MAST20005/MAST90058: Week 4 Problems

Some useful information for many of the problems is shown at end of this problem sheet.

1. Consider the COX2 mouse data from problem 6 in week 2. Here are some descriptive statistics from an analysis of them in R:

```
> x <- c(10.39, 10.43, 9.99, 11.17,  8.91,
+        11.20, 11.38, 7.74, 10.61, 11.11)
> summary(x)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   7.74   10.09   10.52   10.29   11.15   11.38
> sd(x)
[1] 1.159109
```

   Assume that these measurements are a random sample from a normal distribution.

   (a) Provide two estimates of the population mean, one based on the sample mean and one on the sample median.

   (b) What is the standard error of each of these estimates?
   Hint: One of the problems from last week is helpful here.

2. A random sample of size 16 from $N(\mu, 25)$ gave $\bar{x} = 73.8$. Find a 95% CI for μ.

3. A nut shop sells peanuts in '2 kg' bags that are weighed on an old 25 kg scale. Suppose it is known that the standard deviation of weights is 0.12 kg. If a sample of 16 bags of peanuts were carefully weighed in a laboratory and the average weight was 2.09 kg, find an approximate 95% confidence interval for the mean weight of peanuts in the '2 kg' bags sold by the nut shop.

4. To determine whether the bacteria count at St Kilda Beach is at normal levels, $n = 37$ samples of water were taken at the beach and the number of bacteria colonies in 100 milliliters of water from each sample were counted. The sample characteristics were $\bar{x} = 11.95$ and $s = 11.80$, measured in hundreds of colonies. Find the approximate 95% confidence interval for the mean number of colonies (measured in hundreds of colonies) in 100 milliliters of water at the beach.

5. Thirteen tons of cheese is stored in some old gypsum mines, including some wheels labelled '22 kg'. A random sample of 9 of these wheels yields $\bar{x} = 20.9$ and $s = 1.858$. Assuming that the weights of these wheels follows $\mathrm{N}(\mu, \sigma^2)$, find a 95% confidence interval for μ. Is the claim these are '22 kg' wheels reasonable?

6. The length of life of brand X light bulbs is assumed to be $\mathrm{N}(\mu_X, 784)$. The length of life of brand Y light bulbs is assumed to be $\mathrm{N}(\mu_Y, 627)$ and these lifetimes are independent of X. If a random sample of $n = 56$ brand X light bulbs yielded $\bar{x} = 937.4$ hours and a random sample of size $m = 57$ brand Y light bulbs yielded $\bar{y} = 988.9$, find a 95% confidence interval for the difference in mean lifetimes, $\mu_X - \mu_Y$. Is it reasonable to conclude that the two brands of light bulb have the same mean lifetimes?

Quantiles of a standard normal: $\Phi^{-1}(0.975) = 1.960$, $\Phi^{-1}(0.95) = 1.645$
Quantiles of t_{37}: $F^{-1}(0.975) = 2.026$, $F^{-1}(0.95) = 1.687$
Quantiles of t_{36}: $F^{-1}(0.975) = 2.028$, $F^{-1}(0.95) = 1.688$
Quantiles of t_9: $F^{-1}(0.975) = 2.262$, $F^{-1}(0.95) = 1.833$
Quantiles of t_8: $F^{-1}(0.975) = 2.306$, $F^{-1}(0.95) = 1.859$