MATH3411 INFORMATION, CODES & CIPHERS

Test 1, Session 2 2011, SOLUTIONS

Version A

Multiple choice: c,b,c,d,a
True/False: T, F, F, T, T

1. **(c)**:
2. **(b)**: third block has 3 ones and the 20th bit is in that block.
3. **(c)**: The syndrome is 010.
4. **(d)**:
5. **(a)**: create a tree for $\dfrac{1}{4} + \dfrac{1}{8} = (0.011)_2$.

6. (a) **True**. $w = d = 7 = 2 \times 3 + 1$ so three can be corrected.

   (b) **False**: Sphere packing bound implies $|C| = 2^k \le \dfrac{2^n}{1 + n + \frac{1}{2}n(n-1)}$ for 2-error correcting and $n = 8$, $k = 3$ does not satisfy this inequality.

   (c) **False**: $1010 = \mathbf{c}_3 + \mathbf{c}_3 = \mathbf{c}_4 + \mathbf{c}_1$. Alternatively, the Kraft-McMillan constant for this code is $\frac{9}{8}$, so by the Kraft-McMillan theorem no UD code with the parameters of this code exists.

   (d) **True**: $K = \dfrac{1}{3} + 4 \times \dfrac{1}{9} + 2 \times \dfrac{1}{27} < 1$.

   (e) **True**: $M\mathbf{p} = \mathbf{p}$ and sum of entries of $\mathbf{p}$ is 1.

7. (a) In order the codewords are $00, 10, 010, 110, 111, 0110, 0111$.

   (b) Average length is $\dfrac{51}{20}$.

Version B

Multiple Choice: d, d, b, c, d
True/False: F, T, T, F, T

1. **(d)**:
2. **(d)**: last block has 3 ones and the 35th bit is in that block
3. **(b)**: Sphere packing bound implies $|C| = 2^k \leq \dfrac{2^n}{1 + n + \frac{1}{2}n(n-1)}$ for 2-error correcting. The right-hand side of this inequality is $\dfrac{2^7}{29} \approx 4.4$, so $k \leq 2$.
4. **(c)**: Since that codeword has weight 7, $w = d = 7 = 2 \times 3 + 1$ so three can be corrected.

5. **(d)**: The Kraft-Macmillan constant is clearly greater than 1 in the first two cases and for $r = 4$ we get $3 \times \frac{1}{4} + 4 \times \frac{1}{16} + \frac{1}{64} > 1$, but for $r = 5$, $K = 3 \times \frac{1}{5} + 4 \times \frac{1}{25} + \frac{1}{125} < 1$.

6. (a) **False**: second check equation is not satisfied.
   (b) **True**: multiply out the matrix and vector.
   (c) **True**: it's an I-code (draw a tree)

   (d) **False**: $\mathbf{c}_5 = 101$ either by the tree or $\dfrac{1}{4} + 3 \times \dfrac{1}{8} = \dfrac{1}{2} + \dfrac{1}{8} = (0.101)_2$.
   (e) **True**: $M\mathbf{p} = \mathbf{p}$ and sum of entries of $\mathbf{p}$ is 1.
7. (a) In order the codewords are $01, 11, 000, 001, 101, 1000, 1001$
   (b) Average length is $\dfrac{52}{20} = \dfrac{13}{5}$.