Assessing Subjective Probability Distributions

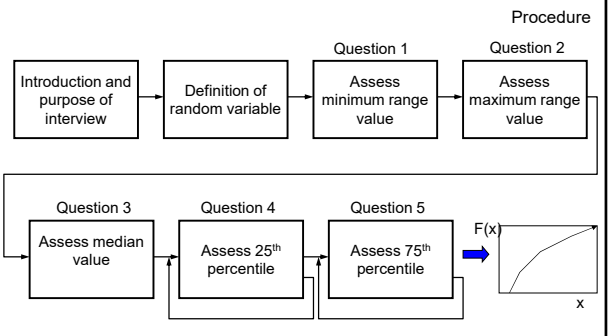
Decision Theory

- There are numerous methods for subjective probability assessment
  - Direct assessment (ask for the value)
  - Odds methods
  - Props (wheels, rulers, software tools)
  - Lottery methods
    - Fixed consequences, variable probabilities
    - variable consequences, fixed probabilities

- Generally:
  - The simpler the method, tendency for lower accuracy with the benefit of speed.
  - The more complex the method, accuracy tends to increase at the expense of more time
  - The method described here falls somewhere between these extremes
    - Easier for the interviewee (binary choices)
    - Captures probabilities (CDF's)
    - Precision of assessment can be selected by the interviewer (3-point, 5-point, 7-point distributions)

Interval splitting method

- 2 questions to bound the range (zero and 100th percentile.
- 1 question to establish the median (50th percentile).
- A series of questions to assess the (25th percentile).
- A series of questions to assess the (75th percentile)



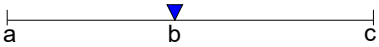
X = name of the random variable

1. *What is the value of [X] for which there is one chance in a hundred that [X] will be less than that value?*
2. *What is the value of [X] for which there is one chance in a hundred that [X] will be greater than that value?*
3. *What is the value of [X] between the above for which there is an equal chance that [X] will be greater than the value and less than the same value? (The median.)*

Questions 4,5:

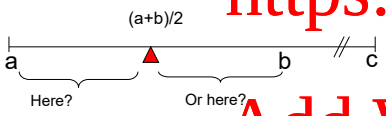
- Respondent is asked to make a choice between A or B where the question is
  - "Do you think the value of x is:
    - A) "less than x?"
    - B) "between y1 and y2?"
- If the answer is A (less than x), then the next set of questions bisects the interval [y1, y2] on the lower side.
- If the answer is B (between), then the next set of questions bisects the interval [y1, y2] on the upper side of the interval.
- So how are the values determined?

- Suppose the answers to questions 1,2,3 were a,c,b

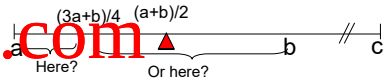


- The questions for the 25th percentile will focus on the lower interval from a to b.
- Compute the midpoint of the lower interval and ask the question:

- Do you think it is more likely the value of x is less than (a+b)/2 or between (a+b)/2 and b?

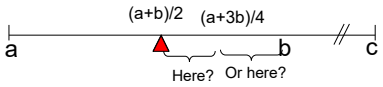


- If the answer is the lower interval, subdivide the lower interval in half and ask the same question:



- Is it more likely that the value is below the midpoint ((3a+b)/4) or in the upper interval [(3a+b)/4, b]?

- If the answer is the upper interval, subdivide the upper interval in half and ask the same question:

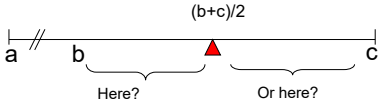


- Is it more likely that the value is below the midpoint ((a+3b)/4) or in the upper interval [(a+3b)/4, b]?

- Repeat this process until the interviewee can't choose (the differences will be too small)
- The value of x when this occurs will be the 25th percentile value.
- $P(X \le x) = 0.25$

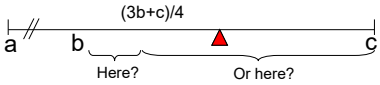
75th percentile

- Do you think it is more likely the value of x is between b and (b+c)/2 or greater than (b+c)/2?



75th percentile

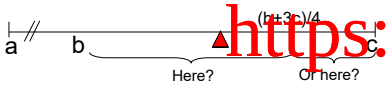
- If the answer is the lower interval, subdivide the lower interval in half and ask the question:



- Is it more likely that the value of x is between b and (3b+c)/4 or greater than (3b+c)/4?

75th percentile

- If the answer is the upper interval, subdivide the upper interval in half and ask the same question:



- Is it more likely that the value is between b and (b+3c)/4 or greater than (b+3c)/4?

75th percentile

- Repeat this process until the interviewee can't choose (the differences will be too small)
- The value of x when this occurs will be the 75th percentile value.
- P(X≤x) = 0.75

Example

- We want to assess the probability distribution of our interviewee's height in inches
- Questions noted in blue
- Answers noted in red

Example

- "What is the value of my height for which there is one chance in a hundred that my height will be less than that value?"
- "56 inches"

Example

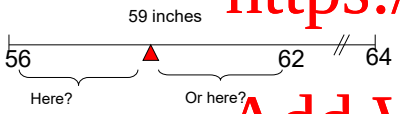
- "What is the value of my height for which there is one chance in a hundred that my height will be more than that value?"
- "Let's say 64 inches"

Example

- "What is the value of height for which there is an equal chance that height will be greater than the value and less than the same value? (The median.)"
- "62 inches."

25th percentile

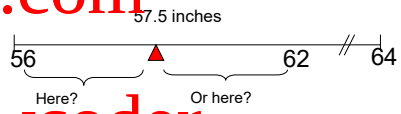
- Do you think it is more likely the value of height is less than 59 inches or between 59 inches and 62 inches?



- "I think its more likely it's less than 59 inches."

25th percentile

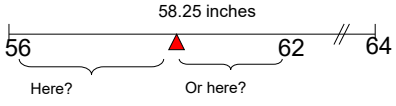
- Do you think it is more likely the value of height is less than 57.5 inches or between 57.5 inches and 62 inches?



- "I think its more likely it's between 57.5 and 62 inches."

25th percentile

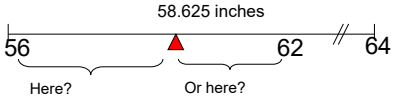
- Do you think it is more likely the value of height is less than 58.25 inches or between 58.25 inches and 62 inches?



- "I think its more likely it's between 58.25 and 62 inches."

25th percentile

- Do you think it is more likely the value of height is less than 58.625 inches or between 58.625 inches and 62 inches?

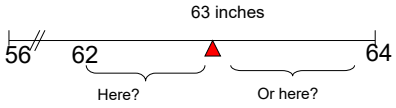


- "I can't really tell; let's say 58.6."

$P(X \leq 58.6) = 0.25$

75th percentile

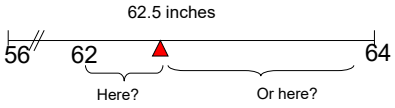
- Do you think it is more likely the value of height is between 62 and 63 inches or greater than 63 inches?



- "It's more likely that its lower than 63."

75th percentile

- Do you think it is more likely the value of height is between 62 and 62.5 inches or greater than 62.5 inches?



- "I can't tell beyond this—say 62.25."

$P(X \le 62.25) = 0.75$

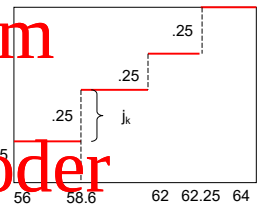
Results

- Now we can assemble the values

Height, x	P(X≤x)
56	0.0
58.6	0.25
62.00	0.50
62.25	0.75
64.00	1.0

Computing the moments

- We can use Riemann-Stieltjes integration to calculate the mean and variance
- Redraw the CDF as series of step functions



Computing the moments

$$\int_a^b g(x)dh(x) = \lim_{n\to\infty}\sum_{k=1}^{m} g(x_k)\,j_k = \sum_{k=1}^{m} g(\eta_k)\,j_k$$

$$so\ \mu = E[x] = \int_{56}^{64}(g(x)=x)dF(x) = \sum_{k=1}^{4} x\cdot j_k$$

$$= 58.6(.25) + 62.0(.25) + 62.25(.25) + 64.0(.25)$$

$$= 61.7$$

$$and\ \sigma^2 = V[x] = E[x^2] - \mu^2$$

$$= 58.6^2(.25) + 62.0^2(.25) + 62.25^2(.25) + 64.0^2(.25) - \left(61.7125^2\right)$$

$$= 3.823$$

$$or\ \sigma = \pm 1.96$$

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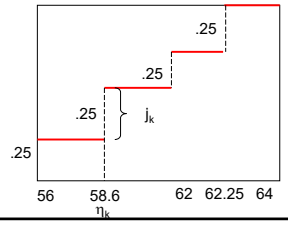

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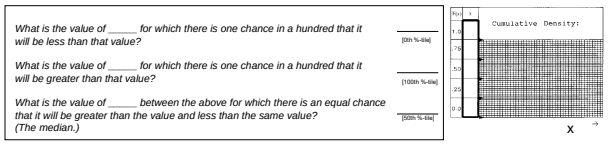

ISE 562; Dr. Smith

- Class Exercise:
  - Assess the probability distribution for Dr. Smith's weight in pounds or kilograms
  - Determine the 0, .25, .50, .75, and 1.0 percentile values; write them down
  - If you choose to use kilograms, convert the percentile values to pounds (multiply by 2.2)

10/15/2022 31