MATH3411 INFORMATION, CODES & CIPHERS

Test 1 Session 2 2014 SOLUTIONS

Version A

Multiple choice: c, b, a, c, b, c, b, d, e, a

1. **(c)**:
2. **(b)**:
3. **(a)**:
4. **(c)**: Columns 2 and 3 are parallel (in fact, they are identical).
5. **(b)**: There must be exactly two 0s and two 1s - in other words, 2 errors.

6. **(c)**: $2^3 = 8$
7. **(b)**: Comma code has $\mathbf{c}_1 = 0$, $\mathbf{c}_2 = 10$, and $\mathbf{c}_4 = 1110$.

8. **(d)**: The Kraft-McMillan number must be at most 1 for UD codes. We get $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^5} = \frac{29}{32}$ so we need $\frac{1}{2^\ell} \leq \frac{3}{32}$, or in other words, $\ell \geq 4$.
9. **(e)**:

10. **(a)**: One dummy symbol is needed, so there is only one combination step: combine the dummy, s_2s_2 and s_2s_1 with combination probability $0 + \frac{1}{25} + \frac{4}{25} = \frac{1}{5}$, so the average length of the code is $\frac{6}{5}$, and the per original source symbol average length is $\frac{3}{5}$.

11. (a) The Kraft-McMillan number is

$$K = \frac{1}{3} + \frac{6}{3^2} + \frac{1}{3^3} = \frac{28}{27} > 1$$

so there is no UD-code.

(b) We find that $s_1s_1 \mapsto 0$, $s_1s_2 \mapsto 11$, $s_2s_1 \mapsto 100$, $s_2s_2 \mapsto 101$. The average length per original source symbol is $\frac{1}{2}\left(\frac{14}{49} + \frac{24}{49} + 1\right) = \frac{87}{98}$ by Knuth's Lemma.

Version B

Multiple Choice: a, c, a, c, d, d, e, c, c, e

1. **(a)**:

2. **(c)**:

3. **(a)**:

4. **(c)**: Columns 1 and 6 are parallel (in fact, they are identical).

5. **(d)**: The Sphere Packing Bound is here $|C|(1+n) \leq 2^n$ where $|C| = 2^k$ and $n = m + k = 3 + k$, so $k \leq 2^3 - 4 = 4$.

6. **(d)**:

7. **(e)**: The comma code has $\mathbf{c}_1 = 0$, $\mathbf{c}_2 = 10$, $\mathbf{c}_3 = 110$, and $\mathbf{c}_4 = 1110$.

8. **(c)**: The Kraft-McMillan number is 1 for minimal codeword length binary UD codes, so $\frac{2}{2^2} + \frac{2}{2^3} + \frac{2}{2^4} + \frac{1}{2^\ell} = 1$; in other words, $\ell = 3$.

9. **(c)**: Last two codewords differ in their last bit only.

10. **(e)**: By Knuth's Lemma, the average length is $1 + \frac{2}{5} = \frac{7}{5}$.

11. (a) For instance:

$$C = \{0, 10, 11, 12, 20, 210, 211, 2120\}$$

    (b) We find that $s_1 s_1 \mapsto 00$, $s_1 s_2 \mapsto 01$, $s_2 s_1 \mapsto 10$, $s_2 s_2 \mapsto 11$ (the two middle codewords could be swapped). The average length per original source symbol is $\frac{1}{2}(1 + \frac{15}{25} + \frac{10}{25}) = \frac{2}{2} = 1$.