MATH3075/3975
Financial Derivatives

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Semester 1, 2020

Tutorial sheet 2

Exercise 1 Consider the portfolios composed of shares and options:
(a) one share of stock and put option with strike $K > 0$,
(b) short position in one share of stock and call option with strike $K > 0$,
(c) put with strike K_1 and short call with strike K_2 where $0 < K_1 < K_2$,
(d) put with strike K_2 and call with strike K_1 where $0 < K_1 < K_2$.
(e) call with strike K_1 and short call with strike K_2 where $0 < K_1 < K_2$.
All options are of European style and have the same maturity date T. Sketch the payoff profile at time T of each portfolio. Can you formulate a conjecture about a level of the initial value of each portfolio?

Exercise 2 Assume that you hold the following portfolio: one share, a short call with strike K_1 and a call with strike K_2. Assume that $K_1 < K_2$ and denote by $\pi_0 = S_0 - C_0(K_1) + C_0(K_2)$ the price you paid for this portfolio's at time 0. Assume that the interest rate equals zero.

(a) Sketch the graph of your profit and loss at time T as a function of S_T.
(b) Find the maximum profit, the maximum loss and the break even point(s).
(c) Determine any necessary condition(s) for this portfolio to have the property that both profits and losses may occur when S_T ranges from 0 to infinity.
(d) What is the market view of an investor holding this portfolio?
(e) Give answers to questions (a)-(d) when a short call with strike K_1 is replaced by two short calls with strike K_1.

Exercise 3 Suppose that you hold the following portfolio: short call with strike K_1 and two short calls with strike K_2 with $K_1 < K_2$. Denote by $\pi_0 = C_0(K_1) + 2C_0(K_2) > 0$ the price you received at time 0 and assume that the simple interest rate between 0 and T equals $r > 0$.

(a) How many shares with initial price S_0 would you need to buy/short at time 0 to augment your portfolio if you expect that the market value at time T of one share will be in the interval (K_1, K_2)?
(b) Compute the profit and loss at time T of your augmented portfolio as a function of the stock price S_T. Find the maximum profit, the maximum loss and the break even point(s) of your portfolio.
(c) Assuming that $C_0(K_1)$ and $C_0(K_2)$ are given, derive a condition on the stock price S_0 for which you would make a profit with such a portfolio when $S_T \in (K_1, K_2)$.

Exercise 4 Let $0 < L < K$ be real numbers. Consider the contingent claim with the payoff X at maturity date $T > 0$ given as

$$X = \min\big(|S_T - K|, L\big), \quad \forall\, S_T \in [0, +\infty).$$

Notice that we interpret here S_T as a real variable, as opposed to the random variable $S_T(\omega)$.

(a) Sketch the profile of the payoff X as a function of the stock price S_T at maturity date T and find the decomposition of the payoff X in terms of the payoffs of standard call and put options with different strikes and expiration date T, the payoff of long/short positions on the stock and a constant payoff of maturity T.
(b) Using the law of one price, deduce from part (a) the decomposition of the price of the payoff X at time $t \in [0, T]$ in terms of the prices at time t of call and put options, the price at time t of the stock and the price at time t of the zero-coupon bond with maturity T.
(c) Give answers to questions (a) and (b) for the payoff $Y = \max(|S_T - K|, L)$.

Exercise 5 (MATH3975) Let $g : \mathbb{R}_+ \to \mathbb{R}$ be an arbitrary continuous and piecewise linear function such that $g(0) = c \in \mathbb{R}$ and g is linear on each interval $[K_i, K_{i+1}]$ for $i = 0, 1, \ldots, n-1$ where $K_0 = 0 < K_1 < K_2 < \cdots < K_{n-1} < K_n = +\infty$ where $n \geq 2$ is any (fixed) natural number. Denote by α_i the derivative of g on the interval (K_i, K_{i+1}) for $i = 0, 1, \ldots, n-1$.

(a) Show that the payoff $X = g(S_T)$ can be represented by the terminal payoff of a static portfolio composed of long/short positions in a finite family of call and put options written on the stock S, with the expiration date T and strikes $K_1, K_2, \ldots, K_{n-1}$. You may also include long/short positions in S (equivalently, call and put options with strike zero) and a constant payoff at time T. Notice that several alternative solutions may exist.
(b) Using your solution from part (a), express the price at time $t \in [0, T]$ of the payoff $X = g(S_T)$ which settles at time T.
(c) What is the price of $X = g(S_T)$ at time $t \in [0, U]$ if the payoff X is determined at time T but it settled at time U (that is, the actual transfer of the cash amount X between the counterparties occurs at time U).