MATH3411 INFORMATION, CODES & CIPHERS

Test 1 2018 S2 SOLUTIONS

Version A

Multiple choice: **a**, **d**, (**a**), **d**, **b**, **a**, **e**, **d**, **d**, **b**

1. **(a)**:

2. **(d)**: The error lies in the 3rd row and 3rd column.

3. **(a)**: Under the pure error-detection strategy, all 5 errors would need to occur in order to be undetected. This was the strategy meant for the question but this was not stated, so other strategies could be chosen here too. There is a range of interpretations possible under those other strategies - so an automatic mark has been given here.

4. **(d)**: $P(0 \text{ received}) = \frac{1}{3} \times 1 + \frac{2}{3} \times \frac{1}{2} = \frac{2}{3}$

5. **(b)**: Calculate the $C(4,2) = 6$ Hamming distances and choose the smallest.

6. **(a)**: We can always detect at least as many errors as we can correct.

7. **(e)**: The binary 5-repetition code $C = \{00000, 11111\}$ has $n = 5$, $k = 1$ and $d(C) = 5$.

8. **(d)**: Use the theorem expressed in Problem 19: there are no zero columns or identical columns, so $w(C) \geq 3$, and there are three linearly dependent columns, say columns 2,3,7, so $w(C) \leq 3$. Hence, $w(C) = 3$.

9. **(d)**: There are $k = 4$ information bits, so there are $2^4 = 16$ possible messages, each of which is encoded by a unique codeword.

10. **(b)**: $S(\mathbf{y}) = H\mathbf{y}^T = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, the 2nd column of H. The error is then in the 2nd position of $\mathbf{y}$.

Version B

Multiple choice: **c**, **c**, **c**, **c**, **c**, **e**, **d**, **c**, **d**, **b**

1. **(c)**:

2. **(c)**: The error lies in the 2nd row and 3rd column. 1, 2, or 3 errors, and $\binom{12}{3} = 220$.

3. **(c)**: Any 1, 2, or 3 errors will get corrected correctly.

4. **(c)**: $P(0 \text{ received}) = \frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} = \frac{1}{2}$

5. **(c)**: Calculate the three distances and choose the smallest.

6. **(e)**: We can always detect at least as many errors as we can correct.

7. **(d)**: Since $d = 2t + 2$, we see that $t = 1$. The (binary) Sphere-Packing Theorem asserts that $|C| \sum_{i=0} \binom{n}{i} \leq 2^n$, which here implies that $2^k(1 + 7) \leq 2^7$, or in other words, $2^{k+3} \leq 2^7$. The largest value of k which satisfies this inequality is $k = 4$, and indeed, the code with basis $\{1000111, 0100111, 0010110, 0001101\}$ is such a code.

8. **(c)**: There are no zero columns but there are two identical columns so, by the theorem expressed by Problem 19, $w(C) = 2$.

9. **(d)**: There are $k = 4$ information bits, so there are $2^4 = 16$ possible messages, each of which is encoded by a unique codeword.

10. **(b)**: Testing each $\mathbf{x}$ in (a)–(d) to see whether $H\mathbf{x}^T = \mathbf{0}$ and that the bits of $\mathbf{m}$ are the 3rd, 5th, 6th and 7th coordinates of $\mathbf{x}$.