Multiattribute Decision Analysis

Decision Theory

Today

- What is multiattribute (multicriteria) decision analysis?
- Relationship between multiattribute decision making and what we have been doing so far?
- How to define effective attributes?

What is multiattribute decision analysis? Also called multiattribute utility theory (MAUT).

- A methodology for providing information to decision makers for comparing and selecting from among complex alternative systems in the presence of uncertainty and risk.

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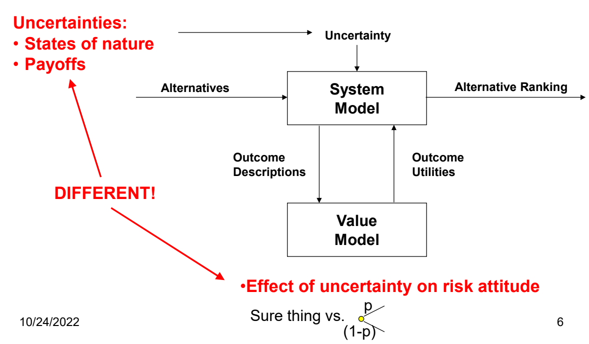
Multiattribute decision analysis?

- Synthesis of techniques from:
  - Operations Research
  - Statistics
  - Economics
  - Mathematics
  - Psychology
- Definitive text: Keeney and Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, 1976.

What is Multiattribute decision analysis?

- A framework that formalizes and synthesizes the relationships between uncertain technical information about alternatives and human values (preferences) that are ultimately used to evaluate the alternatives

Relationship Between the System Model and the Value Model



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Simple example: buying a car based on cost and mpg:

Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32

$\underbrace{\hspace{3em}}$

System Data

Note tradeoff: cheapest car has worst mpg and most expensive has best mpg

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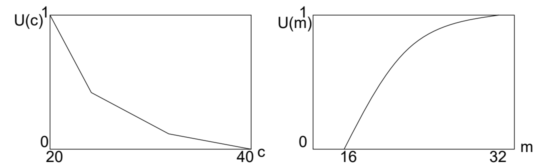
Alternative	Cost,c	Mpg,m
Car A	20	16
Car B	30	25
Car C	40	32



Value Data (utility functions)

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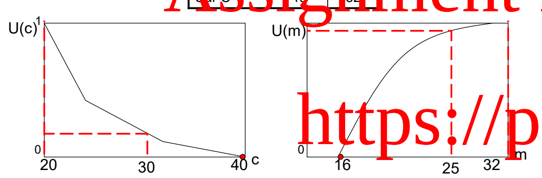
Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32



<u>20≤x≤40</u>
U(x*)=1; U(x°)=0
U(40)=0, U(20)=1
For Car A, U(20)=1.0
For Car B, U(30)=.17
For Car C, U(40)=0.0

<u>16≤x≤32</u>
U(x*)=1; U(x°)=0
U(16)=0, U(32)=1
For Car A, U(16)=0.0
For Car B, U(25)=.93
For Car C, U(32)=1.0

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System Data ($ and mpg)

Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32

Data that describe the alternatives in terms of their physical characteristics (fact-based)

Value Data (attribute utilities)

Alternative	Cost	mpg
Car A	1.0	0.0
Car B	.17	.93
Car C	0.0	1.0

Data that describe the decision maker's values and priorities (importance) to the decision maker (values-based)

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System Data ($ and mpg)

Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32

Value Data (attribute utilities)

Alternative	Cost	mpg
Car A	1.0	0.0
Car B	.17	.93
Car C	0.0	1.0

?

Now what? How do we combine the system data with the value data to evaluate the alternatives?

Answer: compute the *<u>multiattribute</u>* utility function, $U(x_1,\ldots x_n)=f(u(x_1),\ldots,u(x_n))$

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System Data ($ and mpg)

Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32

Value Data (attribute utilities)

Alternative	Cost	mpg
Car A	1.0	0.0
Car B	.17	.93
Car C	0.0	1.0

?

We have the value of each attribute for each alternative which tells us how the decision maker values different levels of each attribute

But, something is missing here…

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System Data ($ and mpg)

Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32

Value Data (attribute utilities)

Alternative	Cost	mpg
Car A	1.0	0.0
Car B	.17	.93
Car C	0.0	1.0

?

So, is Car A = 1+0 =1?
No.

What do you think is missing?

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System Data ($ and mpg)

Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32

Value Data (attribute utilities)

Alternative	Cost	mpg
Car A	1.0	0.0
Car B	.17	.93
Car C	0.0	1.0

?

Which is more important to the decision maker, Cost or mpg?

The evaluation of two attributes requires not only the utility value of the attribute, but also the relative importance of one attribute versus another.

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System Data ($ and mpg)

Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32

Value Data (attribute utilities)

Alternative	Cost	mpg
Car A	1.0	0.0
Car B	.17	.93
Car C	0.0	1.0
	k_1	k_2

?

The measure of relative importance is called the <u>attribute tradeoff scaling constant</u>, k_i.

Sometimes called an importance weight but similar to a marginal rate of substitution in economics that represents DM's tradeoff rate for preference of attribute 1 in response to changes in attribute 2.

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System Data ($ and mpg)

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Car A	20	16
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Value Data (attribute utilities)

Alternative	Cost	mpg
Car A	1.0	0.0
Car B	.17	.93
Car C	0.0	1.0

?

Suppose the attribute tradeoff scaling constants for cost and mpg are
k_1=0.60
k_2=0.40
As you might guess we could calculate the utility of Car A as
U(Car A) = $U(x_1,x_2)$= .60(1)+.40(0) =0.60

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System Data ($ and mpg)

Alternative	Cost	mpg
Car A	20	16
Car B	30	25
Car C	40	32

Value Data (attribute utilities)

Alternative	Cost	mpg
Car A	1.0	0.0
Car B	.17	.93
Car C	0.0	1.0

U(A)=.60
U(B)=.47
U(C)=.40

U(A)= .6(1)+.4(0) = .60

U(B)= .6(.17)+.4(.93) = .47

U(C)= .6(0)+.4(1) = .40

We conclude Car A > Car B > Car C

There are a variety of questions you should be thinking about …

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System Data ($ and mpg)

Alternative	Cost	mpg
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Alternative	Cost	mpg
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U(A)= .6(1)+.4(0) = .60

U(B)= .6(.17)+.4(.93) = .47

U(C)= .6(0)+.4(1) = .40

Why is A preferred? <u>Not</u> because U(A)>U(B) and U(C).

- Because A has the highest utility for the most important attribute (k_{cost} = 0.60 vs. k_{mpg}=0.40) whereas B and C score poorly on cost.

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- What is the theoretical basis for MAU?
- What assumptions are being made to enable the multiattribute utility model?
- Where do the attribute tradeoff scaling constants come from?
- What if the attribute states are uncertain?

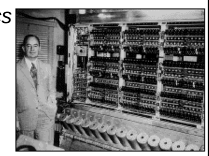
- What is the theoretical basis for MAU?
  – "Specimen Theoriae Novae De Mensura Sortis," 1738, Daniel Bernoulli
  – "Theory of Games and Economic Behavior," John von Neumann and Oskar Morgenstern, 1947.

Our heros!

- Daniel Bernoulli was the son of Johann Bernoulli. He was born in Groningen while his father held the chair of mathematics there. His older brother was Nicolaus(II) Bernoulli and his uncle was Jacob Bernoulli so he was born into a family of leading mathematicians but also into a family where there was unfortunate rivalry, jealousy and bitterness. Johann was determined that Daniel should become a merchant and he tried to place him in an apprenticeship. However Daniel was as strongly opposed to this as his own father had been and soon [illegible] relented but certainly not as far as to let Daniel study mathematics. Johann declared that there was no money in mathematics and so he sent Daniel back to Basel University to study medicine. This Daniel did spending time studying medicine at Heidelberg in 1718 and Strasbourg in 1719. He returned to Basel in 1720 to complete his doctorate in medicine.
- An important work which Daniel produced while in St Petersburg was one on probability and political economy. Daniel makes the assumption that the moral value of the increase in a person's wealth is inversely proportional to the amount of that wealth. He then assigns probabilities to the various means that a person has to make money and deduces an expectation of increase in moral expectation. Daniel applied some of his deductions to insurance.
- Daniel Bernoulli did produce other excellent scientific work during these many years back in Basel. In total he won the Grand Prize of the Paris Academy 10 times, for topics in astronomy and nautical topics. He won in 1740 (jointly with Euler) for work on Newton's theory of the tides; in 1743 and 1746 for essays on magnetism; in 1747 for a method to determine time at sea; in 1751 for an essay on ocean currents; in 1753 for the effects of forces on ships and in 1757 for proposals to reduce the pitching and tossing of a ship in high seas. Another important aspect of Daniel Bernoulli's work that proved important in the development of mathematical physics was his acceptance of many of Newton's theories and his use of these together with the tolls coming from the more powerful calculus of Leibniz. Daniel worked on mechanics and again used the principle of conservation of energy which gave an integral of Newton's basic equations. He also studied the movement of bodies in a resisting medium using Newton's methods.

Source: http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Bernoulli_Daniel.html

"If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.
von Neumann



Von Neumann was a child prodigy, born into a banking family is Budapest, Hungary. When only six years old he could divide eight-digit numbers in his head. He received his early education in Budapest, studied Chemistry, moving his base of studies to both Berlin and Zurich before receiving his diploma in 1925 in Chemical Engineering. He quickly gained a reputation in set theory, algebra, and quantum mechanics. At a time of political unrest in central Europe, he was invited to visit Princeton University in 1930, and when the Institute for Advanced Studies was founded there in 1933, he was appointed to be one of the original six Professors of Mathematics, a position which he retained for the remainder of his life.

Areas of contribution: Quantum mechanics, Hilbert spaces (mathematics), game theory, automata theory, modern computer

Source: http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Von_Neumann.html

1946

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Howard Raiffa

Howard was a cofounder of the Kennedy School of Government, Harvard University and a cofounder of the Negotiation Program of the Harvard Law School and the Kennedy School of Government. He is the Frank P. Ramsey Professor (Emeritus) of Managerial Economics, Harvard University.

One of Howard's best known books is "The Art and Science of Negotiation" (1982), which is still in print and widely used in negotiation courses in schools of business and law. He is the author, co-author or editor of innumerable books.

For his outstanding contributions to the field of decision analysis he was given the Frank P. Ramsey medal by the Operations Research Society of America. He has also been awarded the Distinguished Contributions Award from the Society of Risk analysis, and has been given honorary doctorates by Carnegie Mellon, the University of Michigan, Northwestern University, and Ben Gurion University of the Negev. From the International Association for Conflict Management, he was awarded the Lifetime Achievement Award.

Source: Duke faculty website

Ralph Keeney https://ralphkeeney.com/

EXPERTISE

Professor Keeney's areas of expertise are decision analysis, risk analysis, and management decision-making. He is an authority on decision making with multiple objectives. During the past thirty years, Dr. Keeney has contributed substantially toward the development of decision analysis and risk analysis. His experience includes corporate management problems, risk analyses, energy policy, large-scale siting studies (e.g., airports, power plants), and environmental studies. Dr. Keeney has been a consultant for several organizations including Fair Isaac, Seagate Technology, American Express, British Columbia Hydro, Pacific Gas and Electric, Westinghouse, Kaiser Permanente, Proctor and Gamble, Hewlett-Packard, GTE, Hunton & Williams, the Electric Power Research Institute, Arkansas Power and Light, International Institute of Management (Berlin), Ministry of Public Works (Mexico), U.S. Department of Commerce, U.S. Department of Energy, Environmental Protection Agency, and the Office of Naval Research.

EDUCATION

Ph.D.	Massachusetts Institute of Technology	- 1969. Operations Research.
E.E.	Massachusetts Institute of Technology	- 1968. Electrical Engineering.
S.M.	Massachusetts Institute of Technology	- 1967. Electrical Engineering.
B.S.	University of California, Los Angeles	- 1966. Engineering.

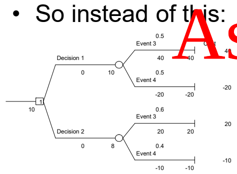
Source: Duke faculty website

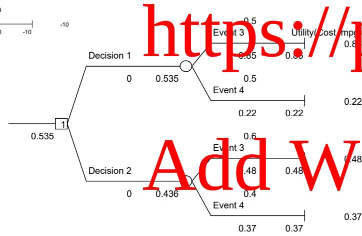
- Bernoulli's contribution—introduced the concept
- Von Neumann's contribution to Keeney-Raiffa multiattribute utility functions:
  - Rationality axioms; greater outcome utility values correspond to more preferred outcomes
  - The utility value to be assigned to a gamble is the expected value of the outcome utilities of the gamble (EU criterion)
- Keeney & Raiffa's contribution: decision analysis framework and theoretical basis for multiple attributes

How is Multiattribute Utility Theory related to what we have learned so far?

- The payoffs at the terminating nodes of a decision tree are multiattribute utility values for the outcome of that node.
- Instead of one attribute with a single payoff or utility, the value for a vector of attributes is collapsed into a single "numeraire" or multiattribute utility for that outcome.

- So instead of this: we have this:





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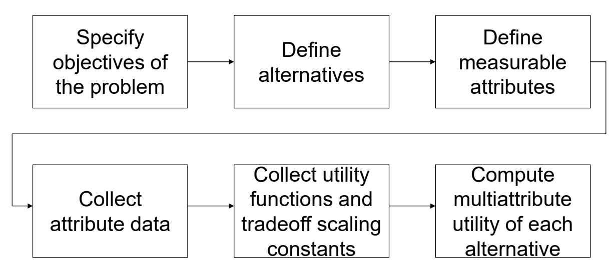
- MAU Pro's
  - More comprehensive by considering multiple factors (attributes) and their tradeoffs
  - Combines system data with decision maker preferences in a rational framework for decision making
  - Organizes the decision problem into logical elements (alternatives, attributes, data, preferences, uncertainty)
  - Model assumptions studied extensively
  - Normative approach—how rational DM <u>should</u> make decision
  - Theoretical foundation

- MAU Con's
  - Assumes decision maker is rational when values of the decision maker may be irrational or non-rational
  - Assessment of utility functions and tradeoff scaling constants can be problematic
    - Labor intensive
    - Subject to biases
  - Normative approach—Model may not capture (describe) how actual decision is made due to external factors (e.g., biases, politics, or hidden attributes)

- Steps in a multiattribute decision problem



- Another view:

What does DM want to do? → What are the choices? → How to measure the choices? → The numbers (data) → The DM's preferences (values) → Rank ordered options

Specify objectives of the problem

- Outcomes that terminate the decision tree can be described using an <u>objectives hierarchy</u>
  - Expresses the preference structure of the DM's from high-level objectives to measurable attributes
  - Constructed in a manner consistent with the quantification and mathematical conditions required by the MAU function of the value model

Objectives Hierarchy Examples

Define Alternatives

- Alternatives must be unique, separable entities whose value can be represented by measurable attributes
  - Non-overlapping
  - Sufficiently different to justify consideration
  - Generally the choice that must be "selected" in order to achieve the goal or objective sought in the objectives hierarchy

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Define measurable attributes

The set of attributes must satisfy the following requirements in order to be a valid representative of the preference structure of the decision makers.

1. <u>Completeness</u>: The set of attributes should characterize all of the factors to be considered in the decision-making process.

2. <u>Comprehensiveness</u>: Each attribute should adequately characterize its associated criterion.

3. <u>Importance</u>: Each attribute should represent a significant criterion in the decision-making process, at least in the sense that the attribute has the potential for affecting the preference ordering of the alternatives under consideration.

Define measurable attributes

The set of attributes must satisfy the following requirements in order to be a valid representative of the preference structure of the decision makers.

4. <u>Measurability</u>: Each attribute should be capable of being objectively or subjectively quantified; technically, this requires that it be possible to establish an attribute utility function for each attribute.

5. <u>Familiarity</u>: Each attribute should be understandable to the decision makers in the sense that they should be able to identify preferences for different states of the attribute for gambles over the states of the attribute.

6. <u>Nonredundancy</u>: Two attributes should not measure the same criterion, thus resulting in double counting.

7. <u>Independence</u>: The value model should be so structured that changes within certain limits in the state of one attribute should not affect the preference ordering for states of another attribute or the preference ordering for gambles over the states of another attribute.

- Steps in a multiattribute decision problem
- Before discussing data collection, let's examine what we are going to do with it.
- That is, how do we obtain the multiattribute utility function, $U(X_1, \ldots, X_n)$?

Collect attribute data → Collect utility functions and tradeoff scaling constants → Compute multiattribute utility of each alternative

- With 2 attributes, x_1, x_2, the trade-offs between them are 1-1 substitutions (x_1 vs. x_2)
- With 3 or more attributes there may be tradeoffs among the attributes. e.g., if we have 3 attributes, the tradeoffs between any pair must consider:
- (x_1, x_2) vs x_3, (x_2, x_3) vs x_1, (x_1, x_3) vs x_2

With 5 attributes, x_1, x_2, x_3, x_4, x_5 the trade-offs between 2 attributes are:

- (x_1, x_2) vs $\{x_3, x_4, x_5\}$
- (x_1, x_3) vs $\{x_2, x_4, x_5\}$
- (x_1, x_4) vs $\{x_2, x_3, x_5\}$
- $\{x_1, x_5\}$ vs $\{x_2, x_3, x_4\}$
- (x_2, x_3) vs $\{x_1, x_4, x_5\}$
- (x_2, x_4) vs $\{x_1, x_3, x_5\}$
- …etc.

(Sets of attributes)

Tradeoff set

Complement set

- Three assumptions are required when there are 3 or more attributes

1. Preferential independence
2. Utility independence
3. Mutual utility independence

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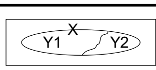
1. Definition: Let x, y, and z be 3 different attributes; the pair of attributes x and y is *preferentially independent* of z if the conditional preferences over the (x,y) space given z, do not depend on z.

Examples:

- If the coffee shop I go to depends on x=cost of coffee, y=quality of coffee, and z=presence of music: and the value tradeoffs between cost of coffee and quality don't depend on presence of music; then {x,y} is preferentially independent of z.
- Similarly, if the value tradeoffs between quality of the coffee and presence of music don't depend on the cost of the coffee, then {y,z} is preferentially independent of x.



Y2=complement of Y1
Y1=complement of Y2

<u>General</u> Definition of Preferential Independence: Let Y1 and X be sets of attributes, $Y \subset X$;

Y1 is *preferentially independent* of its complement Y2 if the preference order of consequences involving only changes in the levels in Y1 does not depend on the levels for which attributes in Y2 are held fixed.

For example

	Y1	Y2	
	x	y	z
Alternative	mpg	Cost	Style (0-10)
Car A	16	20	8
Car B	25	30	9
Car C	32	40	10
Ranges:	16-32	20-40	8-10

Y1 is *preferentially independent* of its complement Y2 if the preference order of consequences involving only changes in the levels in Y1 does not depend on the levels for which attributes in Y2 are held fixed.

How the decision maker values mpg is not affected by differing levels of cost and style

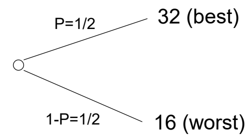
2. Definition: Attribute Y1 is *utility independent* of its complement, Y2, if the conditional preference order for lotteries involving only changes in the levels of attributes in Y1 does not depend on the levels at which the attributes in Y2 are held fixed.

   Note that if Y1 is utility independent, then Y1 is preferentially independent (the reverse is not necessarily true).

The main difference here is that the consequences are not fixed—they are probabilistic outcomes—the independence holds when the outcomes are risky.

For example

Instead of fixed choices for mpg = 16, 25, 32 versus (cost and style) we now have a risky choice:



With cost and style fixed at (a,b)

If we vary (a,b) and the CE for the lottery is unaffected we have utility independence

3. Definition: Attributes $X_1,\ldots,X_n$ are *mutually utility independent* if every subset of $\{X_1,\ldots,X_n\}$ is utility independent of its complement.
   - For the previous example if
     - cost U.I. of {mpg, style}
     - mpg U.I. of {cost, style}
     - Style U.I. of {cost, mpg}
   - Then the attributes {cost, mpg, style} are mutually U.I.

Mutual U.I. allows the construction of the multiattribute utility function

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- Multiattribute utility functions

Let x_i be the level of attribute i; $u_i(x_i)$ be the attribute utility function for attribute i; k_i be the attribute tradeoff scaling constant for attribute i. If the x_i are mutually utility independent then the expression for the multiattribute utility function takes one of the following forms depending on the sum of the k_i:

$$If \sum_{k=1}^{N} k_n = 1.0, \quad then :\Rightarrow U(\vec{x}) = \left\{\sum_{n=1}^{N} k_n \cdot u_n(x_n)\right\} \qquad Eq\ 1$$

$$If \sum_{k=1}^{N} k_n \neq 1.0, \quad then :\Rightarrow U(\vec{x}) = \frac{1}{K}\left\{\prod_{n=1}^{N}[1 + K \cdot k_n \cdot u_n(x_n)] - 1\right\} \qquad Eq\ 2$$

where the master scaling constant, K, is solved from the equation :

$$1 + K = \prod_{n=1}^{N} [1 + K \cdot k_n] \qquad Eq\ 3$$

Notation

Quantity	Variable name
Attribute	x_i
Best (most preferred) attribute state	x_i^*
Worst (least preferred attribute state	x_i^o
Alternative (a vector of attributes)	$<x_1, x_2,\ldots,x_n>$
Single attribute utility function	$u_i(x_i)$
Attribute tradeoff scaling constant	K_i
Multiattribute utility function	$U(x_1, x_2,\ldots,x_n)$

- What is next?
  – How to obtain $U_i()$, k_i?
  – How to check for utility independence?
  – How to calculate the master scaling constant, K?

Value model templates

Example Part 1