Name: .... Student ID: ....

UNSW School of Mathematics and Statistics

MATH3411 Information Codes and Ciphers

2018 S2 **TEST 1** **VERSION A**

- Time Allowed: **30 minutes**

For multiple choice questions, **circle the correct answer**; each question is worth **1 mark**.

1. There may be an error in the **3rd** digit of the ISBN number 0-76-535615-4. The correct 3rd digit is

(a) 2 (b) 3 (c) 5 (d) 6 (e) None of these

2. A message is sent using a 5-character 8-bit ASCII code which encodes characters in blocks of four together with a 5th character which is used as a check character for even parity in rows and columns, similar to the 9-character 8-bit ASCII code.

The message 10101010 10110111 11000100 00111010 11000011 is received. Assuming at most one error, which of the following bits could be incorrect?

(a) 2nd (b) 3rd (c) 11th (d) 19th (e) None of these

3. Consider a binary symmetric channel with bit-error probability p where errors in different positions are independent. Suppose that a codeword $\mathbf{x}$ is sent from a binary repetition code with codewords of length 5. Define

$$u = p^5 \quad v = 5p^4(1-p) \quad w = 10p^3(1-p)^2 \quad x = 10p^2(1-p)^3 \quad y = 5p(1-p)^4 \quad z = (1-p)^5$$

The probability that undetected error(s) occur is

(a) u (b) $v + w$ (c) $x + y$ (d) z (e) None of these

4. Consider a binary channel with probabilities $P(0\text{ sent}) = \frac{1}{3}$, $P(1\text{ received}|0\text{ sent}) = 0$, and $P(0\text{ received}|1\text{ sent}) = \frac{1}{2}$. The probability $P(0\text{ received})$ is

(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{2}{3}$ (e) $\dfrac{5}{6}$

5. The binary code $C = \{011110, 110011, 101101, 111111\}$ has minimum distance

(a) 0 (b) 2 (c) 4 (d) 6 (e) None of these

6. A binary code C has minimum distance $d = 10$. Suppose that this is used to correct a errors and detect b errors. Which of the following pairs (a, b) **does not** give a valid strategy for decoding C?

(a) $(6, 3)$ (b) $(4, 5)$ (c) $(2, 7)$ (d) $(1, 8)$ (e) $(0, 9)$

7. A binary linear code C has $k = 1$ information bit and length $n = 5$. The maximal possible minimum distance $d(C)$ is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

8. Let C be the binary linear code with parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

What is the minimum weight $w(C)$ of C?

(a) 0 (b) 1 (c) 2 (d) 3 (e) None of these

9. Let C be the binary Hamming code with parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

How many codewords are there in C?

(a) 2 (b) 3 (c) 8 (d) 16 (e) 128

10. Suppose that a codeword $\mathbf{x}$ of the code C in Question 9 is sent and that the received word $\mathbf{y} = 1100011$ has 1 error. The codeword $\mathbf{x}$ is then

(a) 0100011 (b) 1000011 (c) 1110011 (d) 1101011 (e) None of these

Name: .... Student ID: ....

UNSW SCHOOL OF MATHEMATICS AND STATISTICS

MATH3411 INFORMATION CODES AND CIPHERS

2018 S2 **TEST 1** **VERSION B**

- Time Allowed: **30 minutes**

For multiple choice questions, **circle the correct answer**;
each question is worth **1 mark**.

1. There may be an error in the **5th** digit of the ISBN number 0-76-535615-4. The correct 5th digit is

(a) 2 (b) 3 (c) 5 (d) 6 (e) None of these

2. A message is sent using a 5-character 8-bit ASCII code which encodes characters in blocks of four together with a 5th character which is used as a check character for even parity in rows and columns, similar to the 9-character 8-bit ASCII code.

The message 10101010 10110110 11000101 00111010 11000011 is received. Assuming at most one error, which of the following bits could be incorrect?

(a) 2nd (b) 3rd (c) 11th (d) 19th (e) None of these

3. Consider a binary symmetric channel with bit-error probability p where errors in different positions are independent. Suppose that a codeword $\mathbf{x}$ is sent from the binary repetition code with codewords of length 8. Define

$$w = (1-p)^8 \quad x = 8p(1-p)^7 \quad y = 28p^2(1-p)^6 \quad z = 56p^3(1-p)^5$$

The probability that one or more errors are correctly corrected using a minimum distance decoding strategy is

(a) x (b) $x+y$ (c) $x+y+z$ (d) $w+x+y$ (e) $w+x+y+z$

4. Consider a binary channel with probabilities $P(0 \text{ sent}) = \frac{1}{3}$, $P(1 \text{ received}|0 \text{ sent}) = \frac{1}{2}$, and $P(0 \text{ received}|1 \text{ sent}) = \frac{1}{2}$. The probability $P(0 \text{ received})$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$ (e) $\frac{5}{6}$

5. The binary code $C = \{10000, 01100, 00111\}$ has minimum distance

(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

6. A binary code C has minimum distance $d = 9$. Suppose that this is used to correct a errors and detect b errors. Which of the following pairs (a, b) **does not** give a valid strategy for decoding C?

(a) $(0,8)$ (b) $(1,7)$ (c) $(2,6)$ (d) $(3,5)$ (e) $(5,3)$

7. A binary linear code C has minimum distance $d = 3$ and length $n = 7$. The maximal possible number of information bits k for such a code is

(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

8. Let C be the binary linear code with parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

What is the minimum weight $w(C)$ of C?

(a) 0 (b) 1 (c) 2 (d) 3 (e) None of these

9. Let C be the binary Hamming code with parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

How many codewords are there in C?

(a) 2 (b) 3 (c) 8 (d) 16 (e) 128

10. For the code C of Question 9, assume that the 1st, 2nd and 4th bits are check bits. The codeword that encodes $\mathbf{m} = 0001$ is then

(a) 0001001 (b) 1101001 (c) 0001110 (d) 1010001 (e) None of these