Decision Problems -1:

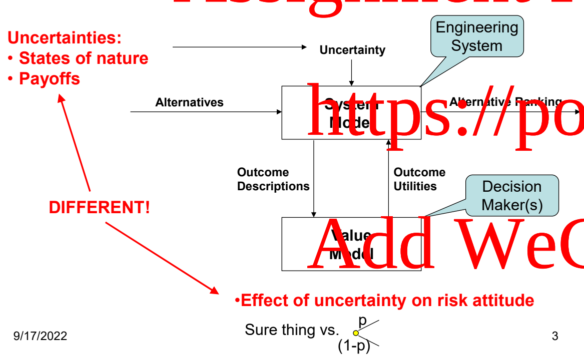
Problem Structuring and Uncertainty

Decision Theory

Status:

- So far
  - Probability review and Bayes theorem
  - Decision criteria under certainty and uncertainty
  - Introduction to utility theory
- Today
  - The decision problem (inputs and definitions)
  - Uncertainty in states of nature vs. payoffs vs. utility lotteries
  - Examples (using expected utility vs. expected monetary value; Mars pathfinder

Relationship Between the System Model and the Value Model



Notation for decision problem inputs

- Set of possible actions or decisions, A
- Let $a \in A$ be an alternative from A
- The set of states of nature called S
- Uncertainty associated with state of nature, θ
- Represent uncertainty with $P(\theta)$ if θ discrete and $f(\theta)$ if θ continuous
  - The distribution may be a prior or posterior
  - i.e., $P'(\theta)$, $f'(\theta)$; $P''(\theta)$, $f''(\theta)$;
- Each (a,θ) maps to a consequence
- There is a utility function that maps each consequence to a utility value, $U(a,\theta)$

A decision problem requires

- A, the set of actions
- S, the set of states of nature
- The probability distribution of the states of nature, ($P(\theta)$ if θ discrete; $f(\theta)$ if continuous)
- $U(a,\theta)$, the utility function that associates a utility with each action and state of the nature
- The decision maker should select the action, a, with highest expected utility of all actions in A, EU(a). Let the best choice be denoted, a*.

To calculate EU:

- If θ discrete, $EU(a)=\Sigma U(a,\theta)P(\theta)$
- If θ continuous, $EU(a)=\int U(a,\theta)f(\theta)d\theta$

- The optimal decision is a* where

  $EU(a^*) \geq EU(a)$ for all $a \in A$

Note that if U() linear (or approx. linear) with respect to money, then payoff \$ (return, $R(a,\theta)$) or loss function, $L(a,\theta)$, can be used instead of $U(a,\theta)$.

For expected payoff:

- If θ discrete, ER(a)=ΣR(a,θ)P(θ)
- If θ continuous, ER(a)=∫R(a,θ)f(θ)dθ
- Find a* such that ER(a*)≥ER(a) for all a

For expected loss

- If θ discrete, EL(a)=ΣL(a,θ)P(θ)
- If θ continuous, EL(a)=∫L(a,θ)f(θ)dθ
- Find a* such that EL(a*)≤EL(a) for all a

NOTE: there is a difference between uncertainty in the states of nature and…

	Interest Rates		
P(state)=	.15	.25	.60
Project	Decline	Stable	Increase
Office park	0.5	1.7	4.5
Office Bldg	1.5	1.9	2.5
Warehouse	1.7	1.4	1.0
Mall	0.7	2.4	3.6
Condos	3.2	1.5	0.6

Uncertainty in payoffs and…

	Interest Rates		
P(state)=	.15	.25	.60
Project	Decline	Stable	Increase
Office park	0.5	1.7	[illegible]
Office Bldg	1.5	1.9	2.5
Warehouse	1.7	1.4	1.0
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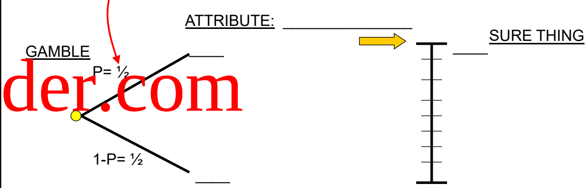
Condos	3.2	[illegible]	0.6

Probability in utility assessments:



NOTE: there is a difference between uncertainty in the states of nature and…

	Interest Rates		
P(state)=	.15	.25	.60
Project	Decline	Stable	Increase
Office pa			
Office Bl			
Warehou			
Mall			
Condos			

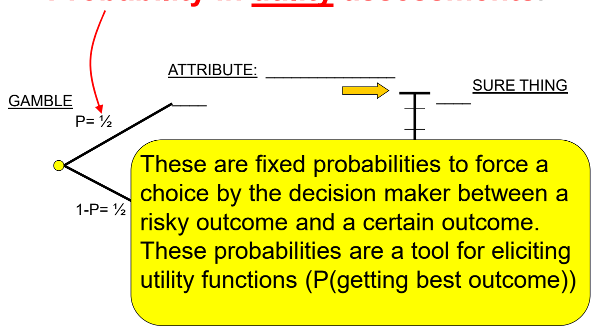
These are uncertainties on outcomes affecting the alternatives. The random variable is the value taken by the state of nature (interest rate)

Uncertainty in payoffs and…

These are uncertainties on the estimated payoffs (precision of knowledge). The random variable is the quantity of the decision variable for the alternative under a state of nature (in this case Profit).

Warehouse	1.7	1.4	1.0
Mall	0.7	2.4	3.6
Condos	3.2	1.5	0.6

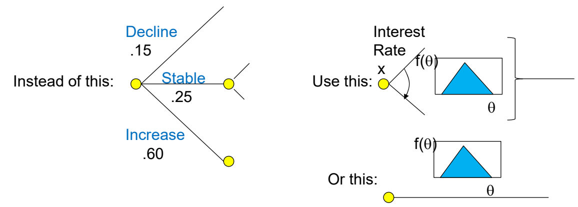
Probability in utility assessments:



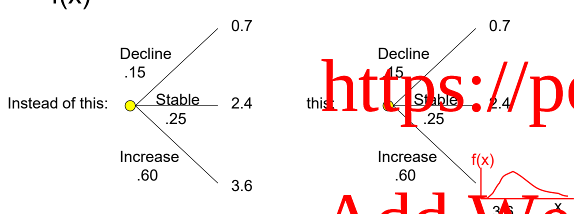
These are fixed probabilities to force a choice by the decision maker between a risky outcome and a certain outcome. These probabilities are a tool for eliciting utility functions (P(getting best outcome))

End note.

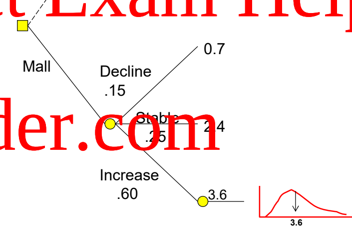
- States of nature can be continuous random variables, f(θ)



- Payoffs as continuous random variables, f(x)

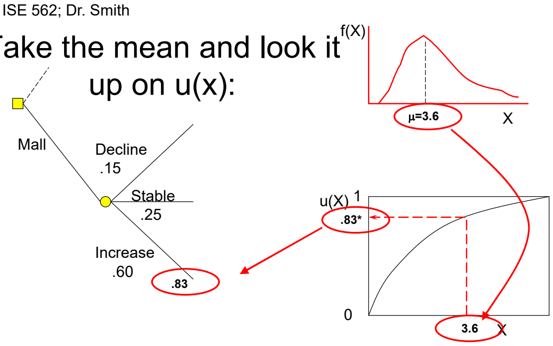


How to calculate the expected value:



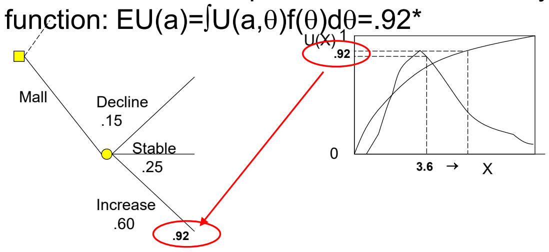
If the utility function for money is linear use EMV
If not linear, could convert into utility like this…

Take the mean and look it up on u(x):



*for example

Even better, take expected value of the utility function: EU(a)=∫U(a,θ)f(θ)dθ=.92*



Convolve the pdf of X against the utility function to calculate the EU weighted by the pdf.

*for example

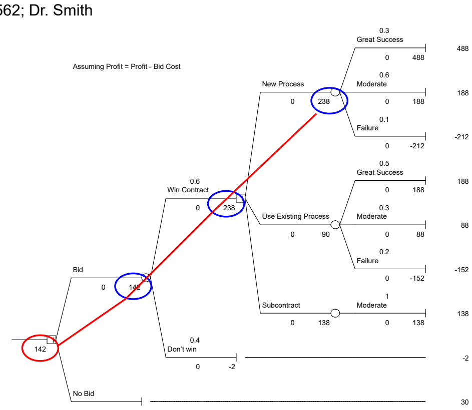
- Note that there may be many initial alternatives that give the appearance of an overwhelming decision problem.
- First step in defining the set {A} includes screening out “inadmissible” alternatives
- *Inadmissible*: those alternatives that are impractical or infeasible for technical, budgetary, or other reasons.

- Decision making under uncertainty: a simple example using utility theory

- Example: Amerco Oil is considering making a bid of $110M for shale oil development. Company estimates it has a 60% chance of winning the contract. If it wins, it can choose one of 3 methods of oil extraction: 1) new method; 2) use existing ineff. process; 3) subcontract to smaller companies. Data are summarized in the following table. Cost of the contract proposal is $2M; If company doesn’t bid, will invest with a return guaranteed to be $30M. Construct the decision tree and identify the correct decision.

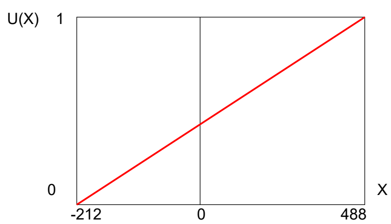
- Example

Develop new process		
Outcomes	Probability	Profit ($M)
Great success	.30	600
Moderate	.60	300
Failure	.10	-100
Use existing (ineff.) process		
Great success	.50	300
Moderate	.30	200
Failure	.20	-40
Subcontract		
Moderate	1.0	250



- The range of net profits is -212M to $488M
- If the decision maker is risk neutral
  - X^o=-212; $U(X^o)$=U(-212)=0
  - X^*=488; $U(X^*)$=U(488)=1
  - The utility function is the line between the pairs (-212, 0) and (488, 1)
  - m=1/(488-(-212))=1/700 (slope)
  - U(X)=1/700 (X+212)
    - (note at X=-212, U(X)=0; X=488, U(X)=1)

- Now use the utility function to compute the utilities for all the dollar values in the decision tree or payoff table

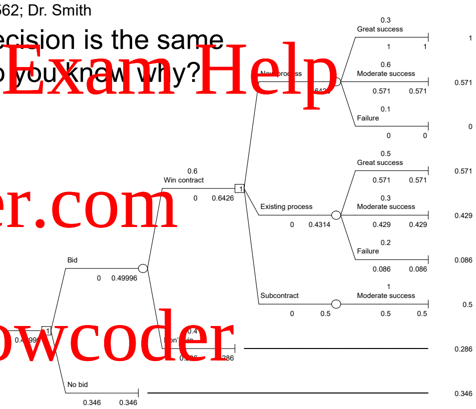


- U(X)=1/700 (X+212)

	Develop new process		
Outcomes	Probability	Net Profit ($M)	Utility
Great success	.30	488	1.000
Moderate	.60	188	0.571
Failure	.10	-212	0.000
Use existing (ineff.) process			
Great success	.50	188	0.571
Moderate	.30	88	0.429
Failure	.20	-152	0.086
Subcontract			
Moderate	1.0	138	0.500

- Now replace the costs in the decision tree with utility values

- Decision is the same
- Do you know why?



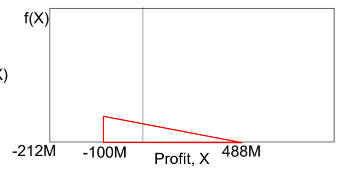
- Suppose one (or more) of the payoffs were represented by a probability distribution.

- Pick this one (arbitrary) for illustration

	Develop new process		
Outcomes	Probability	Net Profit ($M)	Utility
Great success	.30	488	1.000
Moderate	.60	188	0.571
Failure	.10	-212	0.000
Use existing (ineff.) process			
Great success	.50	188	0.571
Moderate	.30	88	0.429
Failure	.20	-152	0.086
Subcontract			
Moderate	1.0	138	0.500

- Instead of \$488M, there is uncertainty about the true profit. Company analysts believe the true net profit will be somewhere between -100M and 488M.
- A triangular pdf is used to represent the uncertainty:

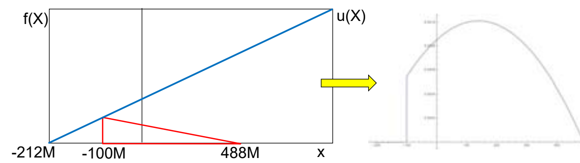
f(X)=(5.786 x 10^{-6})(488 – X)
for -100 ≤ X ≤ 488



So we calculate EU(a)=∫U(a,θ)f(θ)dθ

EU(new process, great success)

$$EU[a = new\ process, \theta = great\ success] = \int_{-100}^{488} U(a,\theta)f(\theta)d\theta = \int_{-100}^{488} \left(\frac{1}{700}(x+212)\right)(5.786\ x\ 10^{-6})(488-x)dx = 0.44$$



Details

$$\int_{-100}^{488} \left(\frac{1}{700}(x+212)\right)(5.786\ x\ 10^{-6})(488-x)dx = 0.44$$

$$= \frac{1}{700 \cdot 172872} \int_{-100}^{488} (x+212)(488-x)dx$$

$$= \frac{1}{700 \cdot 172872} \int_{-100}^{488} (488x - x^2 + 103456 - 212x)dx$$

$$= \frac{1}{700 \cdot 172872} \int_{-100}^{488} (276x - x^2 + 103456)dx$$

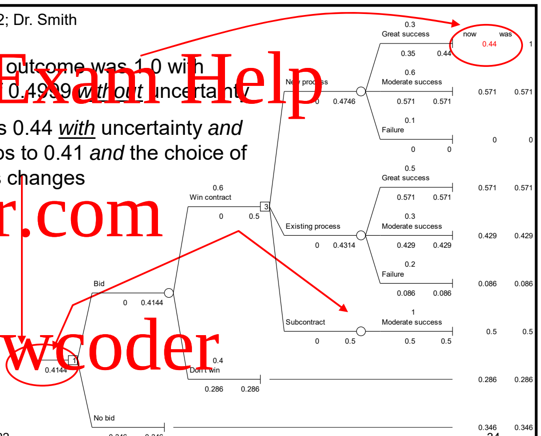
$$= \frac{1}{700 \cdot 172872} \left[276\frac{x^2}{2} - \frac{x^3}{3} + 103456x\right]_{-100}^{488}$$

$$= \frac{1}{700 \cdot 172872}[32863872 - 38738090.67 + 50486528 - 1380000 - 333333.33 + 10345600]$$

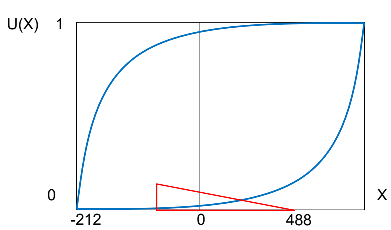
$$= 0.44$$

Original outcome was 1.0 with result of 0.4999 *without* uncertainty

Now it is 0.44 *with* uncertainty *and* EU drops to 0.41 *and* the choice of process changes



- If utility function not linear, e.g.



- Procedure is the same except decision can be very different than EMV.

Example

- Mars Pathfinder Experiment

Modeling cost uncertainty

Now

Decision Problems - 2:

Practice Problems

Decision Theory

- Problem structuring practice (inputs and trees)
- Identifying states of nature and their uncertainty
- Define the decision tree with probabilities and payoffs; find the optimal decision.
- See if you can work the following problems (at least set them up before class.
- (If you are viewing the lecture later, hit “pause” and try the exercises.)

Problem 1:

The management of First Bank was concerned about the potential loss that might occur in the event of a physical catastrophe such as a power failure or fire. The bank estimated the loss from one of these incidents could be as much as $100M, including losses due to interrupted service and customer relations. One project the bank is considering is the installation of an emergency power generator at its operations headquarters. The cost of the emergency generator is $0.8M, and if it is installed no losses from this type of incident will be incurred. However, if the generator is not installed, there is a 10% chance that a power outage will occur during the next year. If there is an outage, there is a 0.05 probability that the resulting losses will be very large, or approximately $80M in lost earnings. Alternatively, it is estimated there is a 0.95 probability of only slight losses of around $1M. Using decision tree analysis should the bank install the generator?

Problem 2: SC is playing UCLA in a major conference game of the season. SC is trailing UCLA 21 to 14, with 7 seconds left in the game when SC scores a touchdown. Still trailing 21 to 20, SC can either go for 2 points and win or go for 1 point to send the game into overtime. The conference championship will be determined by the outcome of the game. If SC wins it will go to the Rose Bowl with a payoff of $7.2M; if it loses it will go to the Sun Bowl with a payoff of $1.7M. If SC goes for 2 points, there is a 33% chance it will be successful and win (and a 67% chance it will fail and lose). If it goes for 1 point, there is a 0.98 probability of success and a tie and a 0.02 probability of failure. If the teams tie, they will play overtime, during which SC believes it has only a 20% chance of winning because of fatigue.

Should SC go for 1 or 2 points? What would SC’s probability of winning in overtime have to be to make SC indifferent to going for 1 or 2 points?

Problem 3: The company has 3 health care plans for staff to choose from:

Plan 1: monthly cost of $32 with a $500 deductible; participants pay the first $500 of medical costs for the year; the insurer pays 90% of all remaining expenses.

Plan 2: monthly cost of $5 but a deductible of $1200 with the insurer paying 90% of medical expenses after the insurer pays the first $1200 in a year.

Plan 3: monthly cost of $24 with no deductible; the participants pay 30% of all expenses with the remainder paid by the insurer.

Mary Jones estimates her annual medical expenses are defined by the following probability distribution:

Annual medical expenses	Probability
$100	.15
500	.30
1500	.35
3000	.10
5000	.05
10000	.05

Which plan should the employee select?

Solutions