Name: .... Student Id: .... Tutorial ....

UNSW **School of Mathematics and Statistics**

MATH3411 Information Codes and Ciphers

Semester 2, 2012 **TEST 1** **VERSION A**

- Time Allowed: **45 minutes**

For the multiple choice questions, **circle the correct answer**; each multiple choice question is worth 2 marks.
For the true/false and written answer questions, use extra paper.
Staple everything together at the end.

1. Consider a binary channel with bit-error probability p, where errors in different positions are independent (that is, white noise). Suppose that a codeword $\mathbf{x}$ is sent from a binary linear code with weight 4 and codewords of length 10, and the word $\mathbf{y}$ is received. Then the probability that **an error is detected**, using a pure error detection strategy, is

(a) $45p^2(1-p)^8$ (b) $10p(1-p)^9+45p^2(1-p)^8+120p^3(1-p)^7$ (c) $1-p-p^2$

(d) $10p(1-p)^9+45p^2(1-p)^8$ (e) none of these

2. A binary linear code C has basis $\{101010100110, 010101011001\}$. Its weight w is

(a) 1 (b) 2 (c) 4 (d) 6 (e) 12

3. Consider a code with codewords $\mathbf{c}_1 = 00$, $\mathbf{c}_2 = 10$, $\mathbf{c}_3 = 1100$, $\mathbf{c}_4 = ?$, where $\mathbf{c}_4$ is to be chosen from the list of four possibilities below. Which choice of $\mathbf{c}_4$, if any, makes the resulting code uniquely decodable?

(a) $\mathbf{c}_4 = 0000$, (b) $\mathbf{c}_4 = 1011$, (c) $\mathbf{c}_4 = 1010$, (d) $\mathbf{c}_4 = 11$, (e) none of these

4. A binary UD-code of has codewords lengths (not necessarily in order) 2, 2, 3, 4, 4, ℓ. What is the minimum value must ℓ take in order for the code to exist?

(a) $\ell = 1$ (b) $\ell = 2$ (c) $\ell = 3$ (d) $\ell = 4$ (e) none of these.

5. A Markov source $S = \{s_1, s_2, s_3\}$ has transition matrix M. The Huffman code for the equilibrium distribution is $\mathrm{Huff}_E = [1, 00, 01]$. (That is, $\mathbf{c}_1 = 1$, $\mathbf{c}_2 = 00$ and $\mathbf{c}_3 = 01$.) Huffman codes for the columns of M are given by $\mathrm{Huff}_1 = [00, 1, 01]$, $\mathrm{Huff}_2 = [0, 10, 11]$ and $\mathrm{Huff}_3 = [11, 10, 0]$. Given the string of source symbols $s_1s_2s_1s_3s_1$, the Markov Huffman encoding is

(a) 0100110 (b) 111011011 (c) 1001011 (d) 1100111

6. **[10 marks]** For each of the following, say whether the statement is true or false, giving a brief reason or showing your working. You will get one mark for a correct true/false answer, and if your true/false answer is correct then you will get one mark for a good reason.

Begin each answer with the word "true" or "false".

i) The code 0-01-616376-2 is a valid ISBN-11 number.

ii) The 8-character 8-bit ASCII burst code can detect all triple errors in a block.

iii) It is possible to construct a binary linear code C with $|C| = 12$ and codewords of length 9 and that can correct 2 errors.

iv) The two binary I-codes $\{0, 100, 110, 1010, 1110\}$ and $\{1, 000, 001, 0100, 0101\}$ are equivalent.

v) If S is a source with two symbols of probabilities $\frac{5}{7}$ and $\frac{2}{7}$ then a binary Huffman coding of S^2 has average length per original source symbol less than 1.

7. **[10 marks]** Let C be the binary linear code with check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix},$$

where the last three columns correspond to information bits.

(i) Find a generator matrix G for the code C.

(ii) Find a basis for the code C.

(iii) Encode 011 with this code C

(iv) Define the minimum distance $d(C)$ of a code C and calculate $d(C)$ for the code above, with explanation.

Name: .... Student Id: .... Tutorial................

UNSW School of Mathematics and Statistics

MATH3411 Information Codes and Ciphers

Semester 2, 2012 **TEST 1** **VERSION B**

- Time Allowed: **45 minutes**

For the multiple choice questions, **circle the correct answer**; each multiple choice question is worth 2 marks. For the true/false and written answer questions, use extra paper. Staple everything together at the end.

1. Consider a binary channel with bit-error probability p, where errors in different positions are independent (that is, white noise). Suppose that a codeword $\mathbf{x}$ is sent from a binary linear code with weight 5 and codewords of length 10, and the word $\mathbf{y}$ is received. Then the probability that any errors are **correctly corrected** with the standard strategy is

(a) $45p^2(1-p)^8$ (b) $10p(1-p)^9+45p^2(1-p)^8+120p^3(1-p)^7$ (c) $1-p-p^2$

(d) $10p(1-p)^9+45p^2(1-p)^8$ (e) none of these

2. Consider a code with codewords $\mathbf{c}_1 = 01$, $\mathbf{c}_2 = 11$, $\mathbf{c}_3 = 1011$, $\mathbf{c}_4 =?$, where $\mathbf{c}_4$ is to be chosen from the list of four possibilities below. Which choice of $\mathbf{c}_4$, if any, makes the resulting code uniquely decodable?

(a) $\mathbf{c}_4 = 0111$, (b) $\mathbf{c}_4 = 1$, (c) $\mathbf{c}_4 = 10$, (d) $\mathbf{c}_4 = 100$, (e) $\mathbf{c}_4 =$ none of these.

3. A radix 3 instantaneous code (I-code) has codeword lengths (not necessarily in order) 1, 3, 4, 4, 4, ℓ and $K = 4/9$. Then ℓ is given by

(a) $\ell = 1$ (b) $\ell = 2$ (c) $\ell = 3$ (d) $\ell = 4$ (e) $\ell = 5$

4. Let $S = \{s_1, s_2\}$ be a source with probabilities $p_1 = \frac{5}{8}$, $p_2 = \frac{3}{8}$,. The average length per original symbol of a **radix 3** Huffman code for the **second extension** $S^{(2)}$ of this source (constructed with the usual strategies) is

(a) $\dfrac{127}{128}$ (b) $\dfrac{103}{64}$ (c) $\dfrac{11}{8}$ (d) $\dfrac{103}{128}$ (e) $\dfrac{11}{16}$

5. A Markov source $S = \{s_1, s_2, s_3\}$ has transition matrix M. The Huffman code for the equilibrium distribution is $\text{Huff}_E = [1, 00, 01]$. (That is, $\mathbf{c}_1 = 1$, $\mathbf{c}_2 = 00$ and $\mathbf{c}_3 = 01$.) Huffman codes for the columns of M are given by $\text{Huff}_1 = [00, 1, 01]$, $\text{Huff}_2 = [0, 10, 11]$ and $\text{Huff}_3 = [11, 10, 0]$. The string 001101100 decodes under the Markov Huffman encoding as

(a) $s_2s_1s_1s_3s_1s_1$ (b) $s_2s_3s_3s_1s_1$ (c) $s_3s_3s_1s_3s_2s_2s_2$ (d) $s_2s_2s_1s_2s_3$

6. [**10 marks**] For each of the following, say whether the statement is true or false and give a brief reason or showing your working. You will get one mark for a correct true/false answer, and if your true/false answer is correct then you will get one mark for a good reason.

Begin each answer with the word "true" or "false".

i) The code 0-00-716376-2 is a valid ISBN-11 number.

ii) The 8-character 8-bit ASCII burst code cannot detect all quadruple errors in a block.

iii) A binary code C with $|C| = 12$ that can correct 2 errors must have codewords of length at least 9.

iv) The binary linear code C with basis $\{101010101001, 010101010101\}$ has weight 5.

v) The two binary I-codes $\{0, 100, 110, 1010, 1110\}$ and $\{0, 101, 110, 1000, 1111\}$ are equivalent.

7. [**10 marks**] Let C be the binary linear code with check matrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

where the last three columns correspond to information bits.

(i) Reduce H to row reduced echelon form and hence find a generator matrix G for the code C.

(ii) Hence or otherwise encode 101 with the code C.

(iii) Define the minimum distance $d(C)$ of a code C and calculate $d(C)$ for the code above, with explanation.