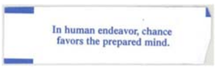
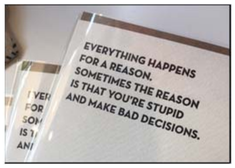
Decision Criteria

Decision Theory





Today

- Decision making criteria
- Decision making under certainty
- Decision making under uncertainty
- Decision analysis and the value of information

Components of decision making

- Decision alternatives-choices
- Decision attributes (profit, cost, etc.)
- Uncertain (future) outcomes (states of nature) that affect the decision
- Payoffs (profits or costs) for each alternative under each state of nature

All of these components can be displayed in a payoff table. Consider problem of farmer considering planting one of 3 crops next year. The return will be determined by whether a trade bill with China passes the Senate. The profits under the two outcomes are displayed in the following payoff table.

Payoff Table

Alternatives

States of nature

Payoff

	Trade Bill	
Crop	Passes	Fails
Corn	$35,000	$8,000
Peanuts	18,000	12,000
Soybeans	22,000	20,000

Decision making under certainty

Decision making criteria: rules for selecting the “best” decision

- Each rule has assumptions
- Uncertainty not considered because:
  - Not available
  - Not important
  - Not enough time

Decision making criteria (no uncertainty):

- Maximax criterion
- Maximin criterion
- Minimax regret criterion
- Hurwicz criterion
- Equal likelihood criterion

Decision making criteria: Maximax criterion

- Maximum of the maximum payoffs
- Overly optimistic and ignores large negative consequences

	Trade Bill	
Crop	Passes	Fails
Corn	$35,000	$8,000
Peanuts	18,000	12,000
Soybeans	22,000	20,000

- **Decision: plant corn**

Decision making criteria: Maximin criterion

- Maximum of the minimum payoffs
- Pessimistic; (Note if table contained costs, would use Minimax)

	Trade Bill	
Crop	Passes	Fails
Corn	$35,000	$8,000
Peanuts	18,000	12,000
Soybeans	22,000	20,000

- **Decision: plant soybeans**

Decision making criteria: Minimax regret criterion (opportunity loss)

- Difference between actual choice and best choice Example.1 Example.2
- Trying to minimize the maximum regret; subtract each state of nature from the maximum payoff state of nature and redraw table
- Then select the alternative with minimum regret.

Subtract each state of nature from the maximum payoff state of nature and select minimum regret value

	Trade Bill	
Crop	Passes	Fails
Corn	35,000-35,000=0	20,000-8,000 =12,000
Peanuts	35,000-18,000 =17,000	20,000-12,000 =8,000
Soybeans	35,000-22,000 =13,000	20,000-20,000=0

Decision: plant corn

Decision making criteria: Hurwicz criterion

- Compromise between maximax/maximin by allowing the degree of optimism to be specified.
- Coefficient of optimism, $0 \le \alpha \le 1$, if $\alpha=0$ decision maker pessimistic; =1 optimistic. $(1-\alpha)$ is coefficient of pessimism.
- For each alternative, calculate
  α (max payoff) + $(1-\alpha)$ (min payoff)
- Select largest (if cost, smallest)

Decision making criteria: Hurwicz criterion with $\alpha=0.30$

	Trade Bill		
Crop	Passes	Fails	
Corn	$35,000	$8,000	H(corn)=.3(35000)+(1-.3)(8000)=16,100
Peanuts	18,000	12,000	H(pnut)=.3(18000)+(1-.3)(12000)=13,800
Soybeans	22,000	20,000	H(soy)=.3(22000)+(1-.3)(20000)=20,600

- **Decision: plant soybeans**

Decision making criteria: Equal likelihood criterion

- Assumes states of nature equally likely
- Multiply by 1/(no states of nature) and sum; Pick maximum

	Trade Bill		
Crop	Passes	Fails	
Corn	$35,000	$8,000	=.5(35000)+(.5)(8000)=21,500
Peanuts	18,000	12,000	=.5(18000)+(.5)(12000)=15,000
Soybeans	22,000	20,000	=.5(22000)+(.5)(20000)=21,000

- **Decision: plant corn**

Recap

- Maximax criterion corn
- Maximin criterion soybean
- Minimax regret criterion corn
- Hurwicz criterion soybeans
- Equal likelihood criterion corn

Which one should decision maker use?

Depends on attitude toward outcomes and payoffs.

Another example. A real estate firm is considering the following development projects. The financial success of these projects depends on interest rate movements in the next 5 years. The financial returns of each project for different interest rate conditions is shown in the payoff table. Determine the best investment using

a. Maximax
b. Maximin
c. Equal likelihood
d. Hurwicz criterion, $\alpha=.30$
e. Minimax regret

Financial returns ($M):

	Interest Rates		
Project	Decline	Stable	Increase
Office park	0.5	1.7	4.5
Office Bldg	1.5	1.9	2.5
Warehouse	1.7	1.4	1.0
Mall	0.7	2.4	3.6
Condos	3.2	1.5	0.6

Maximax criterion

	Interest Rates		
Project	Decline	Stable	Increase
Office park	0.5	1.7	4.5
Office Bldg	1.5	1.9	2.5
Warehouse	1.7	1.4	1.0
Mall	0.7	2.4	3.6
Condos	3.2	1.5	0.6

Decision: Office park

Maximin criterion

	Interest Rates		
Project	Decline	Stable	Increase
Office park	0.5	1.7	4.5
Office Bldg	1.5	1.9	2.5
Warehouse	1.7	1.4	1.0
Mall	0.7	2.4	3.6
Condos	3.2	1.5	0.6

Decision: Office bldg.

$.33(.5)+.33(1.7)+.33(4.5)=2.21$

Equal likelihood criterion

	Interest Rates			
Project	Decline	Stable	Increase	
Office park	0.5	1.7	4.5	2.21
Office Bldg	1.5	1.9	2.5	1.95
Warehouse	1.7	1.4	1.0	1.35
Mall	0.7	2.4	3.6	2.21
Condos	3.2	1.5	0.6	1.75

Decision: Office park or mall

$.30(4.5)+.70(0.5)=1.7$

Hurwicz criterion, $\alpha=0.30$

	Interest Rates			
Project	Decline	Stable	Increase	
Office park	0.5	1.7	4.5	1.70
Office Bldg	1.5	1.9	2.5	1.80
Warehouse	1.7	1.4	1.0	1.21
Mall	0.7	2.4	3.6	1.57
Condos	3.2	1.5	0.6	1.38

Decision: Office bldg

Minimax regret criterion: subtract payoff-max payoff for each state of nature

	Interest Rates		
Project	Decline	Stable	Increase
Office park	3.2-0.5 =2.7	2.4-1.7 = 0.7	4.5-4.5 =0.0
Office Bldg	1.7	0.5	2.0
Warehouse	1.5	1.0	3.5
Mall	2.4	0.0	0.9
Condos	0.0	0.9	3.9

Minimax regret criterion: choose min of max regrets for each alternative

	Interest Rates		
Project	Decline	Stable	Increase
Office park	2.7	0.7	0.0
Office Bldg	1.7	0.5	2.0
Warehouse	1.5	1.0	3.5
Mall	2.4	0.0	0.9
Condos	0.0	0.9	3.9

Decision: Office bldg

Decision making under uncertainty

Decision making under uncertainty

1. Maximum Likelihood Criterion
   - Select highest outcome for most probable state
2. Expected Monetary Value Criterion:
   - Payoffs x probability distribution of outcomes
   - More frequent states of nature receive more weight
   - Less likely states of nature receive less weight
3. Expected Opportunity Loss (EOL) Criterion: Losses x (probability distribution of outcomes)

Mathematically:

- Maximum Likelihood:

$$ML = Max\{probability\ state\ ,\ payoff\ \}$$

- Expected value:

$$E[x] = \sum_{i=1}^{n} x_i P(x_i)$$

- Expected opportunity loss:

$$EOL[x] = \sum_{i=1}^{n} L(x_i)P(x_i),\ L(x_i) = x^* - x_i$$

Maximum likelihood criterion: choose max of most likely state of nature

	Interest Rates		
Project	Decline P=0.15	Stable P=0.25	Increase P=0.60
Office park	0.5	1.7	4.5
Office Bldg	1.5	1.9	2.5
Warehouse	1.7	1.4	1.0
Mall	0.7	2.4	3.6
Condos	3.2	1.5	0.6

Decision: Office Park

Maximum Likelihood Criterion

	Trade Bill		
	P(pass)=.60	P(fails)=0.40	
Crop	Passes	Fails	
Corn	$35,000	$8,000	=.6(35000)+(.4)(8000)=24,200
Peanuts	18,000	12,000	=.6(18000)+(.4)(12000)=15,600
Soybeans	22,000	20,000	=.6(22000)+(.4)(20000)=21,200

Maximum likelihood decision = plant corn

Expected Monetary Value

	Trade Bill		
	P(pass)=.60	P(fails)=0.40	
Crop	Passes	Fails	
Corn	$35,000	$8,000	=.6(35000)+(.4)(8000)=24,200
Peanuts	18,000	12,000	=.6(18000)+(.4)(12000)=15,600
Soybeans	22,000	20,000	=.6(22000)+(.4)(20000)=21,200

Expected value decision = plant corn

Expected Opportunity Loss (choose option with minimum EOL)

	Trade Bill		
	P(pass)=.60	P(fails)=0.40	
Crop	Passes	Fails	
Corn	0	12,000	=.6(0)+(.4)(12000)=4800
Peanuts	17,000	8,000	=.6(17000)+(.4)(8000)=13,400
Soybeans	13,000	0	=.6(13000)+(.4)(0)=7,800

EOL decision = plant corn

Example. Suppose the chance of interest rates declining is .15, .25, and .60. Determine the decision with highest expected value.

	Interest Rates		
P(state)=	.15	.25	.60
Project	Decline	Stable	Increase
Office park	0.5	1.7	4.5
Office Bldg	1.5	1.9	2.5
Warehouse	1.7	1.4	1.0
Mall	0.7	2.4	3.6
Condos	3.2	1.5	0.6

Expected Value

E[Office park]=.15(.5)+.25(1.7)+.60(4.5)=3.20

	Interest Rates			
P(state)=	.15	.25	.60	
Project	Decline	Stable	Increase	
Office park	0.5	1.7	4.5	3.20
Office Bldg	1.5	1.9	2.5	2.20
Warehouse	1.7	1.4	1.0	1.21
Mall	0.7	2.4	3.6	2.87
Condos	3.2	1.5	0.6	1.22

Decision: Office park

Example. Determine the decision with lowest EOL. Take expectations of regrets (opportunity losses)

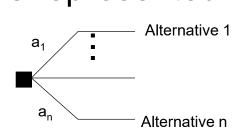
	Interest Rates			
P(state)=	.15	.25	.60	
Project	Decline	Stable	Increase	EOL
Office park	2.7	0.7	0.0	.58
Office Bldg	1.7	0.5	2.0	1.58
Warehouse	1.5	1.0	3.5	2.58
Mall	2.4	0.0	0.9	0.90
Condos	0.0	0.9	3.9	2.57

Decision: Office park

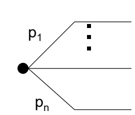
Decision Tree Diagrams

- Used for multi-stage decisions
- Organizes decisions, states of nature (outcomes), and payoffs
- Maximize expected value of decision through "roll-back" of tree from branches to root.
- Time progresses from left to right

- Decisions represented by <u>choice</u> nodes



- States of nature represented by <u>chance</u> nodes



Examples

Car Part Decision

Shoulder Decision

Americo Case

Utility Concepts

Decision Theory

The expected monetary value (EMV) criterion:

- Multiply each payoff by its probability of achieving that payoff and sum.
  - Intuitive
  - Many practical applications
  - Can also be used for cost (minimize expected cost, EC)
  - Can also be used for opportunity cost (EOL)

Mathematically:

- Expected value:

$$EMV[x] = \sum_{i=1}^{n} x_i P(x_i)$$

- Expected opportunity loss:

$$EOL[x] = \sum_{i=1}^{n} L(x_i) P(x_i)$$

$$L(x_i) = x^* - x_i$$

Utility

"Top 15 Things Money Can't Buy
Time. Happiness. Inner Peace. Integrity. Love. Character. Manners. Health. Respect. Morals. Trust. Patience. Class. Common sense. Dignity."
— Roy T. Bennett, The Light in the Heart

"If you want to know what God thinks of money, just look at the people he gave it to."
— Dorothy Parker

"This planet has - or rather had - a problem, which was this: most of the people living on it were unhappy for pretty much of the time. Many solutions were suggested for this problem, but most of these were largely concerned with the movement of small green pieces of paper, which was odd because on the whole it wasn't the small green pieces of paper that were unhappy."
— Douglas Adams, The Hitchhiker's Guide to the Galaxy

"Don't think money does everything or you are going to end up doing everything for money."
— Voltaire

Utility

Problems with Expected monetary value (EMV) and Expected Loss (EL)

- Some decisions do not involve monetary value (safety, number of side effects, time saved, down-time, time-between failure)

- Attitude toward risk

Utility

Consider the following bet (fair coin):

P_w=1/2 — Win \$1

P_L=1/2 — Lose -\$0.75

EMV= ½(1) + ½(-.75) = \$0.125
if you take the bet, zero if you don't

Rational choice is to take the bet.

Utility

Now consider the following bet (fair coin):

P_w=1/2 — Win \$100,000

P_L=1/2 — Lose -\$75,000

EMV= ½(100k) + ½(-75k) = \$12500
if you take the bet, zero if you don't

Rational choice is to take the bet.
But, would you?

Utility

The choice under the EMV criterion is to take both bets. If you don't, it's a violation of the EMV criterion. The monetary payoffs are clear so why the hesitation?

→ It is the risk of the big loss of \$75,000.

- Daniel Bernoulli was the son of Johann Bernoulli. He was born in Groningen while his father held the chair of mathematics there. His older brother was Nicolaus(II) Bernoulli and his uncle was Jacob Bernoulli so he was born into a family of leading mathematicians but also into a family where there was unfortunate rivalry, jealousy and bitterness. Johann was determined that Daniel should become a merchant and he tried to place him in an apprenticeship. However Daniel was as strongly opposed to this as his own father had been and soon Johann relented but certainly not as far as to let Daniel study mathematics. Johann declared that there was no money in mathematics and so he sent Daniel back to Basel University to study medicine. This Daniel did spending time studying medicine at Heidelberg in 1718 and Strasbourg in 1719. He returned to Basel in 1720 to complete his doctorate in medicine.

- An important work which Daniel produced while in St Petersburg was one on probability and political economy. Daniel makes the assumption that the moral value of the increase in a person's wealth is inversely proportional to the amount of that wealth. He then assigns probabilities to the various means that a person has to make money and deduces an expectation of increase in moral expectation. Daniel applied some of his deductions to insurance.

- Daniel Bernoulli did produce other excellent scientific work during these many years back in Basel. In total he won the Grand Prize of the Paris Academy 10 times, for topics in astronomy and nautical topics. He won in 1740 (jointly with Euler) for work on Newton's theory of the tides; in 1743 and 1746 for essays on magnetism; in 1747 for a method to determine time at sea; in 1751 for an essay on ocean currents; in 1753 for the effects of forces on ships; and in 1757 for proposals to reduce the pitching and tossing of a ship in high seas. Another important aspect of Daniel Bernoulli's work that proved important in the development of mathematical physics was his acceptance of many of Newton's theories and his use of these together with the tolls coming from the more powerful calculus of Leibniz. Daniel worked on mechanics and again used the principle of conservation of energy which gave an integral of Newton's basic equations. He also studied the movement of bodies in a resisting medium using Newton's methods.

Source: http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Bernoulli_Daniel.html

Utility

The classic violation of EMV principle:
St. Petersburg paradox

- Toss a fair coin until a head appears
- When head appears on kth toss, you win 2^{k-1} dollars.
- How much would you be willing to pay to play this game?

Utility

- If heads occurs on kth toss, then it is preceded by (k-1) tails.
- So probability of k-1 tails followed by a head is $(\tfrac{1}{2})^{k-1}(\tfrac{1}{2}) = (\tfrac{1}{2})^{k}$
- To get EMV of the gamble:

$$EMV[gamble] = \sum_{k=1}^{\infty} 2^{k-1}\left(\frac{1}{2}\right)^{k} = \sum_{k=1}^{\infty} \frac{1}{2} = \infty$$

- So expected payoff is infinite
- But how much would you be willing to pay to play?

Utility

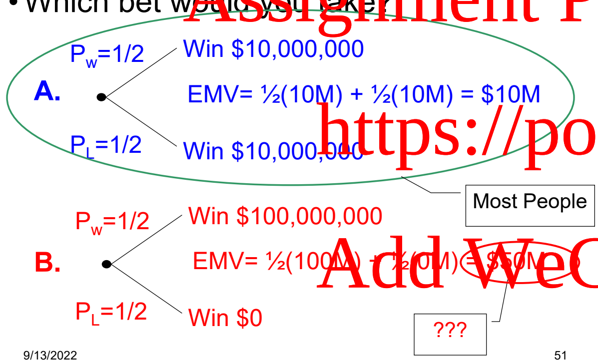
- Suppose you were willing to pay \$127. To break even you would need to win 127 + \$1.
- To determine when heads occurs find k that yields ≥ \$128. $2^{k-1} \geq 128$, so k=8. and P(heads on k=8)=(½ $)^8$=0.004
- So chance of winning \$128 only 0.004

Utility

- What if we lower the entry cost to \$31?
- To break even you would need to win \$31 + \$1.
- To determine when heads occurs find k that yields ≥ \$32. $2^{k-1} \geq 32$, so k=6. and P(heads on k=6)=(½ $)^6$=0.02
- So chance of winning \$32 only 0.02; would you pay \$31 to enter this bet?
- At \$3 probability is .125

Utility

- Which bet would you take?



Utility

- Basic message…
- Value of a dollar varies from person to person (decision maker to decision maker). Deal or No Deal Examples: Example 1; Example 2

- How to address this?
- Theory of expected utility; also called the EU criterion.

Utility

- Utility allows us to measure the relative value of different levels of consequences to the decision maker.
- Consequences can be dollars, hours, reliability, number of fatalities, or other attributes.

Utility

- The utility function, U(X) is a preference relationship with the following axioms:
  1. If payoff X1 is preferred to payoff X2, then U(X1)>U(X2); if X2 preferred to X1, then U(X2)>U(X1); if neither preferred (indifferent), then U(X1)=U(X2)

Utility

2. If you are indifferent between receiving
- a) payoff X1 for certain and
- b) a gamble where you receive payoff X2 with probability p and payoff X3 with probability (1-p), then

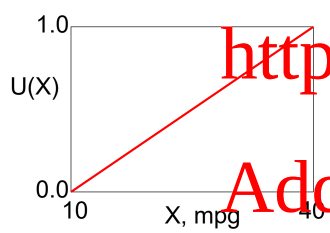
$$U(X1)=pU(X2)+(1-p)U(X3)$$

Utility

Assessment of utility functions
- Suppose we are buying a car and selecting from among 10 different cars with various mileage ratings (mpg).
- Let X^o be the worst mileage = 10 mpg
- Let X^* be the best mileage = 40 mpg
- Define $U(X^o)= 0.0$; $U(X^*)= 1.0$

Utility

If we stop here with 2 points we have the following utility function (risk neutral)



Utility

- To get more precision we offer the decision maker the following choice:

"Which do you prefer?"

A. Receive car with 40 mpg for certain

IF RATIONAL, SHOULD CHOOSE THIS OPTION

OR

B.
- $P_w=1/2$ Receive car with 40 mpg
- $P_L=1/2$ Receive car with 10 mpg

Utility

- Now bump the sure thing up to ¼ the interval from worst value (10+ ¼ (40-10)=17.5)

A. Receive car with 17.5 mpg for certain

THE CHOICE WILL BE BASED ON RISK ATTITUDE

SUPPOSE THEY CHOOSE "B"

OR

B.
- $P_w=1/2$ Receive car with 40 mpg
- $P_L=1/2$ Receive car with 10 mpg

Utility

- Now split the interval from 17.5 to 40 or (17.5+40)/2 = 28.8

A. Receive car with 28.8 mpg for certain

SUPPOSE THEY CHOOSE "A"

OR

B.
- $P_w=1/2$ Receive car with 40 mpg
- $P_L=1/2$ Receive car with 10 mpg

Utility

- Now split the interval from 17.5 to 28.8 or (17.5+28.8)/2 = 23.2

NOW SUPPOSE THEY CANNOT DECIDE—THEY ARE INDIFFERENT BETWEEN A AND B

A. Receive car with 23.2 mpg for certain

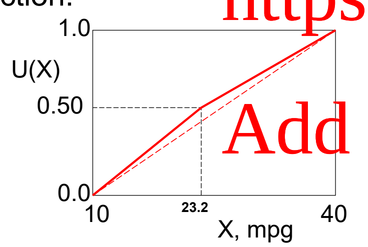
OR

B.
P_w=1/2 Receive car with 40 mpg
P_L=1/2 Receive car with 10 mpg

Utility

- The value of 23.2 is called the "certainty equivalent" for the gamble.
- It is the amount the decision maker is willing to accept to avoid the risk of getting a car with only 10 mpg.
- If the attribute were $ this could also be called a risk premium→what the DM should be willing to pay to avoid the chance of getting the worst case.

If indifferent at 23.2, then
U(23.2)=.5U(10) + .5U(40)
= .5(0) + .5(1) = .50
Graphically we have the midpoint of the function:



- To obtain the utility at 0.25 we work between 10 and 23.2 and repeat the procedure

A. Receive car with 23.2 mpg for certain

IF RATIONAL, SHOULD CHOOSE THIS OPTION

OR

B.
P_w=1/2 Receive car with 23.2 mpg
P_L=1/2 Receive car with 10 mpg

Utility

- Now bump the sure thing up to ¼ the interval from worst value (10+ ¼ (23.2-10)=13.3)

A. Receive car with 13.3 mpg for certain

SUPPOSE THEY CHOOSE "B"

OR

B.
P_w=1/2 Receive car with 23.2 mpg
P_L=1/2 Receive car with 10 mpg

Utility

- Now split the interval from 13.3 to 23.2 or (13.3+23.2)/2 = 18.25

A. Receive car with 18.25 mpg for certain

SUPPOSE THEY CHOOSE "B"

OR

B.
P_w=1/2 Receive car with 23.2 mpg
P_L=1/2 Receive car with 10 mpg

Utility

- Now split the interval from 18.25 to 23.2 or (18.25+23.2)/2 = 20.73

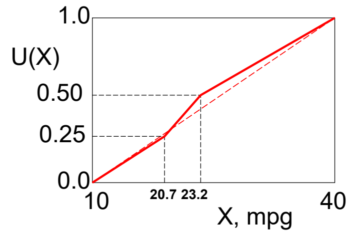
NOW SUPPOSE THEY CANNOT DECIDE—THEY ARE INDIFFERENT BETWEEN A AND B

A. Receive car with 20.73 mpg for certain

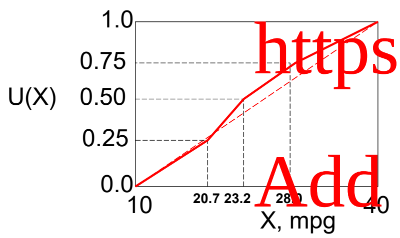
OR

B.
P_W=1/2 Receive car with 23.2 mpg
P_L=1/2 Receive car with 10 mpg

If indifferent at 20.7, then U(20.7)= 0.25
U(20.7)=.5U(10) + .5U(23.2)
= .5(0) + .5(.50) = .25
Graphically we now have 4 points of the function:



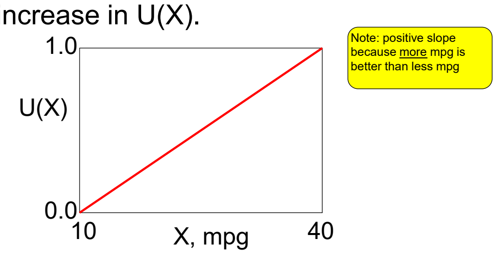
If we repeat for interval from 23.2 to 40 we will elicit the U(X)=0.75 value; suppose it is 28.0. Then we have:



- Once assessed, the advantage of the utility function is the value to the decision maker of any level of attribute X can be "looked up" on the function.
- Because the utility function is the decision maker's preference function for the attribute, it reflects their attitude toward risk depending on the shape of the function.

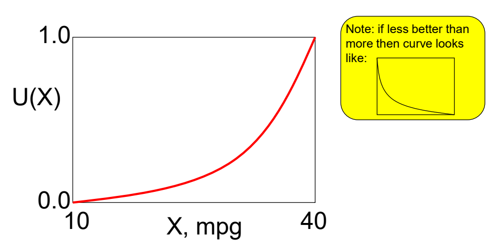
There are 3 attitudes toward risk

- Risk neutral (no preference to gamble or avoid risk); shape of utility function is linear. One unit increase in X yields one unit increase in U(X).



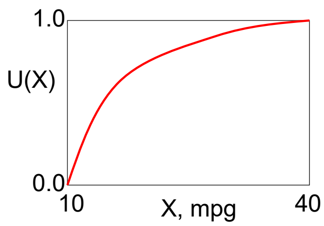
There are 3 attitudes toward risk

- Risk seeking (willing to gamble for chance at receiving best case).

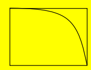


There are 3 attitudes toward risk

- <u>Risk averse</u> (prefers avoiding risk by taking lesser value rather than chance of worst case.



Note: if <u>less</u> better than more then curve looks like:



- What if risk attitude not a factor?

Note on value functions