Name: .... Student ID: ....

UNSW School of Mathematics and Statistics

MATH3411 Information Codes and Ciphers

2015 S2 **TEST 1** **VERSION A**

- Time Allowed: **45 minutes**

For multiple choice questions, **circle the correct answer**; each multiple choice question is worth **1 mark**. For written answer questions, **use extra paper**. Staple all papers together when finished.

1. There may be an error in the check digit in the ISBN number 0-19-861133-X. The correct check digit is

(a) 1 (b) 4 (c) 7 (d) X (e) None of these

2. Consider a binary channel with bit-error probability p where errors in different positions are independent. Suppose that a codeword $\mathbf{x}$ is sent from the binary repetition code with codewords of length 6. Define

$$w = (1-p)^6 \qquad x = 6p(1-p)^5 \qquad y = 15p^2(1-p)^4 \qquad z = 20p^3(1-p)^3$$

The probability that one or more errors are correctly corrected using a minimum distance decoding strategy is

(a) x (b) $x+y$ (c) $x+y+z$ (d) $w+x+y$ (e) $w+x+y+z$

3. Let C be the binary linear code with generator matrix

$$G = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

How many codewords are there in C?

(a) 4 (b) 16 (c) 64 (d) 256 (e) 1024

4. For the code C of Question 4, assume that the first four bits are check bits. The codeword that encodes $\mathbf{m} = 1011$ is then

(a) 11001011 (b) 10011011 (c) 11101011 (d) 11011011 (e) 10101011

5. Let C be the code of all vectors $\mathbf{x} = x_1x_2x_3x_4 \in \mathbb{Z}_5^4$ satisfying the check equations

$$\begin{aligned} x_1 \quad\quad\;\; + \; x_3 + x_4 &\equiv 0 \pmod 5 \\ 2x_2 + 3x_3 + 2x_4 &\equiv 0 \pmod 5 \end{aligned}$$

Which, if any, of the following is a valid code word?

(a) 1212 (b) 1221 (c) 3434 (d) 3443 (e) None of these

6. Consider a compression code with codewords $\mathbf{c}_1 = 1$, $\mathbf{c}_2 = 10$, $\mathbf{c}_3 = 100$, $\mathbf{c}_4 = ?$ where $\mathbf{c}_4$ is to be chosen from the list of four possibilities below.
Which choice, if any, of $\mathbf{c}_4$ makes the resulting code uniquely decodable?

(a) $\mathbf{c}_4 = 0$ (b) $\mathbf{c}_4 = 011$ (c) $\mathbf{c}_4 = 000$ (d) $\mathbf{c}_4 = 1010$ (e) None of these

7. The minimum radix that would be needed to create a UD-code for the source

$$S = \{s_1, s_2, \ldots, s_9\}$$

with codeword lengths $1, 1, 1, 2, 2, 2, 2, 3, 4$, respectively, is

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

8. Consider the standard binary I-code with codeword lengths 2, 2, 3, 3, 4, 4.
The codeword $\mathbf{c}_6$ corresponding to symbol s_6 is given by

(a) 0011 (b) 1111 (c) 1100 (d) 1110 (e) 1101

9. Let $S = \{s_1, s_2\}$ be a source with probabilities $p_1 = \frac{5}{7}$, $p_2 = \frac{2}{7}$. The average length of a **radix 3** Huffman code for the **second extension** $S^{(2)}$ of this source is

(a) $\dfrac{10}{7}$ (b) $\dfrac{6}{7}$ (c) $\dfrac{9}{7}$ (d) $\dfrac{12}{7}$ (e) $\dfrac{73}{49}$

10. A Markov source $S = \{s_1, s_2, s_3\}$ has transition matrix M. The Huffman code for the equilibrium distribution is $\text{Huff}_E = (1,\ 00,\ 01)$ (so $\mathbf{c}_1 = 1$, $\mathbf{c}_2 = 00$ and $\mathbf{c}_3 = 01$).
The Huffman codes for the columns of M are given by

$$\text{Huff}_1 = (01,\ 00,\ 1) \qquad \text{Huff}_2 = (10,\ 0,\ 11) \qquad \text{Huff}_3 = (11,\ 0,\ 10)$$

The Markov Huffman encoding of the string of source symbols $s_2s_1s_3s_2s_3$ is

(a) 00101011 (b) 000110010 (c) 00101101 (d) 00110011 (e) 00100111

11. [**5 marks**] Let C be the binary linear code with check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

(i) Find a generator matrix G for the code C.

(ii) Write down all the codewords in C.

(iii) Find Hamming weights of each codeword in C.

(iv) Find the Hamming distances between each pair of distinct codewords in C.

(v) What are the error correcting and error detecting capabilities of C?

Name: .... Student ID: ....

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MATH3411 Information Codes and Ciphers

2015 S2 **TEST 1** **VERSION B**

- Time Allowed: **45 minutes**

For multiple choice questions, **circle the correct answer**; each multiple choice question is worth **1 mark**. For written answer questions, **use extra paper**. Staple all papers together when finished.

1. There may be an error in the check digit in the ISBN number 0-245-58345-9. The correct check digit is

(a) 0 (b) 3 (c) 6 (d) 9 (e) None of these

2. Let C be the code of all vectors $\mathbf{x} = x_1x_2x_3x_4 \in \mathbb{Z}_5^4$ satisfying the check equations

$$\begin{array}{ccccccccc} x_1 & + & x_2 & + & x_3 & & & \equiv & 0 \pmod 5 \\ x_1 & + & 3x_2 & & & + & 2x_4 & \equiv & 0 \pmod 5 \end{array}$$

Which, if any, of the following is a valid code word?

(a) 1122 (b) 2121 (c) 4343 (d) 3344 (e) None of these

3. Consider a binary channel with bit-error probability p where errors in different positions are independent. Suppose that a codeword $\mathbf{x}$ is sent from a binary repetition code with codewords of length 4. The probability that undetected error(s) occur is

(a) $4p^3(1-p)$ (b) $6p^2(1-p)^2$ (c) p^4 (d) $4p^3(1-p)+p^4$ (e) 0

4. Let C be the binary linear code with generator matrix

$$G = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

How many codewords are there in C?

(a) 8 (b) 16 (c) 32 (d) 64 (e) 128

5. For the code C of Question 4, assume that the first four bits are check bits. The codeword that encodes $\mathbf{m} = 011$ is then

(a) 0101011 (b) 0010011 (c) 0110011 (d) 1100011 (e) 0100011

6. Consider a code with codewords $\mathbf{c}_1 = 1$, $\mathbf{c}_2 = 01$, $\mathbf{c}_3 = 001$, $\mathbf{c}_4 = ?$
where $\mathbf{c}_4$ is to be chosen from the list of four possibilities below.
Which choice, if any, of $\mathbf{c}_4$ makes the resulting code **not** uniquely decodable?

(a) $\mathbf{c}_4 = 0001$ (b) $\mathbf{c}_4 = 000$ (c) $\mathbf{c}_4 = 00$ (d) $\mathbf{c}_4 = 0000$ (e) None of these

7. The minimum radix that would be needed to create a UD-code for the source

$$S = \{s_1, s_2, s_3, s_4, \ldots, s_{10}\}$$

with codeword lengths $1, 1, 1,\ 2, 2, 2,\ 3, 3, 3,\ 3$, respectively is

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

8. Consider the standard binary I-code with codeword lengths 1, 3, 3, 4, 4, 4.
The codeword $\mathbf{c}_6$ corresponding to symbol s_6 is given by

(a) 0011 (b) 1110 (c) 1100 (d) 1101 (e) 1111

9. Let $S = \{s_1, s_2\}$ be a source with probabilities $p_1 = \frac{4}{7}$, $p_2 = \frac{3}{7}$. The average length of a **radix 3** Huffman code for the **second extension** $S^{(2)}$ of this source is

(a) $\frac{82}{49}$ (b) $\frac{9}{14}$ (c) $\frac{6}{7}$ (d) $\frac{9}{7}$ (e) $\frac{10}{7}$

10. A Markov source $S = \{s_1, s_2, s_3\}$ has transition matrix M. The Huffman code for the equilibrium distribution is $\text{Huff}_E = (1,\ 00,\ 01)$ (so $\mathbf{c}_1 = 1$, $\mathbf{c}_2 = 00$ and $\mathbf{c}_3 = 01$). The Huffman codes for the columns of M are given by

$$\text{Huff}_1 = (01,\ 00,\ 1) \qquad \text{Huff}_2 = (10,\ 0,\ 11) \qquad \text{Huff}_3 = (11,\ 0,\ 10)\,.$$

The Markov Huffman encoding of the string of source symbols $s_3 s_2 s_1 s_2 s_3$ is

(a) 010100001 (b) 010100011 (c) 10001010 (d) 01001011 (e) 01010001

11. [**5 marks**] Let C be the binary linear code with parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

(i) Find a generator matrix G for the code C.

(ii) Write down all the codewords in C.

(iii) Find Hamming weights of each codeword in C.

(iv) Find the Hamming distances between each pair of distinct codewords in C.

(v) What are the error correcting and error detecting capabilities of C?