MATH3075/3975
Financial Derivatives

School of Mathematics and Statistics
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Tutorial sheet 1

Background: Chapter 6 – Probability Review.

Exercise 1 Assume that the joint probability distribution of the two-dimensional random variable (X, Y), that is, the set of probabilities

$$\mathbb{P}(X = i, Y = j) = p_{i,j} \quad \text{for } i, j = 1, 2, 3,$$

is given by:

$$\begin{aligned} &p_{1,1} = 1/9, \quad && p_{1,2} = 1/9, \quad && p_{1,3} = 0, \\ &p_{2,1} = 1/3, && p_{2,2} = 0, && p_{2,3} = 1/6, \\ &p_{3,1} = 1/9, && p_{3,2} = 1/18, && p_{3,3} = 1/9. \end{aligned}$$

(a) Compute $\mathbb{E}_{\mathbb{P}}(X \,|\, Y)$, that is, $\mathbb{E}_{\mathbb{P}}(X \,|\, Y = j)$ for $j = 1, 2, 3$.
(b) Show that the equality $\mathbb{E}_{\mathbb{P}}(X) = \mathbb{E}_{\mathbb{P}}[\,\mathbb{E}_{\mathbb{P}}(X \,|\, Y)]$ holds.
(c) Check if the random variables X and Y are independent.

Exercise 2 The joint probability density function $f_{(X,Y)}$ of random variables X and Y is given by

$$f_{(X,Y)}(x, y) = \frac{1}{y} e^{-x/y} e^{-y}, \quad \forall (x, y) \in \mathbb{R}^2_+,$$

and $f_{(X,Y)}(x, y) = 0$ otherwise.
(a) Check that $f_{(X,Y)}$ is a two-dimensional probability density function.
(b) Show that $\mathbb{E}_{\mathbb{P}}(X \,|\, Y = y) = y$ for all $y \in \mathbb{R}_+$.

Exercise 3 Let X be a random variable uniformly distributed over $(0, 1)$. Compute the conditional expectation $\mathbb{E}_{\mathbb{P}}(X \,|\, X < 1/2)$.

Exercise 4 Let X be an exponentially distributed random variable with parameter $\lambda > 0$, that is, with the probability density function $f_X(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}$ for all $x > 0$. Compute the conditional expectation $\mathbb{E}_{\mathbb{P}}(X \,|\, X > 1)$.

Exercise 5 We assume that $\mathbb{P}(X = \pm 1) = 1/4,\ \mathbb{P}(X = \pm 2) = 1/4$ and we set $Y = X^2$. Check whether the random variables X and Y are correlated and/or dependent.

Exercise 6 (MATH3975) Let U and V have the same probability distribution and let $X = U + V$ and $Y = U - V$. Examine the correlation and independence of the random variables X and Y (provide relevant examples).