MATH3075/3975
Financial Derivatives

School of Mathematics and Statistics
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Tutorial sheet 10

Background: Section 4.4 – American Options in the CRR Model.

Exercise 1 Assume the CRR model $\mathcal{M} = (B, S)$ with $T = 3$, the stock price $S_0 = 100$, $S_1^u = 120$, $S_1^d = 90$, and the risk-free interest rate $r = 0.1$. Consider the American put option on the stock S with the maturity date $T = 3$ and the constant strike price $K = 121$.

(a) Find the arbitrage price P_t^a of the American put option for $t = 0, 1, 2, 3$.

(b) Find the rational exercise times τ_t^*, $t = 0, 1, 2, 3$ for the holder of the American put option.

(c) Show that there exists an arbitrage opportunity for the issuer if the option is not rationally exercised by its holder.

Exercise 2 Assume the CRR model $\mathcal{M} = (B, S)$ with $T = 3$, the stock price $S_0 = 100$, $S_1^u = 120$, $S_1^d = 90$, and the risk-free interest rate $r = 0$. Consider the American call option with the expiration date $T = 3$ and the running payoff $g(S_t, t) = (S_t - K_t)^+$, where the variable strike price equals $K_0 = K_1 = 100$, $K_2 = 105$ and $K_3 = 110$.

(a) Find the arbitrage price X_t^a of the American call option for $t = 0, 1, 2, 3$ and show that it is a strict supermartingale under $\widetilde{\mathbb{P}}$.

(b) Find the holder's rational exercise times τ_0^* for the American call option.

(c) Find the issuer's replicating strategy for the American call option up to the rational exercise time τ_0^*

Exercise 3 (MATH3975) Consider the CRR binomial model $\mathcal{M} = (B,\ S)$ with the initial stock price $S_0 = 9$, the interest rate $r = 0.01$ and the volatility equals $\sigma = 0.1$ per annum. Use the CRR parametrization for u and d, that is,

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = \frac{1}{u},$$

with the time increment $\Delta t = 1$.

We consider call and put options with the expiration date $T = 5$ years and strike $K = 10$.

(a) Compute the price process C_t, $t = 0, 1, \ldots, 5$ of the European call option using the binomial lattice method.

(b) Compute the price process P_t, $t = 0, 1, \ldots, 5$ for the European put option.

(c) Does the put-call parity relationship hold for $t = 0$?

(d) Compute the price process P_t^a, $t = 0, 1, \ldots, 5$ for the American put option. Will the American put option be exercised before the expiration date $T = 5$ by its rational holder?

Exercise 4 (MATH3975) Consider the game option (See Section 4.5) with the expiration date $T = 12$ and the payoff functions $h(S_t)$ and $\ell(S_t)$ where

$$H_t = h(S_t) = (K - S_t)^+ + \alpha$$

and

$$L_t = \ell(S_t) = (K - S_t)^+$$

where $\alpha = 0.02$ and $K = 27$. Assume the CRR model with $d = 0.9, u = 1.1, r = 0.05$ and $S_0 = 25$.

(a) Compute the arbitrage price process $(X_t^g)_{t=0}^T$ for the game option using the recursive formula, for $t = 0, 1, \ldots, T - 1$,

$$X_t^g = \min\Big\{h(S_t),\ \max\Big[\ell(S_t),\ (1+r)^{-1}\big(\widetilde{p}X_{t+1}^{gu} + (1-\widetilde{p})X_{t+1}^{gd}\big)\Big]\Big\}$$

with $\pi_T(X^g) = \ell(S_T)$.

(b) Find the optimal exercise times τ_0^* and σ_0^* for the holder and the issuer of the game option. Recall that

$$\tau_0^* = \inf\big\{t \in \{0, 1, \ldots, T\} \,|\, X_t^g = \ell(S_t)\big\}$$

and

$$\sigma_0^* = \inf\big\{t \in \{0, 1, \ldots, T\} \,|\, X_t^g = h(S_t)\big\}.$$