
EV(C) = .4(-2) + .4(3) + .2(8) =2.0

EVgivenPI = .4(-2) + .4(6) + .2(12) = 4.0

So

EVPI = 4.0 – 2.4 = 1.6

Another example

Payoff Table	States of Nature, estimated minutes of travel time		
	.60	.30	.10
Alternative Route	Normal traffic	Rush hour traffic	Accident
A: 105-110-10	52	125	98
B: 105-710-10	62	132	75
C: 105-605-10	48	105	68

EV(A) = .6(52) + .3(125) + .1(98) =78.5
EV(B) = .6(62) + .3(132) + .1(75) =84.3
EV(C) = .6(48) + .3(105) + .1(68) =67.1 = EV(a*)

EVgivenPI = .6(48) + .3(105) + .1(68) = 67.1

So

EVPI = 67.1 – 67.1 = 0. Why? Because C dominates the other alternatives. No information would add any benefit.

2. Expected Value of Sample Information

- There is another (common) situation where the value of information can be computed—the expected value of sample information, EVSI
- It is the difference in expected value with and without additional information
- EVSI = $EV_{\text{with info}} - EV_{\text{without info}}$
- Because we have gathered new information we need to update our probabilities … but how?

- Reverend Bayes is back!
- Need to compute posterior probabilities after sampling
- Let's work a problem.

$$posterior\ \ probability = \frac{(prior\ prob.)(likelihood)}{\sum(prior\ prob.)(likelihood)}$$

Investment problem

Decision to invest in apartments, office Buiding, or warehouse. Payoffs in thousands

	States of nature	
P(state)=	.60	.40
Project	Good	Poor
Apartments	50	30
Office Bldg	100	-40
Warehouse	30	10

The basic calculations:

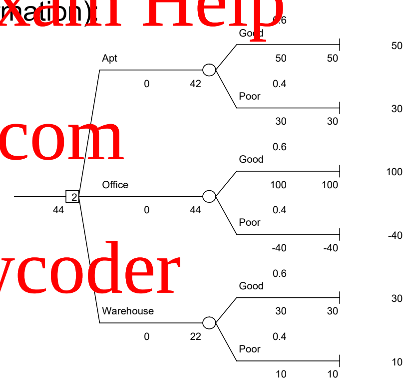
	States of nature		
P(state)=	.60	.40	
Project	Good	Poor	
Apartments	50	30	=.6(50)+.4(30)=42
Office Bldg	100	-40	=.6(100)+.4(-40)=44
Warehouse	30	10	=.6(30)+.4(10)=22

The basic calculations: max EV=44

EV given PI = .60(100)+.40(30)=72

EVPI = EV given PI – EV = 72-44=28

The decision tree with payoffs (and no added information):



Now the decision maker decides to obtain sample information by hiring an economic analyst to forecast future economic conditions. A report will be provided indicating a positive result for good economic conditions or negative for poor economic conditions.

- Based on consultant's past record of forecasting, the decision maker has estimated the conditional probabilities for the following events:

  Let
  - g=event good economic conditions
  - p=poor economic conditions
  - P=positive economic report
  - N=negative economic report

- The consultant's record:
  - P(P|g)=.80
  - P(N|g)=.20
  - P(P|p)=.10
  - P(N|p)=.90
- These values represent the likelihood part of Baye's rule

$$posterior\ probability = \frac{(prior\ prob.)(likelihood)}{\sum (prior\ prob.)(likelihood)}$$

- From other sources (e.g., published forecasts of indices), the decision maker also has estimates of the prior probabilities for the states of nature:
  - P(g)=.60
  - P(p)=.40
- Given the prior and likelihood values, we can compute the posterior probabilities for the states of nature using Bayes rule.

$$posterior\ probability = \frac{(prior\ prob.)(likelihood)}{\sum (prior\ prob.)(likelihood)}$$

- Posterior probabilities (from Bayes rule)

$$posterior\ probability = \frac{(prior\ prob.)(likelihood)}{\sum (prior\ prob.)(likelihood)}$$

- Bayes rule: $P(X_r \mid Y) = \dfrac{P(Y \mid X_r)P(X_r)}{\sum_i P(Y \mid X_i)P(X_i)}$
- For this example:

$$P(g \mid P) = \frac{P(P \mid g)P(g)}{\left(P(P \mid g)P(g) + P(P \mid p)P(p)\right)}$$

$$= \frac{.80(.60)}{\left(.80(.60) + .10(.40)\right)} = \frac{.48}{.52} = 0.923$$

And the other states:

$$P(g \mid N) = \frac{P(N \mid g)P(g)}{(P(N \mid g)P(g) + P(N \mid p)P(p))} = \frac{.20(.60)}{(.20(.60) + .90(.40))} = \frac{.12}{.48} = 0.25$$

$$P(p \mid P) = \frac{P(P \mid p)P(p)}{(P(P \mid p)P(p) + P(P \mid g)P(g))} = \frac{.10(.40)}{(.10(.40) + .80(.60))} = \frac{.04}{.52} = 0.077$$

$$P(p \mid N) = \frac{P(N \mid p)P(p)}{(P(N \mid p)P(p) + P(N \mid g)P(g))} = \frac{.90(.40)}{(.90(.40) + .20(.60))} = \frac{.36}{.48} = 0.75$$

Left hand side values are in P(A|B) form (conditional)

With the posterior probabilities we can calculate the probabilities on the branches of the decision tree.

Starting with conditional probability equation: P(AB)=P(A|B)P(B)

We want P(P) and P(N) so

- P(P)=denominator of bayes calculation for branch P
- P(P)=P(Pg)+P(Pp)=P(g)P(P|g) + P(p)P(P|p) =0.52

- P(N)=denominator of bayes calculation for branch N
- P(N)=P(Ng)+P(Np)=P(g)P(N|g) + P(p)P(N|p) =0.48

Compute branch probabilities using table:

$$posterior\ probability = \frac{(prior\ prob.)(likelihood)}{\sum (prior\ prob.)(likelihood)}$$

	States of nature	Prior prob.	Conditional (likelihoods)	Prior prob x likelihood	Posterior prob.
Pos	Good Conditions	P(g) =.60	P(P|g) =.80	P(Pg) =.48	=.48/.52 =.923
	Poor Conditions	P(p) =.40	P(P|p) =.10	P(Pp) =.04	=.04/.52 =.077
	totals	1.00		.52	1.000
Neg	Good Conditions	P(g) =.60	P(N|g) =.20	P(Ng) =.12	=.12/.48 =.25
	Poor Conditions	P(p) =.40	P(N|p) =.90	P(Np) =.36	=.36/.48 =.75
	totals	1.00		.48	1.000

P(P)=denominator of bayes calculation for branch “P” = .52

P(N)=denominator of bayes calculation for branch “N” = .48

Now we can revise the decision tree to show the effect of (new) sample information.

Display tree

Now (finally) the EVSI can be calculated:

- EVSI = $EV_{\text{with info}} - EV_{\text{without info}}$
- From example, the expected value of the decision is $63.194 with sample info
- Without the sample information we use the original value of 44 See tree A
- So EVSI = 63.194 – 44 = 19.194
- This implies the decision maker should be willing to pay the economic analyst up to $19,194 for the forecast.

- Note that the cost of consultant was zero in this example.
- What if hiring consultant cost $5k?
- Where does it go in the tree? See tree B
- There are two ways
  - Calculate net payoff by subtracting cost at end of tree branches
  - Wait until rollback of tree calculations and subtract the test cost when test is performed
- Like this…

The EVSI does NOT include cost of sampling, CS

Net payoff approach (Tree C)

EVSI = [58.194+5*] – 44 = 19.194

EMV cost with SI

EMV cost without SI

Cost at point of expense (Tree D)

EVSI = [63.194**] – 44 = 19.194

Same result

*need to add the CS back to get correct EVSI if net payoffs used
**CS not included

- To determine if sampling is cost effective compute expected net gain of sampling, ENGS:
  - If positive, sample
  - If negative, not worth the sampling cost
- Let CS = "cost of sampling"

ENGS = EVSI – CS

Net payoff approach (Tree C)

ENGS = [58.194*] – 44 = 14.194

CS included

Cost at point of expense (Tree D)

CS included

ENGS = [63.194**] – 44 – 5 = 14.194

Same result

*included in net payoffs
**not included in gross payoffs so subtract here

EVPI and EVSI Part II

Decision Theory

- More on utility—how to find the certainty equivalent
- How to identify risk attitude
- Other methods for calculating EV of information; ENGS

Machine shop problem:

A machine shop owner is attempting to decide whether to purchase a new drill press, a lathe, or a grinder. The return from each will be determined by whether the company succeeds in getting a government military contract. The profit or loss from each purchase and the probabilities is in the following table.

	Profit	
	P = 0.40	P=0.60
State of nature	Contract	No Contract
Polisher	$40000	$-8000
Lathe	20000	4000
Stamper	12000	10000

The machine shop owner is considering hiring a military consultant to ascertain whether the shop will get the government contract. The consultant is a former military officer who uses various personal contacts to find out such information. By talking to other shop owners who have hired the consultant, the owner has estimated a .70 probability that the consultant would present a favorable report, given that the contract is awarded to the shop, and a .80 probability that the consultant would present an unfavorable report, given that the contract is not awarded. Using decision tree analysis, determine the decision strategy the owner should follow, the expected value of this strategy, and the maximum fee the owner should pay the consultant.

Consider the machine shop problem

	Profit	
	P = 0.40	P=0.60
State of nature	Contract	No Contract
Polisher	$40000	$-8000
Lathe	20000	4000
Stamper	12000	10000

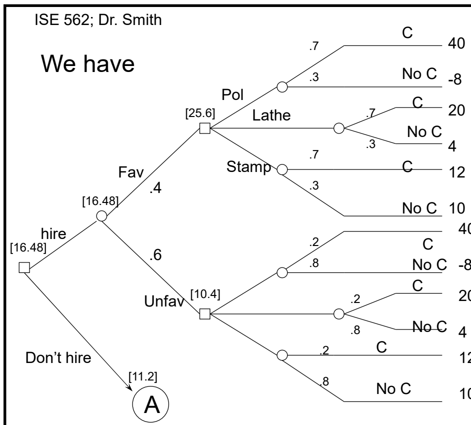
EVPI = EV given PI – EV(a*)

EV given PI = .40(40) + .60(10) = 22

EV(a*)= .4(40) – .60(-8) = 11.2

So EVPI = 22 – 11.2 = 10.8

We have



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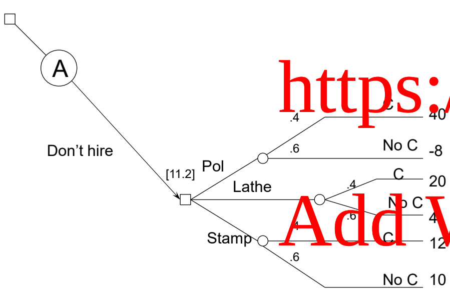


Bayes Rule Calculation With <u>Imperfect</u> Information

Favorable Case

State, θ	Prior(θ)	P(F|θ)	P(F|θ) P(θ)	Post(θ|F)
Contract	0.4	0.7	0.28	0.7
No Contract	0.6	0.2	0.12	0.3
			0.40	1.00

Unfavorable Case

State, θ	Prior(θ)	P(U|θ)	P(U|θ) P(θ)	Post(θ|U)
Contract	0.4	0.3	0.12	0.2
No Contract	0.6	0.8	0.48	0.8
			0.60	1.00

Bayes Rule Calculation With <u>Perfect</u> Information

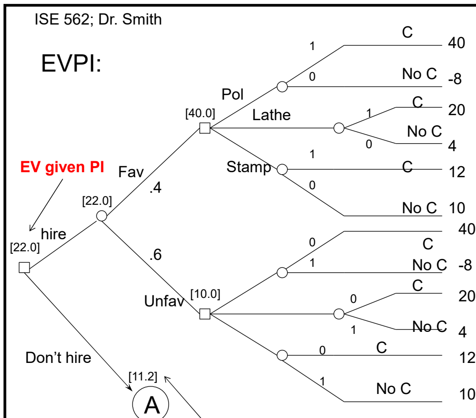
Favorable Case

State, θ	Prior(θ)	P(F|θ)	P(F|θ) P(θ)	Post(θ|F)
Contract	0.4	1.0	0.40	1.0
No Contract	0.6	0.0	0	0
			0.40	1.00

Unfavorable Case

State, θ	Prior(θ)	P(U|θ)	P(U|θ) P(θ)	Post(θ|U)
Contract	0.4	0.0	0	0
No Contract	0.6	1.0	.60	1.0
			0.60	1.00

EVPI:



EV(a*) without PI

Difference = EVPI = 22 – 11.2 = 10.8

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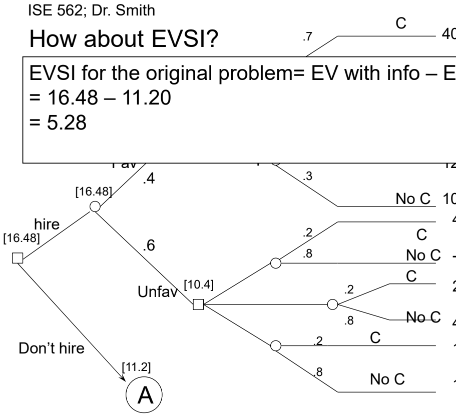
How about EVSI?

EVSI for the original problem= EV with info – EV without info
= 16.48 – 11.20
= 5.28



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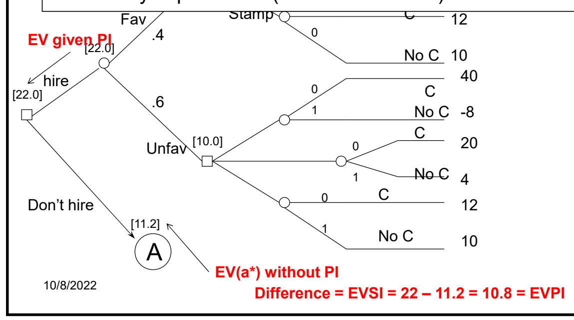
EVSI with perfect information = EV with info – EV without info = 22 – 11.20 = 10.8 = EVPI for the original problem—we have maxed out the EVSI because the prediction is perfect; the EVSI is lower for the original problem because of the uncertainty in prediction (the value is lower).



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How to find the certainty equivalent

- What is it?
  - The point where the decision maker is indifferent between a lottery (gamble) versus certainty of receiving some payoff value.

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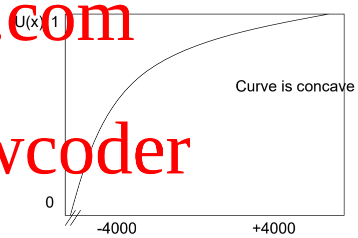
Example: CE

- Consider the positive increasing utility function over the range -4000 to +4000:



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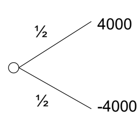
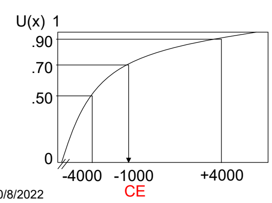
- Now suppose we have the lottery:



The EU
=1/2(.90)+1/2(.50)=0.70



The value of .70 corresponds to x= –1000

So DM is indifferent between lottery and – 1000 for certain

The CE = -1000

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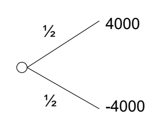
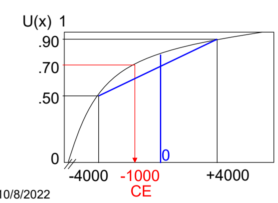
- Note that if CE<EMV, DM is risk averse:



The EU
=1/2(.90)+1/2(.50)=0.70

EMV= ½ (4000)+ ½(-4000)=0



CE = -1000 < 0 so risk averse

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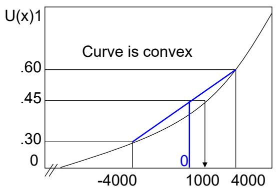
Now suppose utility function looks like



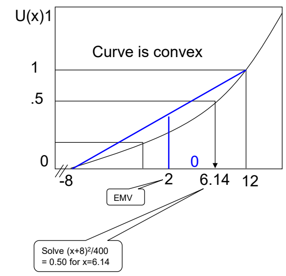
EMV= ½ (4000) + ½(-4000)=0

EU = ½ (.6)+ ½ (.3) =0.45

The value of .45 corresponds to +1000 so CE=+1000

In this case CE > EMV so DM is a risk taker

Wednesday exam:



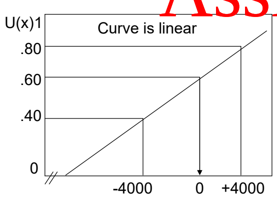
EMV= ½ (-8) + ½(12)= 2

EU = ½ (0)+ ½ (1) =0.50

The value of .50 corresponds to +6.14 so CE=+6.14

In this case CE > EMV so DM is a risk taker

Finally, suppose utility function looks like



EMV= ½ (4000) + ½(-4000)=0

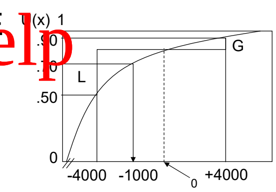
EU = ½ (.8)+ ½ (.4) =0.60

The value of .60 corresponds to 0 so CE=0

In this case CE = EMV so DM is risk neutral

Method in book for CE

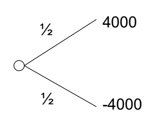
G= gain in utility if lottery is won
L= loss in utility if lottery is lost



Curve is concave

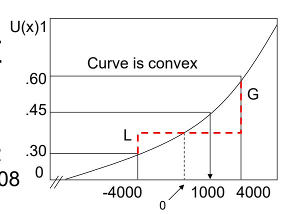
G =U(4000) – U(0)= .9 -.8 = .10
L =U(0) – U(-4000) = .8 - .5 = .30

If G-L < 0 then risk averse
If G-L > 0 then risk taker
If G-L = 0 then risk neutral



Since G-L<0, risk averse

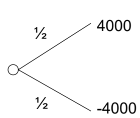
Method in book for CE



Curve is convex

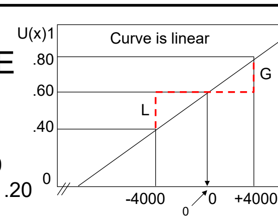
G =U(4000) – U(0)= .60 -.38 = .22
L =U(0) – U(-4000) = .38 - .30 = .08

If G-L < 0 then risk averse
If G-L > 0 then risk taker
If G-L = 0 then risk neutral



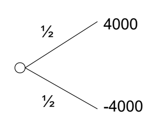
Since G-L>0, risk taker

Method in book for CE



G =U(4000) – U(0)= .8 -.60 = .20
L =U(0) – U(-4000) = .60 - .40 = .20

If G-L < 0 then risk averse
If G-L > 0 then risk taker
If G-L = 0 then risk neutral



Since G-L=0, risk neutral

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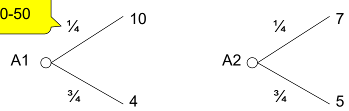
Question 2

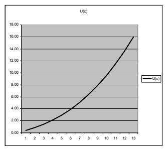
Suppose we have two alternatives:

Not 50-50



And utility function: $U(x)=\frac{1}{36}(x-4)^2 \quad 4\le x\le 10$

$and \quad x=6\sqrt{U(x)}+4$



Find the CE's

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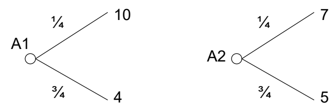
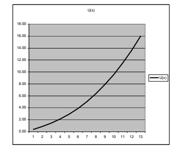
Question 2

First calculate EV's

$U(x)=\frac{1}{36}(x-4)^2 \quad 4\le x\le 10$

$and \quad x=6\sqrt{U(x)}+4$





EV(A1)= ¼ (10)+ ¾ (4) = 5.5
EV(A2)= ¼ (7)+ ¾ (5) = 5.5

EU(A1)= ¼ (1)+ ¾ (0)= +.25
EU(A2)= ¼ (.25)+ ¾ (.0278)= +0.083
Now find the CE's

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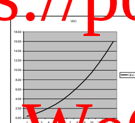
Question 2

Find CE(A1) by computing the value of x for which U(x)=0.25 and 0.083

$x=6\sqrt{U(x)}+4 \qquad x=6\sqrt{U(x)}+4$

$x=6\sqrt{.25}+4 \qquad x=6\sqrt{.083}+4$

$=7 \qquad =5.73$



The CE(A1)=7 and CE(A2)=5.73
Based on CE choose A1.

Note that G=U(10)-U(5.5)=1-(0.0625)= 0.9375 and L=U(5.5)-U(4)= 0.0625 – 0 = 0.0625; G-L>0: risk taker

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Representing utility functions

- Suppose we would like to "fit" a mathematical function for the utility of a decision attribute.
- Must be adjustable for risk attitude
- Relatively easy to calibrate

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Consider the following:

For monotonically increasing utility (more is better than less)

$$u(x)=\begin{cases}\dfrac{\exp\left[\dfrac{-(x-Low)}{\rho}\right]-1}{\exp\left[\dfrac{-(High-Low)}{\rho}\right]-1} & \rho\ne\infty\\[2ex] \dfrac{x-Low}{High-Low} & otherwise\end{cases}$$

For monotonically decreasing utility (less is better than more)

$$u(x)=\begin{cases}\dfrac{\exp\left[\dfrac{-(High-x)}{\rho}\right]-1}{\exp\left[\dfrac{-(High-Low)}{\rho}\right]-1} & \rho\ne\infty\\[2ex] \dfrac{High-x}{High-Low} & otherwise\end{cases}$$

where Low and High are the endpoints of the range and
$\rho<\alpha(High-Low)$ where $\alpha=0.10$ indicates highly risk averse
$\rho>\alpha(High-Low)$ where $\alpha=10$ indicates risk neutral
$\alpha<0$ indicates risk seeking

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Consider the following:

- Utility function for miles per gallon varies from 15 to 30 mpg
- Low = 15
- High = 30
- CE = 22.5; so u(22.5) = 0.50
- Monotonically increasing utility (more mpg preferred to less mpg)

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Then

For monotonically increasing utility (more is better than less)

$$u(x)=\begin{cases}\dfrac{\exp\left[\dfrac{-(x-15)}{\rho}\right]-1}{\exp\left[\dfrac{-(30-15)}{\rho}\right]-1} = \dfrac{\exp\left[\dfrac{-(x-15)}{\rho}\right]-1}{\exp\left[\dfrac{-15}{\rho}\right]-1}\end{cases}$$

where Low and High are the endpoints of the range and
$\rho < \alpha(High - Low)$ where $\alpha = 0.10$ indicates highly risk averse
$\rho > \alpha(High - Low)$ where $\alpha = 10$ indicates risk neutral
$\alpha < 0$ indicates risk seeking

We want the function to go through the endpoints and the CE

Demo for above utility function

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Then

For monotonically increasing utility (more is better than less)

Demo for $U(x)=(x-4)^2/36$ $4\le x \le 10$

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Decision analysis and value of information

- Other methods for calculating EV of information
- So far
  - EVPI = (EV given PI) – EV(a*)
  - EVSI = (EV with SI) – (EV without SI)

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Could also use

Expected value of PI = EL(a*)

1. Compute loss table from payoff table
2. Calculate EL(a_i)
3. Optimal decision = minimum expected loss which occurs at EL(a*)
4. EVPI = EL(a*)

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To calculate EVPI we used

Expected value given PI =.6(35000)+(.4)(20000)=29,000
Expected value(a*) =.6(35000)+(.4)(8000)=24,200

	Trade Bill	
	P(pass)=.60	P(fails)=0.40
Crop	Passes	Fails
Corn	$35,000	$8,000
Peanuts	18,000	12,000
Soybeans	22,000	20,000

So EVPI
=[(EV given PI) – EV]
=29000 – 24200
=$4,800

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Like this

Expected value of PI = Expected loss(a*)
=.6(0)+(.4)(12000)=4,800

Regret	Trade Bill		
	P(pass)=.60	P(fails)=0.40	
Crop	Passes	Fails	
Corn	$0	$12,000	EL = $4800 =EL(a*)
Peanuts	17,000	8,000	EL = $13400
Soybeans	13,000	0	EL = $7800

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Could also use

- EVPI = ΣVPI(θ)P(θ)
- Where VPI(θ)=R(a,θ) – R(a*,θ)
- Or, if continuous case:
- EVPI = $\int$VPI(θ)f(θ)dθ
- Note: EVPI can <u>never</u> be negative since
  - R(a_θ,θ) – R(a*,θ) $\geq$ 0 for all a; a_θ=opt. with perfect info.
  - P(θ) and f(θ) $\geq$ 0 (axiom of probability)

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2. Expected Value of Sample Information

- EVSI = $EV_{\text{with info}} - EV_{\text{without info}}$

Another way:

- EVSI = ΣVSI(y)P(y) where y is the sample result

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- EVSI = $EV_{\text{with info}} - EV_{\text{without info}}$
- Another way:
- EVSI = ΣVSI(y)P(y) where y is the sample result

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- Another value to consider is the calculation of expected net gain of sampling ENGS = EVSI – CS
- Just the EVSI minus the cost of sampling
- If ENGS>0, should take the sample.
- If ENGS<0, then cost of sampling is higher than EVSI is worth.

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<u>Current glossary of terms</u>

- a*: optimal decision
- CE: certainty equivalent
- CS: cost of sampling
- DM: decision maker
- E[·]: expectation operator
- EV: expected value
- EMV: expected monetary value
- ENGS: expected net gain of sampling
- EL: expected loss
- EOL: expected opportunity loss
- ERPI; EVPI: expected return/payoff under perfect information
- EU: expected utility
- R(a): book notation for return (payoff) of alternative a.
- Utility: functional assignment of numerical preference value to levels of an attribute (U(x): x$\rightarrow$[0, 1])
- RP: risk premium
- VPI(θ): value of perfect information

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