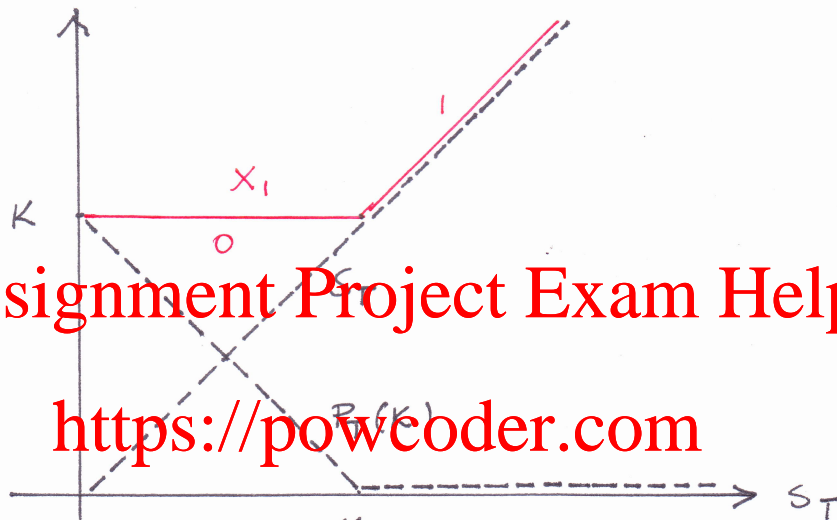


MATH3075/3975 Financial Derivatives

Tutorial 2: Solutions

Exercise 1 It suffices to sketch the graphs of functions representing the portfolio payoff as a function of the terminal price S_T , which may take any nonnegative value.

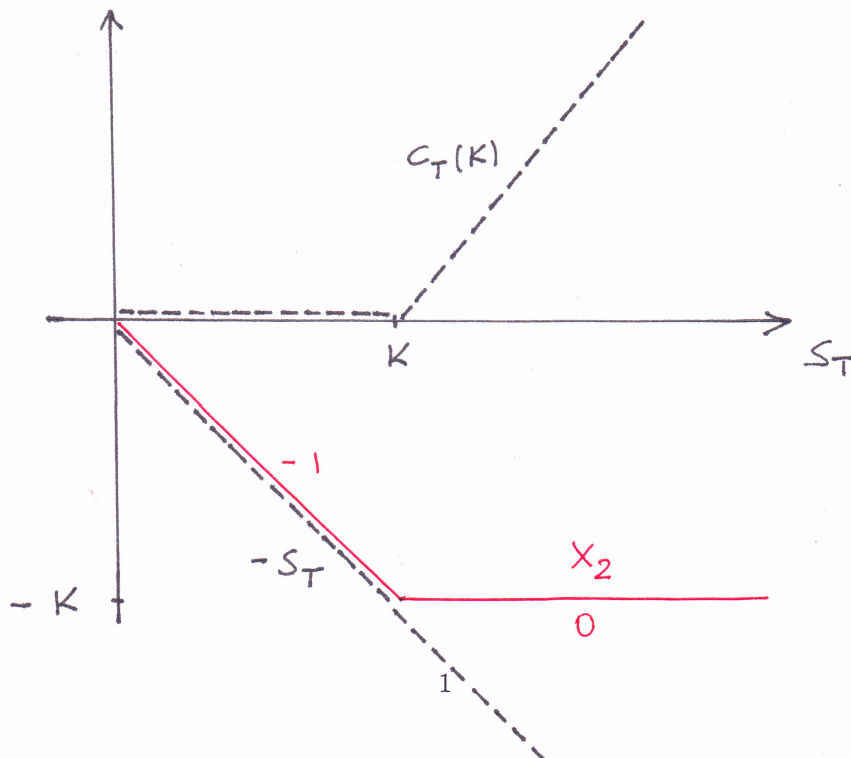
(a) $X_1 = f_1(S_T) = S_T + (K - S_T)^+$ (the initial value should be strictly positive)



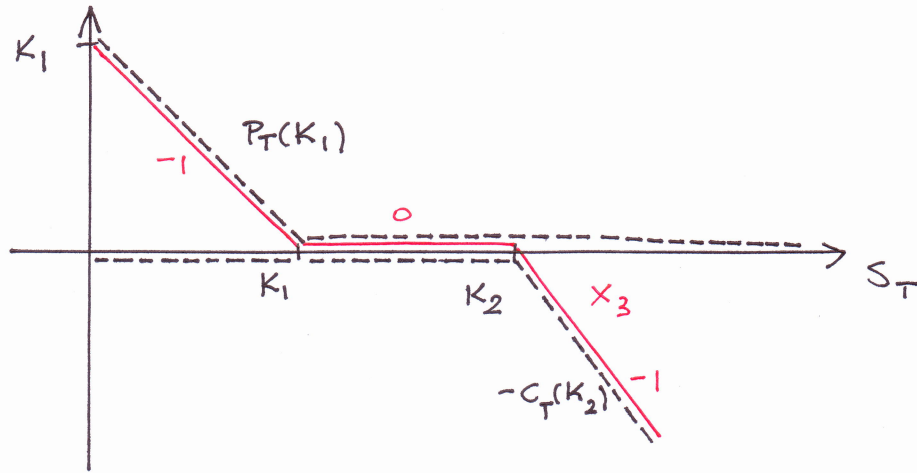
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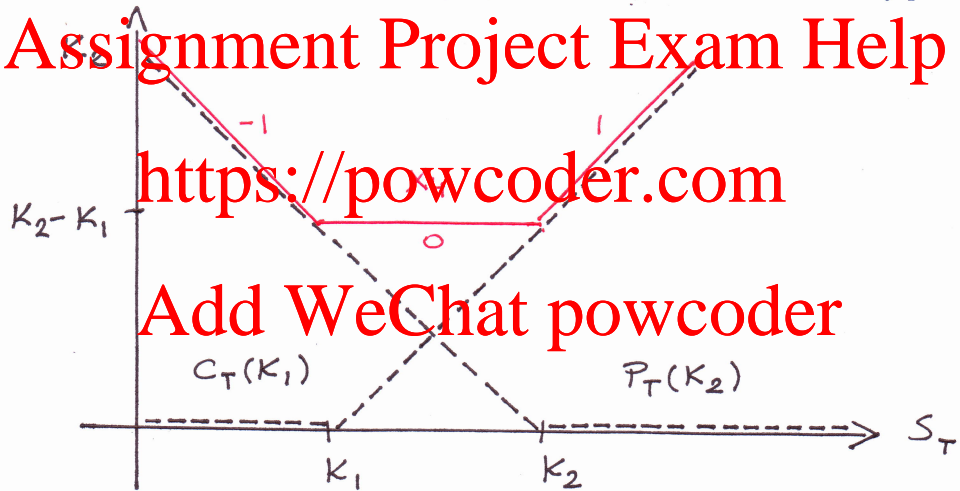
(b) $X_2 = f_2(S_T) = -S_T + (S_T - K)^+$ (the initial value should be strictly negative)



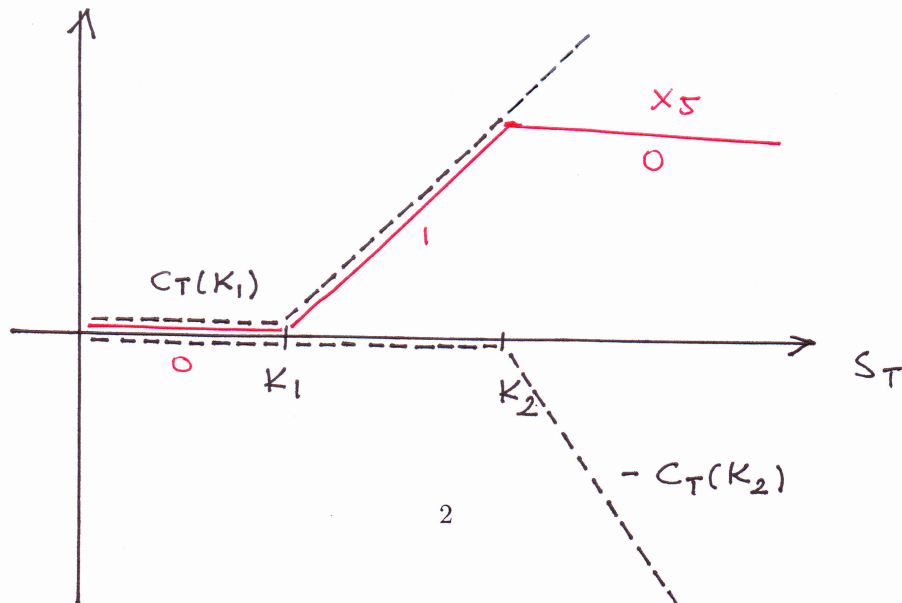
(c) $X_3 = f_3(S_T) = (K_1 - S_T)^+ - (S_T - K_2)^+$ (the sign of the initial value is undetermined)



(d) $X_4 = f_4(S_T) = (K_2 - S_T)^+ + (S_T - K_1)^+$ (the initial value should be strictly positive)



(e) $X_5 = f_5(S_T) = (S_T - K_1)^+ - (S_T - K_2)^+$ (the initial value should be strictly positive)

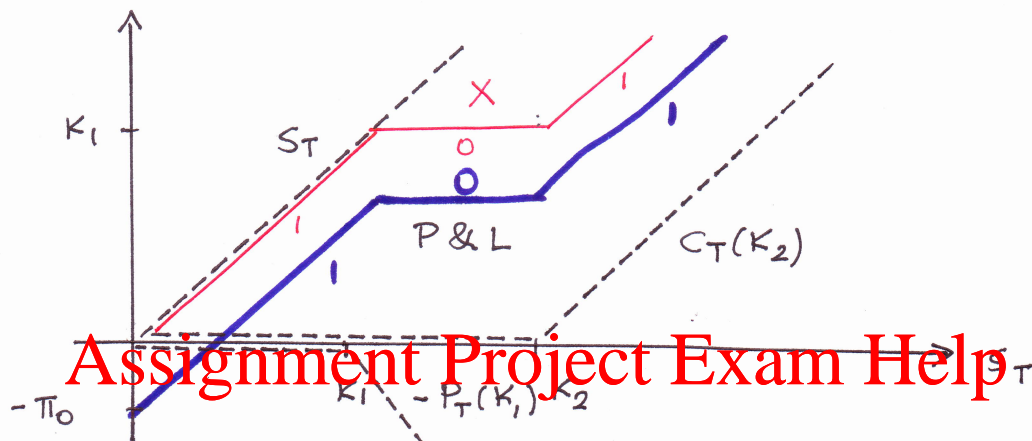


Exercise 2 (a) We note that the portfolio's payoff at time T equals

$$X = S_T - C_T(K_1) + C_T(K_2) = S_T - (S_T - K_1)^+ + (S_T - K_2)^+ \quad (1)$$

where $0 < K_1 < K_2$. Hence the payoff X at time T is strictly positive for every value of $S_T > 0$. Therefore, the price $\pi_0 = S_0 - C_0(K_1) + C_0(K_2)$ paid for the portfolio at time 0 should be strictly positive (but less than S_0 since $X \leq S_T$ for $S_T \geq 0$) and the profits/losses at time T satisfy

$$P\&L = S_T - (S_T - K_1)^+ + (S_T - K_2)^+ - \pi_0. \quad (2)$$



(b) the maximum loss occurs when $S_T = 0$ and equals $-\pi_0$. The maximum profit equals $+\infty$ when $S_T \rightarrow +\infty$, that is, the profit is unlimited. The unique break even point is $S_T = \pi_0$.

(c) We assume that S_T may take any nonnegative value. Then profits and losses occur at time T whenever the following inequality holds $\pi_0 = S_0 - C_0(K_1) + C_0(K_2) > 0$, that is, if $S_0 > C_0(K_1) - C_0(K_2)$. Since $K_1 < K_2$, we also expect that $C_0(K_1) > C_0(K_2)$, since $C_T(K_1) \geq C_T(K_2)$ and $C_T(K_1) > C_T(K_2)$ for $S_T > K_1$. Hence, assuming that S_0 is known, we obtain the following conditions $0 < C_0(K_1) - C_0(K_2) < S_0$.

(d) The view of the investor is bullish.

(e) One can perform an analysis similar to (a)–(d), by noting that we now have

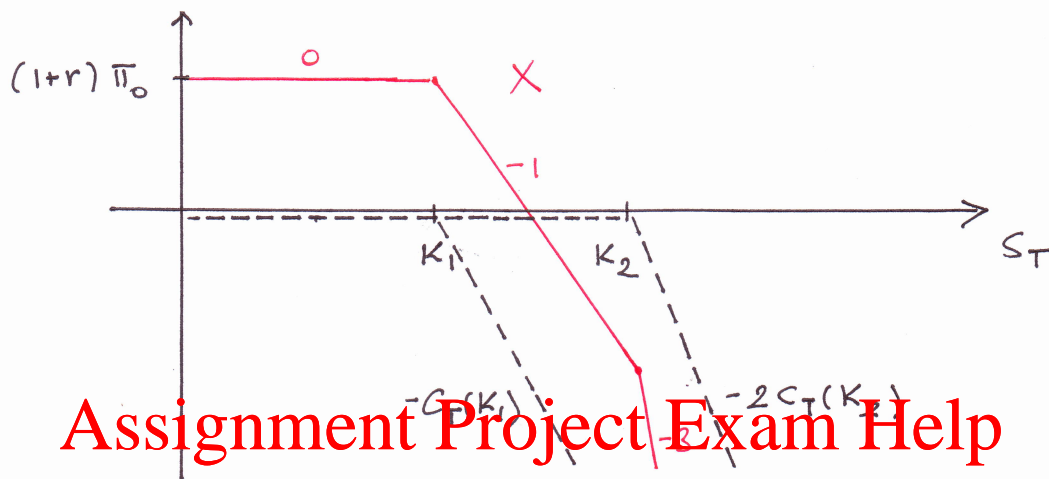
$$X = S_T - 2C_T(K_1) + C_T(K_2) = S_T - 2(S_T - K_1)^+ + (S_T - K_2)^+. \quad (3)$$

Exercise 3 We consider the portfolio with the payoff

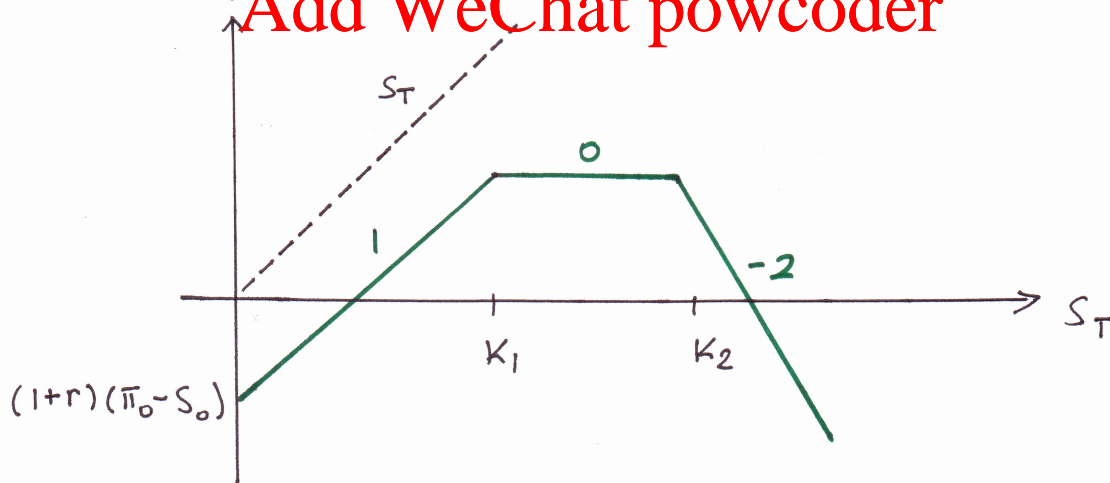
$$X = -C_T(K_1) - 2C_T(K_2) = -(S_T - K_1)^+ - 2(S_T - K_2)^+ \quad (4)$$

with the strictly positive price $\pi_0 = C_0(K_1) + 2C_0(K_2) > 0$ received at time 0, which becomes $(1+r)\pi_0$ at time T . Hence the profits/losses at time T are given by

$$P\&L = (1+r)\pi_0 - C_T(K_1) - 2C_T(K_2) = (1+r)\pi_0 - (S_T - K_1)^+ - 2(S_T - K_2)^+. \quad (5)$$



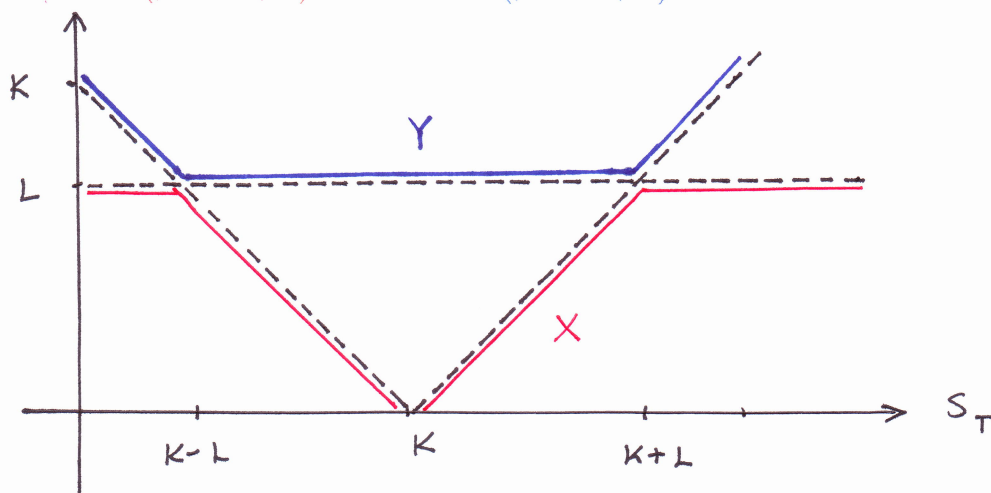
(a) Suppose that at time 0 we buy/short α shares of stock S (notice that $\alpha > 0$ means that we buy shares and $\alpha < 0$ means that we sell short shares). Then the initial and terminal values of the combined portfolio become $-\alpha S_0 + C_0(K_1) + 2C_0(K_2)$ and $\alpha S_T - C_T(K_1) - 2C_T(K_2)$, respectively. If an investor wishes to make profits when S_T is in the interval (K_1, K_2) then he/she should take $\alpha = 1$, that is, buy one share of the stock S .



(b) If we buy one share of the stock S at time 0, then the maximum profit at time T will be $K_1 + (1+r)\pi_0 - (1+r)S_0$ when $S_T \in (K_1, K_2)$. The loss is unlimited when S_T tends to infinity. Break even points are $S_T = (1+r)(S_0 - \pi_0)$ and $S_T = 0.5(K_1 + 2K_2 + (1+r)(\pi_0 - S_0))$.

(c) Since the maximum profit equals $K_1 + (1+r)\pi_0 - (1+r)S_0$, it is strictly positive whenever $S_0 < \pi_0 + (1+r)^{-1}K_1$.

Exercise 4 Recall that $0 < L < K$ are arbitrary real numbers. It is easy to sketch the graph of the payoffs $X = \min(|S_T - K|, L)$ and $Y = \max(|S_T - K|, L)$ as a function of the stock price S_T .



(a) The decomposition of X in terms of long/short positions in standard call options combined with a constant payoff L is

$$X = L - C_T(K - L) + 2C_T(K) - C_T(K + L).$$

Notice that other decompositions of the payoff X are possible (for instance, if we include in our portfolio long/short positions in the put option $P_T(K)$ and the stock S_T).

(b) From the law of one price, we deduce that the price of X at time $t \in [0, T]$ satisfies

$$\pi_t(X) = LB(t, T) - C_t(K - L) + 2C_t(K) - C_t(K + L).$$

(c) The method is analogous to the case of the payoff X . The decomposition of Y in terms of long/short positions in standard call and put options reads

$$Y = P_t(K) + C_t(K - L) - C_t(K) + C_t(K + L),$$

so that the price of Y at time $t \in [0, T]$ satisfies

$$\pi_t(Y) = P_t(K) + C_t(K - L) - C_t(K) + C_t(K + L).$$

Exercise 5 (a) It suffices to generalise the approach used in Exercise 4 by first mimicking the payoff function on the interval $[K_0, K_1] = [0, K_1]$. Then the ‘initial portfolio’ should be adjusted by a suitable number of long/short positions of call options with strikes K_1, K_2, \dots, K_{n-1} to match the slope of the payoff on each interval $[K_i, K_{i+1}]$ for $i = 1, 2, \dots, n - 1$. As soon as the portfolio in the first step is chosen, the solution to the second step (that is, when we consider the interval $[K_1, \infty)$) is unique.

(b) It suffices to apply the law of one price to your solution to part (a).

(c) Since the payoff X is already known at time T , its price at time $t \in [T, U]$ equals $\pi_t(X) = B(t, U)g(S_T)$. Hence before time T , the price satisfies, for every $t \in [0, T]$,

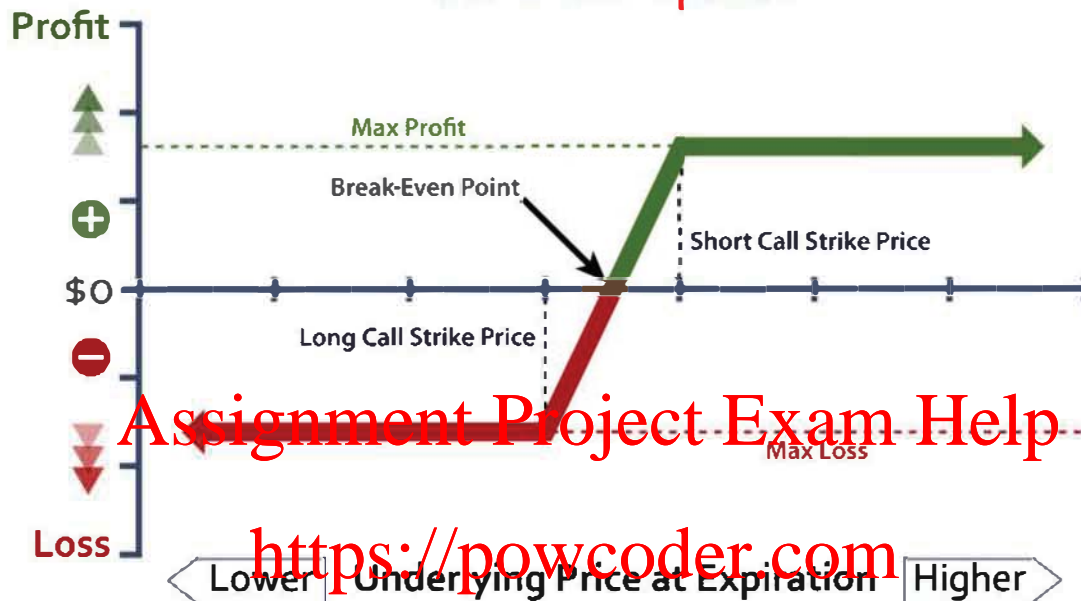
$$\pi_t(X) = \pi_t(B(T, U)g(S_T)) = B(T, U)\pi_t(g(S_T))$$

where the second equality holds only under the assumption that $B(T, U)$ is deterministic (for instance, $B(T, U) = e^{-r(U-T)}$ if the continuously compounded short-term rate is constant). Notice that our approach hinges on the ‘backward induction’ argument.

Basic Options Strategies

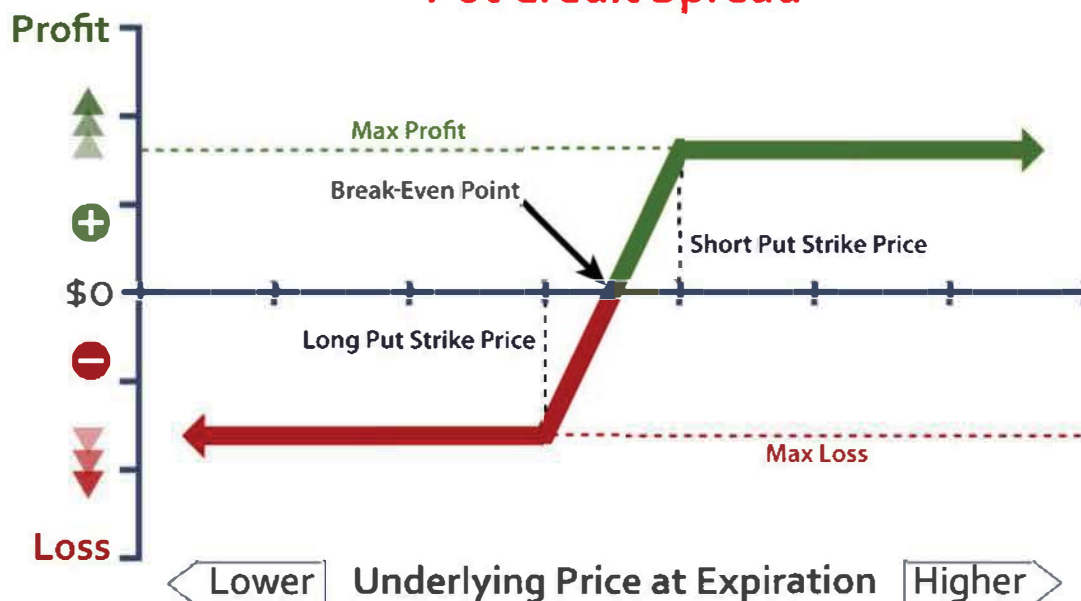
Bullish Strategies

Call Debit Spread



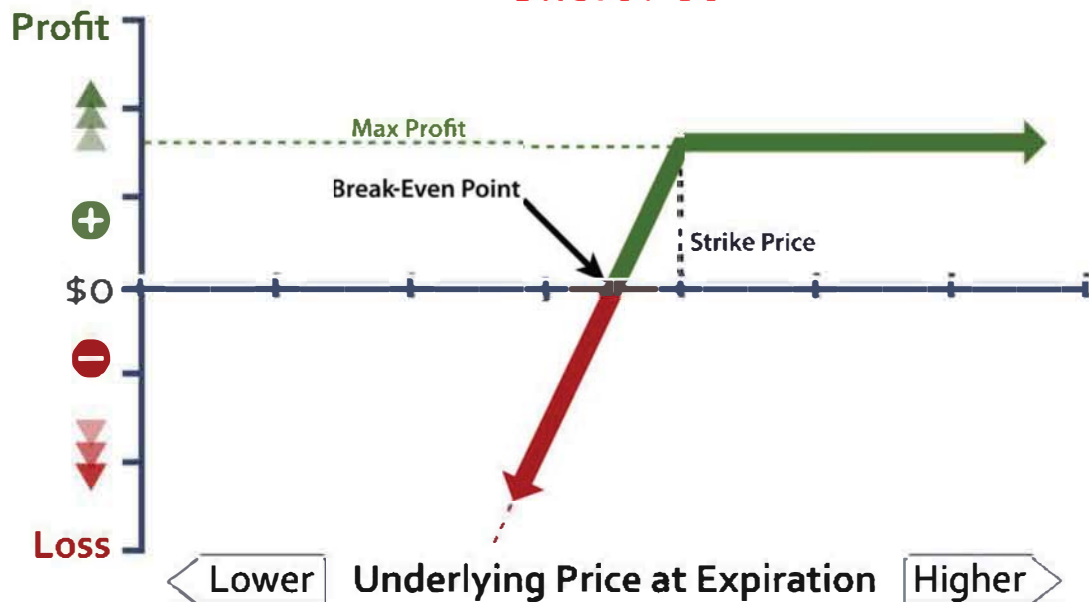
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Put Credit Spread



Put Credit Spread

Short Put



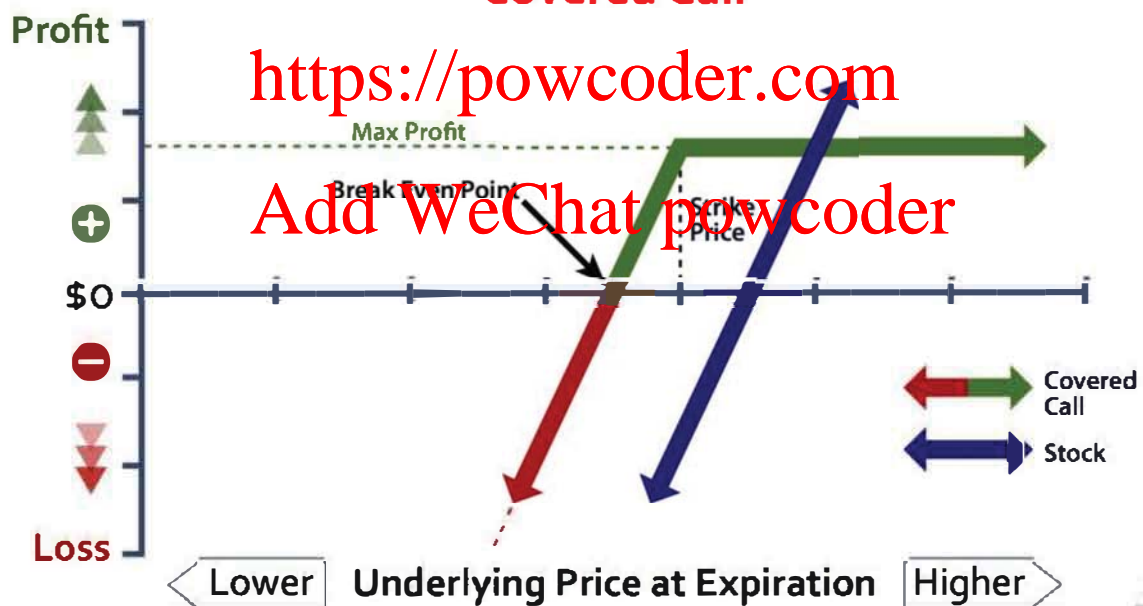
Short Put

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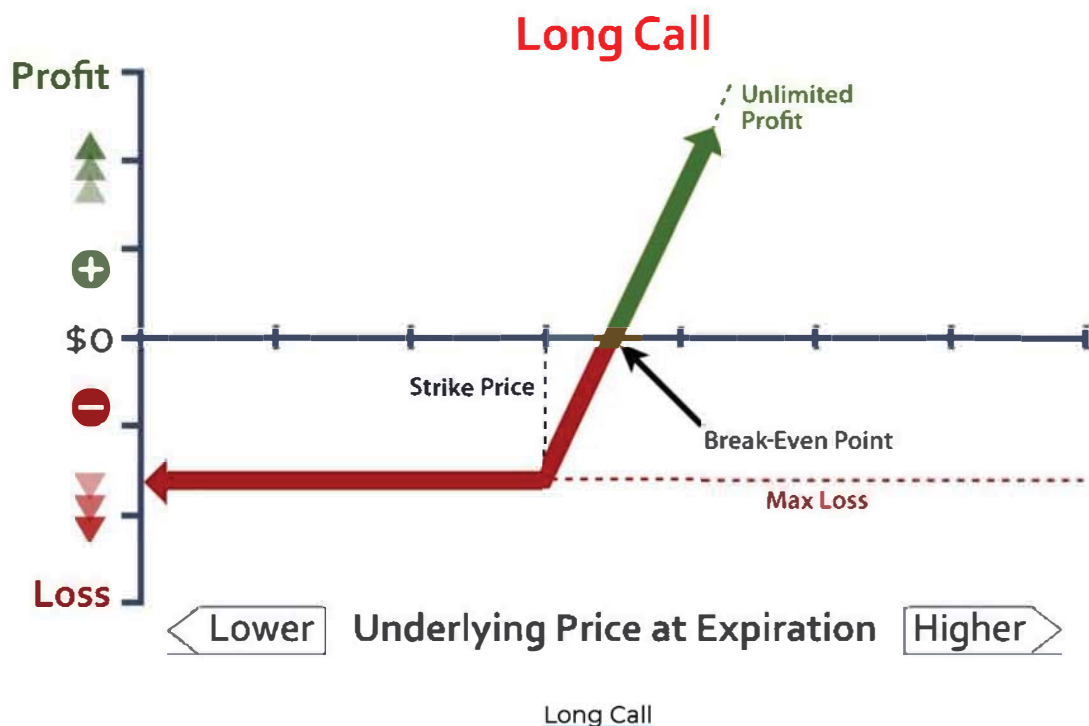
Covered Call

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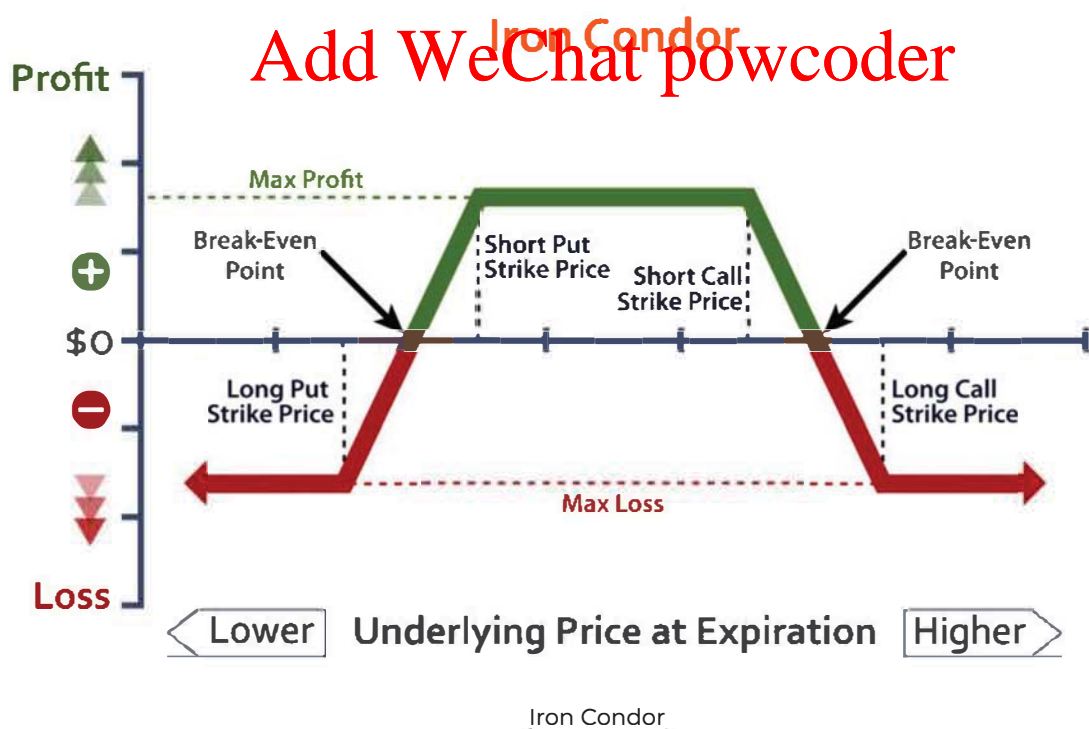
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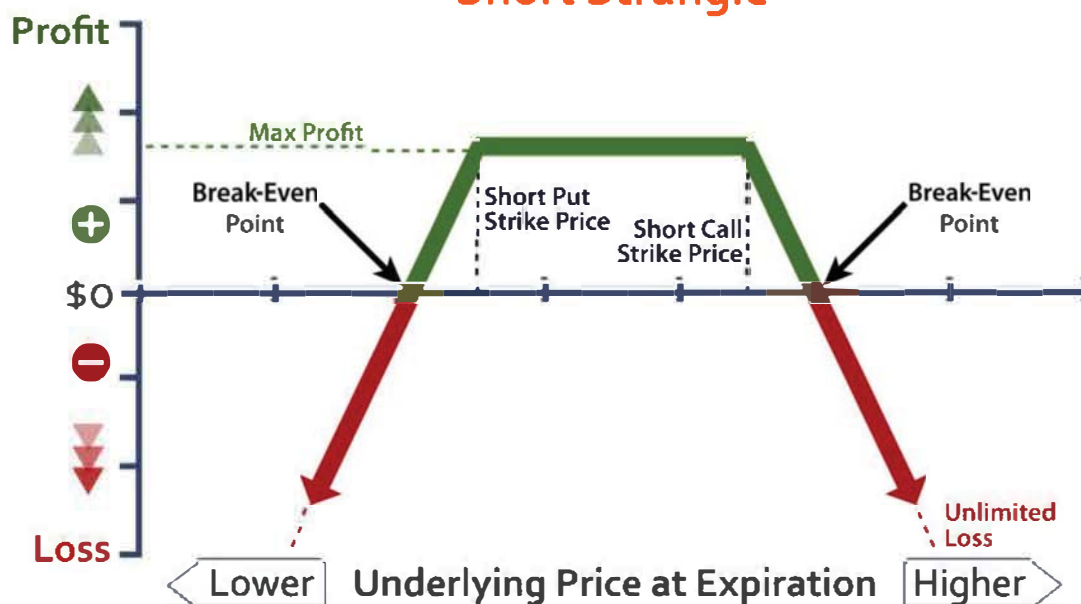
Covered Call



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Neutral Strategies
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Short Strangle



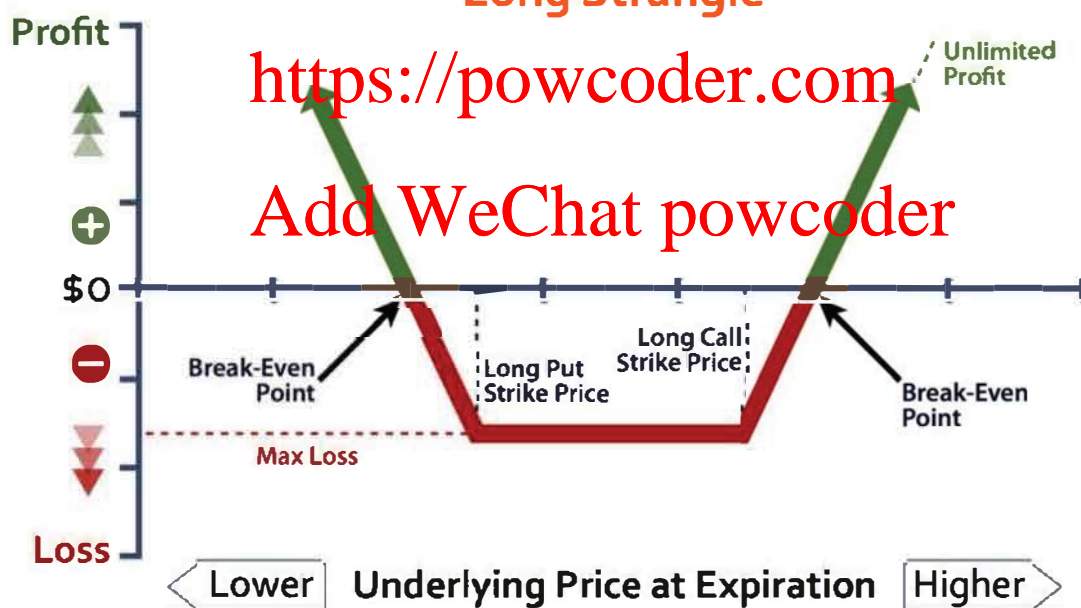
Short Strangle

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Long Strangle

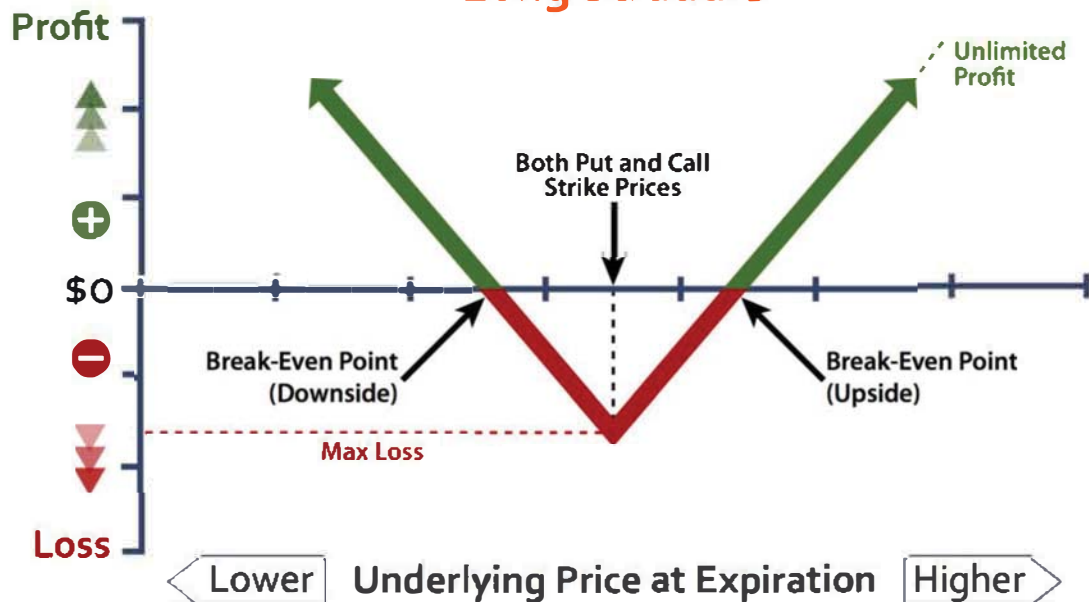
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Long Strangle

Long Straddle



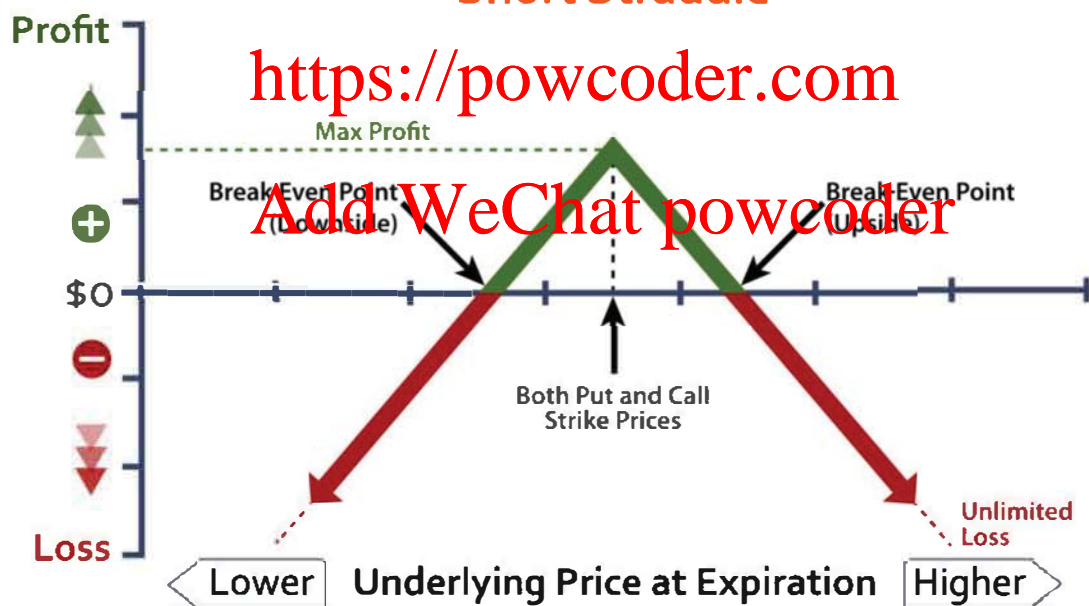
Long Straddle

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Short Straddle

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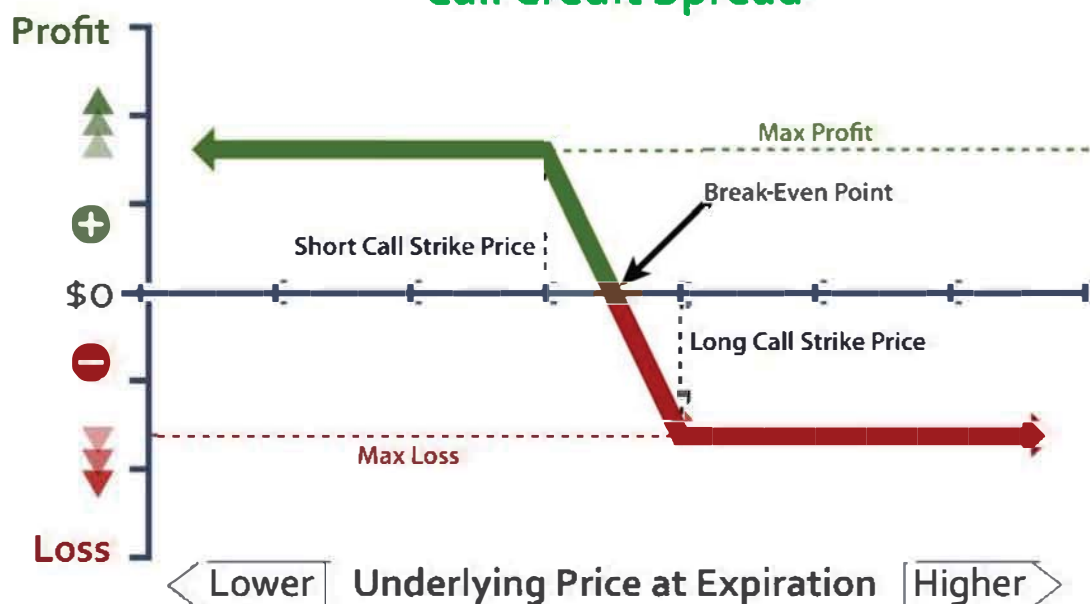
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Short Straddle

Bearish Strategies

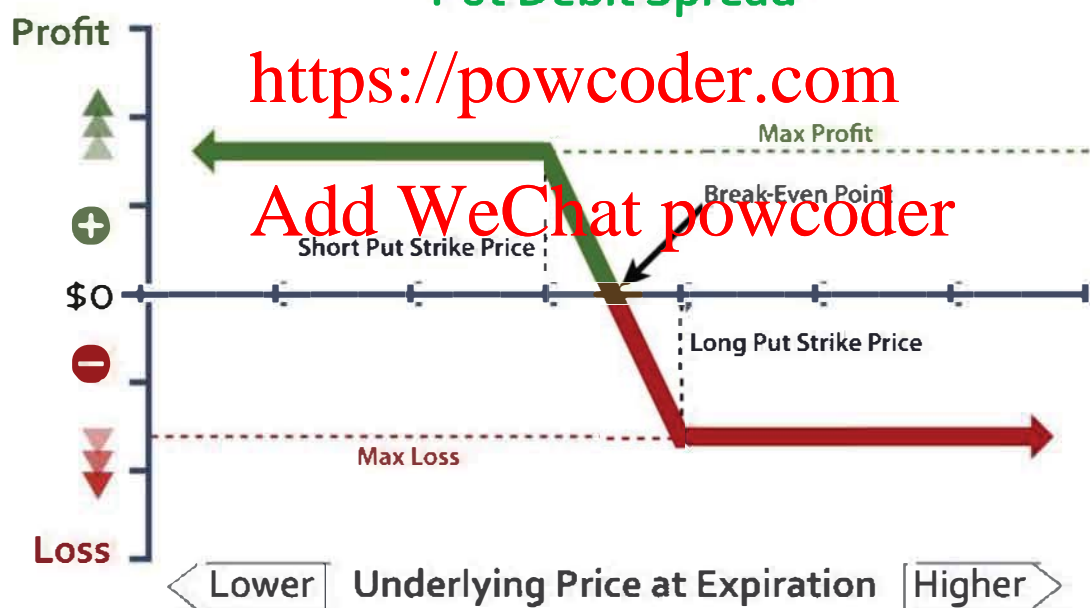
Call Credit Spread



Call Credit Spread

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Put Debit Spread



Put Debit Spread

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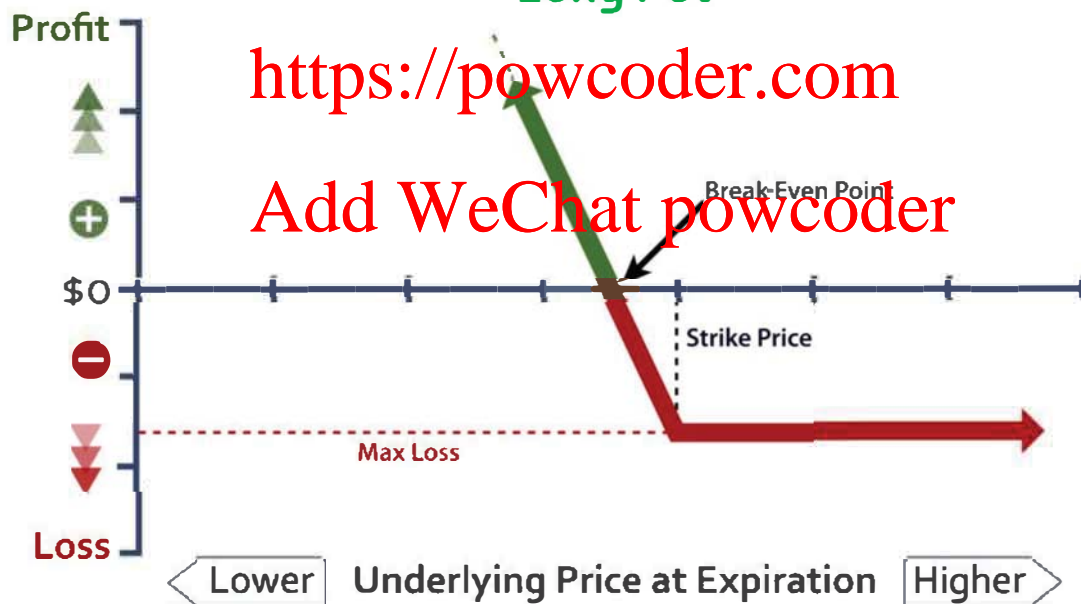
Short Call

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Long Put



Long Put