MATH3411 INFORMATION, CODES & CIPHERS

Test 1 2016 S2 SOLUTIONS

Version A

Multiple Choice: d, a, e, c, d, b, e, b, e, c

1. **(d)**: One or two errors.

2. **(a)**: Test the words with $x_1 = 1$, $x_2 = 0$ to see whether they are codewords.

3. **(e)**: There are $3^4 = 81$ codewords: the 81 linear combinations of the rows of G.

4. **(c)**: There is just one codeword starting with 1021, namely the sum of rows 1, 3 (twice) and 4 of G.

5. **(d)**: By the Sphere-Packing Theorem with $t = 1$, $2^k \times (1 + 7) \leq 2^7$, so $k \leq 4$, and indeed, the binary Hamming $(7, 4)$ code has these parametres.

6. **(b)**:

7. **(e)**: None since the message 11 cannot be decoded unambiguously.

8. **(b)**: The Kraft-McMillan number $K = \sum \frac{1}{r^{l_i}}$ must be at most 1 for UD codes. Testing values of $r = 2, 3, \ldots$ gives us that $r = 3$ is the minimum radix that satisfies this. (You could also draw a decision tree.)

9. **(e)**: Draw a decision tree in the standard way.

10. **(c)**: One dummy symbol is needed.

11. (a) The Kraft-McMillan number is

$$K = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{3}{2^4} = \frac{17}{16} > 1$$

so there is no UD-code.

(b) We find that $s_1s_1 \mapsto 0$, $s_1s_2 \mapsto 11$, $s_2s_1 \mapsto 100$, $s_2s_2 \mapsto 101$.
The average length per original source symbol is
$\frac{1}{2}\left(\frac{5}{25} + \frac{9}{25} + 1\right) = \frac{39}{50}$ by Knuth's Lemma.

Version B

Multiple choice: b, d/e, c, a, b, b, e, c, b, c

1. **(b)**:

2. **(d/e)**: None of the three words containing 10 (in correct order) are codewords. However, you could argue that we could be allowed to specify a different order for the message digits, in which case, (d) is valid.

3. **(c)**: There are $2^4 = 16$ codewords: the 16 linear combinations of the rows of G.

4. **(a)**: There is just one codeword starting with 1011, namely the sum of the first and last rows G.

5. **(b)**: By the Sphere-Packing Theorem with $t = 2$, $2^k \times (1 + 8 + 36) \leq 2^8$, so $k \leq 2$, and indeed, the code with basis $\{11111000, 00000111\}$ has these parametres.

6. **(b)**:

7. **(e)**:

8. **(c)**: The Kraft-McMillan number $K = \sum \frac{1}{r^{\ell_i}}$ must be at most 1 for UD codes. Testing values of $r = 2, 3, \ldots$ gives us that $r = 4$ is the minimum radix that satisfies this. (You could also draw a decision tree.)

9. **(b)**: The codewords are 00, 01, 100, 101, 1100, 1101.

10. **(c)**: One dummy symbol is needed.

11. (a) The Kraft-McMillan number is

$$K = \frac{2}{3} + \frac{3}{3^2} + \frac{1}{3^3} = \frac{28}{27} > 1$$

so there is no UD-code.

(b) We find that $s_1 s_1 \mapsto 0$, $s_1 s_2 \mapsto 11$, $s_2 s_1 \mapsto 100$, $s_2 s_2 \mapsto 101$.
The average length per original source symbol is
$\frac{1}{2}\left(\frac{6}{36} + \frac{11}{36} + 1\right) = \frac{53}{72}$ by Knuth's Lemma.