Utility Concepts 2

Decision Theory

But first another EVSI example:

You are thinking about investing $1000 in one of 3 stocks, A, B, or C. The return on investment of these stocks depends on the probability that each company will release their latest product in the next 6 months.

Let R = new product release

NR = no product release

The net returns on the investments after 1 year are estimated to be:

Stock	R	NR
A	200	0
B	400	-100
C	300	-200

With prior probabilities of product release:

P(R) = 0.70

P(NR) = 0.30

You have the option of buying an on-line investment newsletter that costs $10/month that tracks whether the new product releases of companies are on schedule or not. Their track record is summarized by the following likelihoods. Let

S = on schedule

L = late/not on schedule

P(S| R) =0.85

P(S|NR) =0.10

P(L|R) =0.15

P(L|NR) =0.90

- What is the EVPI?
- What is the EVSI?
- What is your optimal decision strategy?

- What is the EVPI?

Prob =	0.70	0.30	
Stock	R	NR	EV
A	200	0	140
B	400	-100	250*
C	300	-200	150

- EVPI=EV given PI – EV(a*)

= .7(400)+.3(0) – 250 = 30

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- What is the EVSI?

  = EV with SI – EV without SI
- Need to compute posterior probabilities
- and the decision tree…

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S=sched	P(R)	P(S|R)	Product	Post(R|S)
R	.70	.85	.595	.952
NR	.30	.10	.03	.048
			.625	
L=late		P(L|R)		Post(R|L)
R	.70	.15	.105	.28
NR	.30	.90	.270	.72
			.375	

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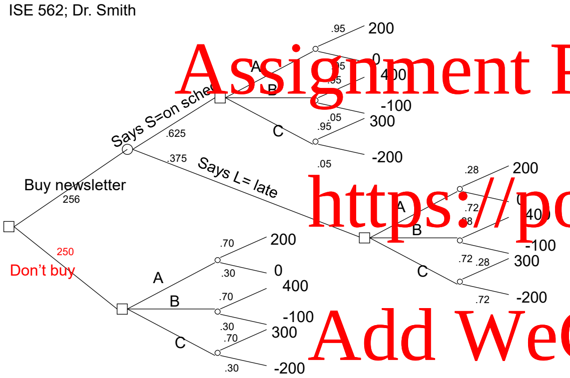

Note: tree does not include cost of sampling

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- EVSI = EV with SI – EV without SI
- EVSI = 256 – 250 = 6
- It appears that the value of additional information (newsletter) is positive. Now the EVSI must be compared to the cost of the information (cost of sampling, CS)
- That is ENGS = EVSI – CS= 6 –10 = -4
- Optimal strategy: don't buy newsletter since EVSI = 6 < CS = 10*

*Does not matter if cost for one month or year; still a negative ENGS

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Back to Utilities…

Utility function, U(X)

- Maps decision attribute X to the interval [0,1] where U(worst case)=0.0 and U(best case)=1.0
- Captures risk attitude
  - Risk neutral
  - Risk seeking
  - Risk averse

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Note that if decision maker is risk neutral, the utility function can be written as:

U(X)=a+bX where X=return of money

- From definition of expected value:

E[U(X)]=E[a+bX]=E[a]+E[bX]=E[a]+bE[X]

- Expected value of constant = constant so,

E[U(X)]=a+bE[X] and E[X]=expected monetary value so,

- EU=a+bEMV b>0, so EU a maximum when EMV a maximum

If utility function is <u>linear</u> with respect to $, then EMV and EU criterion are <u>equivalent</u>

- Assessing utility function from decision maker interviews is labor-intensive
- Worthwhile for high-value or high-cost consequence decisions
- Provides precision beyond simple risk-neutral assumption
- If time, cost, or payoffs limited in scope, are there simplifications available?

Yes

- Risk-neutral case (already discussed)
- 3-point utility function (only assess 0.50 certainty equivalent)
- Use mathematical functions to represent utility
  - $U(X)= aX+b$ (piecewise linear)
  - $U(X)=a-e^{-bX}$ (exponential)
  - $U(X)=a \log(X+b)$

- Suppose we have investments with profits between \$15k and \$30k (more > less); we ask the decision maker, "Which would you prefer?"

IF RATIONAL, SHOULD CHOOSE OPTION B

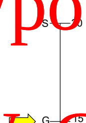
A. Receive profit= 15k for certain

OR

$P_w=1/2$ Receive profit= 30k

B.

$P_L=1/2$ Receive profit= 15k



- Now bump the sure thing up to ¼ the interval from worst value (15+ ¼ (30 – 15)~ 18)

SUPPOSE THEY CHOOSE "B"

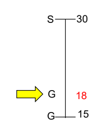
A. Receive profit= 18k for certain

OR

$P_w=1/2$ Receive profit= 30k

B.

$P_L=1/2$ Receive profit= 15k



- They chose B (gamble) so next value between 18 and 30: (18 + 30)/2~ 24)

SUPPOSE THEY CHOOSE "B"

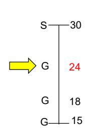
A. Receive profit= 24k for certain

OR

$P_w=1/2$ Receive profit= 30k

B.

$P_L=1/2$ Receive profit= 15k



- Chose gamble so next value between 24 and 30: (24 + 30)/2~ 27)

Suppose they choose B

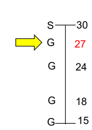
A. Receive profit= 27k for certain

OR

$P_w=1/2$ Receive profit = 30k

B.

$P_L=1/2$ Receive profit = 15k



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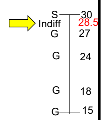
- Chose gamble so next value between 27 and 30: (27 + 30)/2~ 28.5)

NOW SUPPOSE THEY ARE INDIFFERENT BETWEEN A AND B
THE CERTAINTY EQUIVALENT OF THE GAMBLE IS 28.5

THE EXPECTED VALUE OF THE GAMBLE IS .50(15)+.50(30) = 22.5



A. Receive profit = 28.5k for certain

OR

B. P_W=1/2 Receive profit= 30k

P_L=1/2 Receive profit= 15k

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We can now calculate the risk premium—the amount the decision maker should be willing to pay to avoid the risk of the worst case if positive, or take the risk of getting the best case if negative; it is the difference between the expected value and the certainty equivalent (indifference point):

RP=EV – CE

For the example RP= 22.5 – 28.5 = -6

- The negative value indicates the DM is willing to pay to gamble; i.e., they are willing to pay $6,000 to take the risk of receiving a profit of $30,000 (pay to play).

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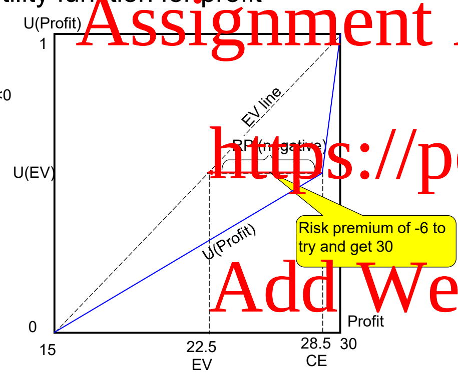
The utility function for profit

RP=EV – CE <0
Risk seeking



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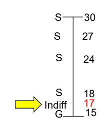
Suppose the decision maker had been indifferent at a CE of $17,000:



Then RP= 22.5 – 17 = +5.5

- The positive value indicates the DM wants to avoid the risk of only receiving 15).
- In this case the decision maker would be willing to pay $5,500 to avoid the chance of receiving a profit of only $15,000.

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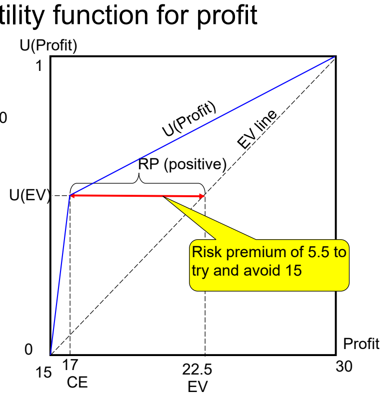
The utility function for profit

RP=EV – CE >0
Risk averse



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Examples

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Example: Risk attitude

Suppose we have the following utility function:

$$U(x) = \{ x^2 \quad 20 \le x \le 100$$

Identify the risk attitude.

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Using the risk premium, RP=EV-CE where

if RP < 0 then risk seeking

if RP > 0 then risk averse

if RP = 0 then risk neutral

Need to compute CE and EV…

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Step 1: find the CE

CE: from expected utility

EU=pU(bestcase) + (1-p)U(worstcase)

= ½ U(100) + ½ U(20)

= ½ (10000) + ½ (400) = 5200

So CE=inverse of utility function at 5200

U(x)=5200=x^2 so x= 72.1

So CE=72.1

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Step 2: find the EV

EV: from expected value

EV=p(bestcase) + (1-p)(worstcase)

= ½ (100) + ½ (20)

= (50) + (10) = 60

So EV = 60

And RP=EV-CE= 60 – 72.1 = – 12.1

Since RP<0, utility function is risk seeking

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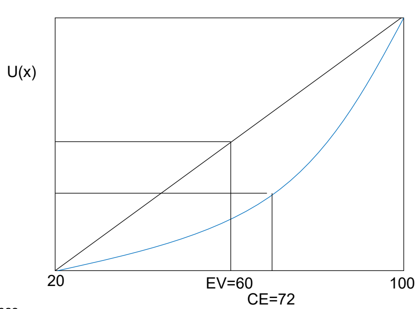
RP=EV-CE= 60 – 72.1 = – 12.1

Since RP<0, utility function is risk seeking



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Method 2: using G and L

Gain G =U(best case) – U(EV)

=U(100) – U(60)

=10000 – 3600

=6400

Loss L =U(EV) – U(worst case)

=U(60) – U(20)

=3600 – 400

=3200

Method 2: using G and L

If G-L > 0 risk seeking

If G-L < 0 risk averse

If G-L = 0 risk neutral

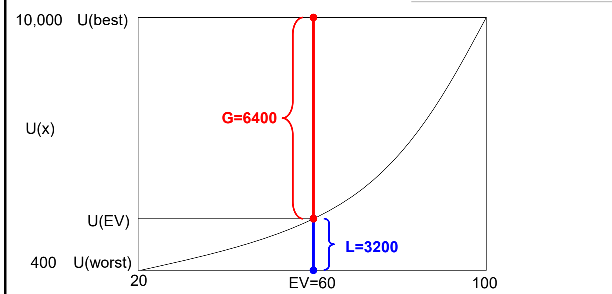
So

G – L =6400 – 3200 = 3200 > 0

so risk seeking

G=6400 L=3200

Since G-L>0, utility function risk seeking



Another example: Risk attitude

Suppose we have the following utility function:

$$U(x) = \left\{ \frac{\sqrt{x-16}}{\sqrt{20}} \quad 16 \le x \le 36 \right.$$

general case:

$$U(x) = \left\{ \frac{\sqrt{x-a}}{\sqrt{b-a}} \quad a \le x \le b \right.$$

Identify the risk attitude

Using the risk premium, RP=EV-CE where

if RP < 0 then risk seeking

if RP > 0 then risk averse

if RP = 0 then risk neutral

Need to compute CE and EV…

CE: from expected utility

EU=pU(bestcase) + (1-p)U(worstcase)

= ½ U(36) + ½ U(16)

= ½ (1) + ½ (0) = 0.50

So CE=inverse of utility function at 0.50

U(x)=0.50= $\frac{\sqrt{x-16}}{\sqrt{20}}$ solving for x= 21

So CE=21

EV: from expected value

EV=p(bestcase) + (1-p)(worstcase)

= ½ (36) + ½ (16)

= (18) + (8) = 26

So EV = 26

And RP=EV-CE= 26 – 21 = +5

Since RP>0, utility function is risk averse

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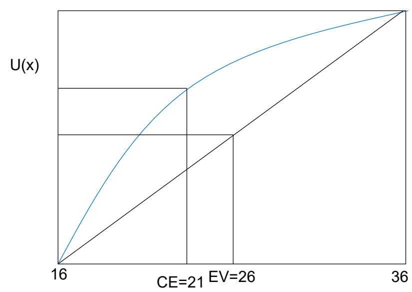
RP=EV-CE= 26 – 21 = +5

Since RP>0, utility function is risk averse



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Method 2: using G and L

Gain G =U(best case) – U(EV)
=U(36) – U(26)
=1 – 0.707
=0.293

Loss L =U(EV) – U(worst case)
=U(26) – U(16)
=0.707 – 0
=0.707

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Method 2: using G and L

If G-L > 0 risk seeking

If G-L < 0 risk averse

If G-L = 0 risk neutral

So

G – L =0.293 – 0.707 = – 0.414 < 0

so risk averse

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Let's work a real example:

- Suppose you are going to buy a car with 10 possible models to consider.
- The range of mpg is 28 to 42 across the 10 vehicles.
- We want the utility function for mpg.
- I need a volunteer!

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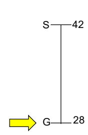
"Which would you prefer?"

A. Receive mpg = 28 for certain

OR

B. P_W=1/2 Receive mpg = 42

P_L=1/2 Receive mpg = 28k



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Utility Function

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Utility theory is based on a set of axioms of coherence for rational decision making. Satisfaction of these axioms implies maximization of expected utility as the decision making criterion.

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1. Completeness Axiom. Existence of a preference ordering. Given any 2 payoffs X1 and X2, the decision maker can decide whether X1 preferred to X2; X2 preferred to X1; or indifference between X1 and X2

- In other words, the decision maker can make up their mind (not indecisive)

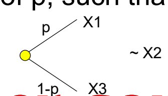
2. Transitivity of preferences. If the decision maker prefers X1 to X2 and X2 to X3, then they should prefer X1 to X3. (same for indifference)

- In other words, the decision maker has a logical preference ordering (logically consistent)

3. Continuity of preferences. If the decision maker prefers X1 to X2 and X2 to X3, then a value of p can be found such that pX1 and (1-p)X3 is preferred to X2; and another value of p* can be found such that X2 is preferred to p*X1 and (1-p*)X3; and another value of p' such that the DM is indifferent between X2 and p'X1 and (1-p')X3.

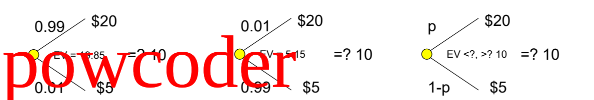
- In other words, an *indifference point* can be found between the ranges of X1 and X2; X2 and X3; and X1 and X3 (X1 cannot be so good that the transitivity of X1 to X3 is violated (X2>pX1 and (1-p)X3).

Continuity (cont). We can always find a value of p, such that if X1 > X2 > X3, then:



That is, L(X1,X3,p) = X2; we can find a value of p that produces indifference between the lottery and X2.

Example: you can receive \$20 (=X1), \$10 (=X2), or \$5 (=X3). So X1 > X2 > X3. Continuity states a value of p exists where you will be indifferent. Consider:



Take the gamble | Take the \$10 | Where would you switch?

4. Independence. If the decision maker prefers X1 to X2 and X3 is some other payoff, then any pX1 and (1-p)X3 is preferred to the same pX2 and (1-p)X3.

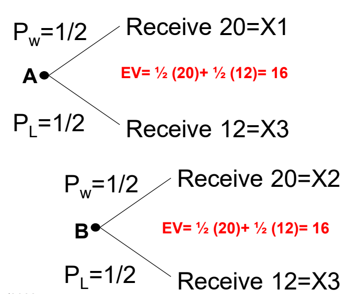
same

$pX1+(1-p)X3 > pX2+(1-p)X3$

- In other words, the decision maker's preference between X1 and X2 is independent of other consequences like X3 using the same probability, p.

4. Independence example 1

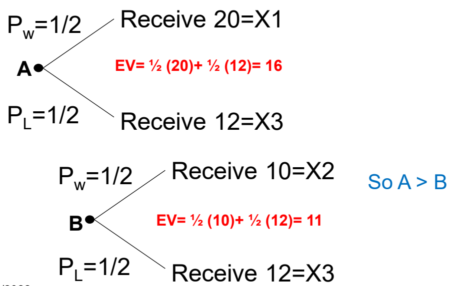
X1=receive \$20; X2= receive \$20 (X1 = X2); let X3= receive \$12. X1 is equal to X2 so lottery A is equivalent to the same lottery with X2 and X3; e.g.,



So A = B

4. Independence example 1

X1=receive \$20; X2= receive \$10 (X1 > X2); let X3= receive \$12. X1 is preferred to X2 and the lottery A is preferred to the same lottery with X2 and X3; e.g.,

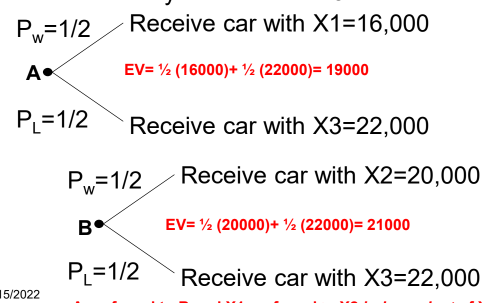


A preferred to B and X1 preferred-> X2 *independent* of X3

4. Independence example 2

X1=car costs \$16,000; X2= \$20,000; X3=\$22,000

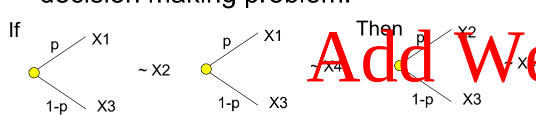
X1 is preferred to X2 and the lottery A is preferred to the same lottery with X2 and X3



A preferred to B and X1 preferred-> X2 *independent* of X3

5. Substitutability. If the decision maker is indifferent between X1 and X2 then they may be substituted for each other as payoffs in any decision making problem.

- In other words, equally preferable payoffs can be substituted for each other in any decision making problem.



6. Monotonicity. If the decision maker prefers consequence X1 to X2, then pX1 and (1-p)X2 is preferred to qX1 and (1-q)X2 if and only if p>q.

- In other words, preference for a lottery involving 2 consequences is determined by the probabilities _and_ the consequences.

- Do the axioms hold?
- Big question in fields of decision theory, economics, and psychology
- Under what conditions are they violated?
- What happens if they are violated?
- How can such problems be mitigated?

- We will address this later in the course during the “Decision biases” lectures.

- What if the axioms hold?
  - Decision maker preferences can be described by a utility function under the axioms of utility.
  - Subjective judgments about uncertain quantities can be described by a probability distribution satisfying the 3 axioms of probability
  - The rational decision maker should make decisions by maximizing expected utility.

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- Utility and decision making implications
  - Payoffs at terminating branches of decision tree may have more than one attribute
  - Can use multiattribute utility to represent decision maker preferences for different attributes.
    - Additive multiattribute decision model
    - Multiplicative multiattribute decision model
  - Group decision making
    - Group decision rules based on utility
- These topics will be addressed soon

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