

MATH3411 INFORMATION, CODES & CIPHERS

Test 1, Session 2 2012, SOLUTIONS

Version A

Multiple choice: b,d,e,b,d

True/False: F, T, F, F, T

1. **(b)**: code can detect up to 3 errors. In fact, there is not really enough information to answer this, which was a mistake on my part, so I accepted (e) as an answer.
2. **(d)**: the two basis words have weight 6 and their only non-trivial combination has weight greater than 6
3. **(e)**: all the given options lead to ambiguities. With (a) $\mathbf{c}_4 = \mathbf{c}_1 + \mathbf{c}_1$; with (b) $\mathbf{c}_1 + \mathbf{c}_1 = \mathbf{c}_2 + \mathbf{c}_3$; with (c) $\mathbf{c}_4 = \mathbf{c}_1 + \mathbf{c}_2$; with (d) $\mathbf{c}_3 = \mathbf{c}_1 + \mathbf{c}_1$.
4. **(b)**:

5. **(d)**: <https://powcoder.com>
6. (i) **False** Syndrome is 10.
(ii) **True**. in a triple error at least one row or column has an odd number of errors.
(iii) **False**: The Sphere Packing Bound $|C| \leq \frac{2}{1 + n + \frac{1}{2}n(n-1)}$ for $t = 2$, $n = 9$, and $|C| = 12$ is not satisfied for $n \leq 9$.
(iv) **False**: draw the trees. The first one has 2 non-decision nodes, the second only one so they cannot be isomorphic rooted trees.
(v) **True**: $L = \frac{87}{98}$

7. (i) $G = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$

- (ii) The three rows of G will do: 011100, 101010, 110001.
- (iii) 011011
- (iv) See Notes for definition. Here $d(C) = 3$: no column is zero and clearly no two columns of H are multiples, but the first, second and last together are linearly dependent.

Version B

Multiple Choice: d, d, c, e, b

True/False: T, T, F, F, T

1. **(d)**: code can correct up to two errors.
2. **(d)**: makes the code a prefix code. Also all the given options lead to ambiguities. With (a) $\mathbf{c}_4 = \mathbf{c}_1 + \mathbf{c}_2$; with (b) $\mathbf{c}_2 = \mathbf{c}_4 + \mathbf{c}_4$; with (c) $\mathbf{c}_4 = \mathbf{c}_2 + \mathbf{c}_1$; with (e) $\mathbf{c}_3 = \mathbf{c}_4 + \mathbf{c}_2$.
3. **(c)**:
4. **(e)**: note the code was radix 3, needs 1 dummy symbol and you need to divide by 2 at the end.
5. **(b)**:
6. (i) **True**: Syndrome is 0.
 (ii) **True**: errors could occur as the corners of a rectangle
 (iii) **False**: sphere packing bound implies $|C| \leq \frac{2^n}{1 + n + \frac{1}{2}n(n-1)}$ for 2-error correcting and $n = 9$, $k = 2$, $|C| = 12$ does not satisfy this inequality.
 (iv) **False**: it is 6. The two basis words have weight 6 and the only non-trivial linear combination has greater weight.
 (v) **True**: The decision trees are isomorphic, or for the two words beginning 10 flip the third bit and also flip the last bit on the word 1111.

$$7. \quad (i) \quad H \sim \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \text{ so } G = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

- (ii) 0101101
- (iii) See Notes for definition. Here $d(C) = 4$: no column is zero, clearly no two columns of H are multiples, and for the sum of any three columns the first entry is one, but the sum of the first three columns and the last column is zero.