Name: .... Student ID: ....

UNSW SCHOOL OF MATHEMATICS AND STATISTICS

MATH3411 INFORMATION CODES AND CIPHERS

2016 S2 **TEST 1** **VERSION A**

- Time Allowed: **45 minutes**

For multiple choice questions, **circle the correct answer**; each multiple choice question is worth **1 mark**. For written answer questions, **use extra paper**. Staple all papers together when finished.

1. Consider a binary channel with bit-error probability p, where errors in different positions are independent. Suppose that a codeword $\mathbf{x}$ is sent from a binary linear code with minimum distance 6 and codeword length 9. The probability that one or more errors are correctly corrected using a minimum distance decoding strategy is

$$w = (1-p)^9 \qquad x = 9p(1-p)^8 \qquad y = 36p^2(1-p)^7 \qquad z = 84p^3(1-p)^6$$

The probability that one or more errors are correctly corrected using a minimum distance decoding strategy is

(a) w (b) $w+x$ (c) $w+x+y$ (d) $x+y$ (e) $x+y+z$

2. Let C be the code of all vectors $\mathbf{x} = x_1x_2x_3x_4 \in \mathbb{Z}_7^4$ satisfying the check equations

$$\begin{array}{ccccccccccc} x_1 & + & & + & x_3 & + & x_4 & \equiv & 0 & \pmod 7 \\ & & 2x_2 & + & 2x_3 & + & 3x_4 & \equiv & 0 & \pmod 7 \end{array}$$

Assuming that x_1 and x_2 are the information bits, find the codeword which encodes the message 10.

(a) 1042 (b) 1024 (c) 4210 (d) 2410 (e) None of these

3. Let C be the ternary linear code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 \end{pmatrix}$$

How many codewords are there in C?

(a) 4 (b) 7 (c) 16 (d) 27 (e) 81

4. For the code C of Question 3, assume that the last three bits are check bits. The codeword that encodes $\mathbf{m} = 1021$ is then

(a) 1021021 (b) 1011021 (c) 1021200 (d) 1021122 (e) None of these

5. A binary linear code C has minimum distance $d = 3$ and length $n = 7$. The maximal possible number of information bits k for such a code is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

6. Consider a linear code C with a 7×10 parity check matrix H. The number of codewords in a basis for C is

(a) 1 (b) 3 (c) 7 (d) 10 (e) none of these

7. Consider a compression code with codewords $\mathbf{c}_1 = 1$, $\mathbf{c}_2 = 11$, $\mathbf{c}_3 = 100$, $\mathbf{c}_4 = ?$ where $\mathbf{c}_4$ is to be chosen from the list of four possibilities below. Which choice, if any, of $\mathbf{c}_4$ makes the resulting code uniquely decodable?

(a) $\mathbf{c}_4 = 0$ (b) $\mathbf{c}_4 = 011$ (c) $\mathbf{c}_4 = 000$ (d) $\mathbf{c}_4 = 1010$ (e) None of these

8. The minimum radix that would be needed to create a UD-code for the source

$$S = \{s_1, s_2, \ldots, s_7\}$$

with codeword lengths 1, 2, 2, 2, 2, 3, 4 respectively, is

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

9. Consider the standard binary I-code with codeword lengths 1, 2, 3, 4, 4. The codeword $\mathbf{c}_5$ corresponding to symbol s_5 is given by

(a) 0011 (b) 0110 (c) 1100 (d) 1110 (e) 1111

10. Let $S = \{s_1, s_2\}$ be a source with probabilities $p_1 = \frac{5}{6}$, $p_2 = \frac{1}{6}$. The average length of a **radix 3** Huffman code for the **second extension** $S^{(2)}$ of this source is

(a) $\frac{53}{36}$ (b) $\frac{53}{72}$ (c) $\frac{7}{6}$ (d) $\frac{7}{12}$ (e) $\frac{53}{12}$

11. **[5 marks]**

(a) Show that there is no uniquely decodable binary code with codeword lengths $1, 2, 3, 4, 4, 4$, respectively.

(b) Symbol s_1 of the source $S = \{s_1, s_2\}$ occurs with probability $4/5$ and symbol s_2 occurs with probability $1/5$. Find a uniquely decodable binary code of minimal average length for S^2, assuming that successive symbols occur independently, and state the average length per original source symbol of the code.

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MATH3411 INFORMATION CODES AND CIPHERS

2016 S2 **TEST 1** **VERSION B**

- Time Allowed: **45 minutes**

For multiple choice questions, **circle the correct answer**; each multiple choice question is worth **1 mark**. For written answer questions, **use extra paper**. Staple all papers together when finished.

1. You are given the following 7-bit ASCII codewords:

C	1000011	o	1101111	d	1100100	e	1100101
\$	0100100	–	0101101	i	1101001	Z	1011010

Define a 5-character 8-bit ASCII burst code by encoding characters in blocks of four together with a 5th character which is used as a check codeword.
(This is similar to the 9-character 8-bit ASCII code studied in lectures.)

The message "Code" together with its check character is given by:

(a) Code\$ (b) Code- (c) Codei (d) CodeZ (e) None of these

2. Let C be the code of all vectors $\mathbf{x} = x_1x_2x_3x_4 \in \mathbb{Z}_7^4$ satisfying the check equations

$$\begin{array}{rcrcrcrcl} 2x_1 & + & x_2 & + & 2x_3 & + & x_4 & \equiv & 0 \pmod 7 \\ x_1 & & & + & x_3 & + & 4x_4 & \equiv & 0 \pmod 7 \end{array}$$

There are two information bits but you are not told in which positions they lie. Which of the following codewords could possibly encode the message 10?

(a) 1050 (b) 1500 (c) 5100 (d) 0015 (e) None of these

3. Let C be the binary linear code with generator matrix

$$G = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

How many codewords are there in C?

(a) 4 (b) 8 (c) 16 (d) 32 (e) 256

4. For the code C of Question 3, assume that the first four bits are information bits. The codeword that encodes $\mathbf{m} = 1011$ is then

(a) 10110001 (b) 10001011 (c) 11001011 (d) 10111111 (e) None of these

5. A binary linear code C has minimum distance $d = 5$ and length $n = 8$.
The maximal possible number of information bits k for such a code is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

6. Consider a linear code C with a 6×11 parity check matrix H.
The number of codewords in a basis for C is

(a) 1 (b) 5 (c) 6 (d) 11 (e) none of these

7. A uniquely decodable code has codewords $\mathbf{c}_1 = 1$, $\mathbf{c}_2 = 01$, $\mathbf{c}_3 = 001$, $\mathbf{c}_4 = ?$.
Which of the following codewords could $\mathbf{c}_4$ be?

(a) $\mathbf{c}_4 = 0$ (b) $\mathbf{c}_4 = 00$ (c) $\mathbf{c}_4 = 10$ (d) $\mathbf{c}_4 = 11$ (e) None of these

8. The minimum radix that would be needed to create a UD-code for the source

$$S = \{s_1, s_2, \ldots, s_7\}$$

with codeword lengths $1, 1, 2, 2, 2, 2, 2$, respectively is

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

9. Consider the standard binary I-code with codeword lengths 2, 2, 3, 3, 4, 4.
The codeword $\mathbf{c}_5$ corresponding to symbol s_5 is given by

(a) 0011 (b) 1100 (c) 1101 (d) 1110 (e) 1111

10. Let $S = \{s_1, s_2\}$ be a source with probabilities $p_1 = \frac{4}{5}$, $p_2 = \frac{1}{5}$. The average length of a **radix 3** Huffman code for the **second extension** $S^{(2)}$ of this source is

(a) $\dfrac{39}{25}$ (b) $\dfrac{39}{50}$ (c) $\dfrac{6}{5}$ (d) $\dfrac{3}{5}$ (e) 1

11. [**5 marks**]

(a) Show that there is no uniquely decodable **ternary** (i.e. radix 3) code with codeword lengths $1, 1, 2, 2, 2, 3$, respectively.

(b) Symbol s_1 of the source $S = \{s_1, s_2\}$ occurs with probability $5/6$ and symbol s_2 occurs with probability $1/6$. Find a uniquely decodable binary code of minimal average length for S^2, assuming that successive symbols occur independently, and state the average length per original source symbol of the code.