MATH3411 INFORMATION, CODES & CIPHERS

Test 1 Session 2 2013 SOLUTIONS

Version A

Multiple choice: b, b, a, b, d, e, e, c, b, a.

1. **(b)**:

2. **(b)**:

3. **(a)**:

4. **(b)**: The syndrome is a column of H.

5. **(d)**: Sphere packing bound is $37|C| \leq 256$, so $|C| \leq \lfloor \frac{256}{37} \rfloor = 6$.

6. **(e)**: We must have $a + b \leq d - 1 = 6$ with $a \leq b$.

7. **(e)**: Comma code has $\mathbf{c}_1 = 0$, $\mathbf{c}_2 = 10$, $\mathbf{c}_3 = 110$, $\mathbf{c}_4 = 1110$ and $\mathbf{c}_5 = 1111$.

8. **(c)**: The Kraft-McMillan number must be ≤ 1 for UD codes. We get $\frac{1}{2} + \frac{2}{8} + \frac{2}{16} = \frac{7}{8}$ so we need $\ell \geq 3$.

9. **(b)**: Last two codewords differ in their last bit only.

10. **(a)**: One dummy symbol is needed, so there is only one combination step: combine the dummy, s_2s_2 and s_2s_1 with combination probability $0 + \frac{4}{25} + \frac{6}{25} = \frac{10}{25} = \frac{2}{5}$, so the average length of the code is $\frac{7}{5}$, and the per original source symbol average length is $\frac{7}{10}$.

11. (a) The Kraft-McMillan number is

$$K = 2 \times \frac{1}{3} + 2 \times \frac{1}{9} + 3 \times \frac{1}{27} + \frac{1}{81} > 1$$

so there is no UD-code.

(b) We find $s_1s_1 \mapsto 1$, $s_1s_2 \mapsto 01$, $s_2s_1 \mapsto 000$, $s_2s_2 \mapsto 001$. Average length per original source symbol is $\frac{1}{2}\left(\frac{24}{64} + \frac{39}{64} + 1\right) = \frac{127}{128}$ from the coding process, or $\frac{1}{2}\left(\frac{25}{64} + 2 \times \frac{15}{64} + 3 \times \frac{15}{64} + 3 \times \frac{9}{64}\right) = \frac{127}{128}$ from the definition.

Version B

Multiple Choice: b, e, a, c, d, d, a, a, e, c

1. **(b)**: Columns 2 and 7 show errors, and a burst starting in the 7th column (in 10101011) and going to the 2nd in the next row (11001011) could produce this.

2. **(e)**:

3. **(a)**:

4. **(c)**: The syndrome is not a column but is the sum of two columns.

5. **(d)**: Sphere packing bound is $2^k(1 + k + 3) \leq 2^{k+3}$, so $k + 4 \leq 2^3$; therefore, $k \leq 4$.

6. **(d)**: We must have $a + b \leq d - 1 = 11$ with $a \leq b$.

7. **(a)**: Comma code has $\mathbf{c}_1 = 0$, $\mathbf{c}_2 = 10$, $\mathbf{c}_3 = 110$, $\mathbf{c}_4 = 1110$ and $\mathbf{c}_5 = 1111$.

8. **(a)**: The Kraft-McMillan number must be ≤ 1 for UD codes. We get $2 \times \frac{1}{4} + \frac{1}{8} + 2 \times \frac{1}{16} = \frac{3}{4}$ so we need $\ell \geq 2$.

9. **(e)**: Last two codewords differ in their last bit only.

10. **(c)**: We need to introduce one dummy symbol. There is one combining phase of the coding: combine the dummy, s_6, s_5 and s_4 with a combination probability of $0 + \frac{1}{17} + \frac{1}{17} + \frac{2}{17} = \frac{4}{17}$, so the average length is $1 + \frac{4}{17} = \frac{21}{17}$.

11. (a) For instance:

$$C = \{0, 10, 11, 12, 20, 2100, 2101, 2102\}$$

(b) We find $s_1 s_1 \mapsto 1$, $s_1 s_2 \mapsto 01$, $s_2 s_1 \mapsto 000$, $s_2 s_2 \mapsto 001$ (middle two could be swapped). Average length per original source symbol is $\frac{1}{2}\left(\frac{30}{100} + \frac{51}{100} + 1\right) = \frac{181}{200}$ from the coding process, or $\frac{1}{2}\left(\frac{49}{100} + 2 \times \frac{21}{100} + 3 \times \frac{21}{100} + 3 \times \frac{9}{100}\right) = \frac{181}{200}$ from the definition.