Name: ..... Student Id: ..... Tutorial .....

UNSW **School of Mathematics and Statistics**

MATH3411 Information Codes and Ciphers

Semester 2, 2011 TEST 1 VERSION A

- Time Allowed: **45 minutes**

For the multiple choice questions, **circle the correct answer**; each multiple choice question is worth 2 marks.
For the true/false and written answer questions, use extra paper.
Staple everything together at the end.

1. There may be an error in the check digit in the ISBN number 0-752-87712-8. The correct check digit is

(a) 0 (b) 4 (c) 7 (d) X (e) None of these

2. A message is sent using a 5-character 8-bit ASCII code, which encodes characters in blocks of four together with a 5th character which is used as a check character for even parity in rows and columns. (This is similar to the 9-character 8-bit and 8-character 8-bit ASCII codes studied in lectures.)

The message 11010100 01101111 01100100 01100101 10101010 is received. Assuming at most one error in each bit is wrong:

(a) 10th (b) 20th (c) 30th (d) 40th (e) no error

3. In the Hamming (7,4) code with the usual parity check matrix the codeword 0110110 decodes to:

(a) 1101 (b) 1011 (c) 1110 (d) 1111 (e) 1001

4. Let C be the code consisting of all vectors $\mathbf{x} = x_1x_2x_3x_4 \in {\mathbb{Z}_5}^4$ satisfying the check equations

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &\equiv 0 \pmod 5, \\ 2x_1 + 3x_2 + 4x_3 + x_4 &\equiv 0 \pmod 5 \end{aligned}$$

Assuming that x_3 and x_4 are the information bits, the codeword which encodes the message 21 is:

(a) 1221 (b) 1321 (c) 4421 (d) 2421 (e) 4121

5. Consider the standard binary I-code with codeword lengths $\ell_1 = 2$, $\ell_2 = 3$, $\ell_3 = 3$, $\ell_4 = 3$, $\ell_5 = 3$. The codeword $\mathbf{c}_3$ corresponding to symbol s_3 is given by

(a) 011 (b) 110 (c) 100 (d) 010 (e) 111

6. **[10 marks]** For each of the following, say whether the statement is true or false, giving a brief reason or showing your working. You will get one mark for a correct true/false answer, and if your true/false answer is correct then you will get one mark for a good reason.

Begin each answer with the word "true" or "false".

i) A binary linear code with weight $w = 7$ can be used to correct all triple errors in a codeword.

ii) There is a binary linear code C with $|C| = 8$ and codewords of length 8 that can correct 2 errors.

iii) The binary code $\mathbf{c}_1 = 0$, $\mathbf{c}_2 = 01$, $\mathbf{c}_3 = 10$, $\mathbf{c}_4 = 101$ is a UD code.

iv) There is a ternary (radix 3) I-code with codewords of lengths $1, 2, 2, 2, 2, 3, 3$.

v) The Markov matrix $M = \frac{1}{10}\begin{pmatrix} 7 & 2 & 7 \\ 1 & 5 & 2 \\ 2 & 3 & 1 \end{pmatrix}$ has equilibrium vector $\mathbf{p} = \frac{1}{5}\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$.

7. **[10 marks]** Consider the source $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ with probabilities

$$p_1 = 3/10, \quad p_2 = 1/4, \quad p_3 = 3/20, \quad p_4 = 1/10, \quad p_5 = 1/10, \quad p_6 = 1/20, \quad p_7 = 1/20.$$

(i) Find the standard binary Huffman code for the source S. Show your working.

(ii) Calculate the average length L for this code. Show your working. (Leave your answer as a fraction.)

Name: ..... Student Id: ..... Tutorial .....

UNSW **School of Mathematics and Statistics**

MATH3411 Information Codes and Ciphers

Semester 2, 2011 TEST 1 VERSION B

- Time Allowed: **45 minutes**

For the multiple choice questions, **circle the correct answer**; each multiple choice question is worth 2 marks.
For the true/false and written answer questions, use extra paper.
Staple everything together at the end.

1. There may be an error in the check digit in the ISBN number 0-752-87721-8. The correct check digit is

(a) X (b) 3 (c) 5 (d) 6 (e) None of these

2. A message is sent using a 5-character 8-bit ASCII code, which encodes characters in blocks of four together with a 5th character which is used as a check character for even parity in rows and columns. (This is similar to the 9-character 8-bit and 8-character 8-bit ASCII codes studied in lectures.)

The message 11110110 01101111 01110100 01100101 10101000 is received. Assuming at most one error, which bit is wrong:

(a) 5th (b) 15th (c) 25th (d) 35th (e) no error

3. What is the maximum number of information bits k in a binary 2-error correcting code C with codewords of length $n = 7$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

4. In a binary linear code C the codeword 001010101100101 has minimum weight among non-zero codewords. The maximum number of errors that C can correct is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

5. The minimum radix that would be needed to create a UD-code for the source

$$S = \{s_1, s_2, s_3, s_4, \ldots, s_8\}$$

with codeword lengths $1, 1, 1, 2, 2, 2, 2, 3$, respectively is

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

6. **[10 marks]** For each of the following, say whether the statement is true or false and give a brief reason or showing your working. You will get one mark for a correct true/false answer, and if your true/false answer is correct then you will get one mark for a good reason.

Begin each answer with the word "true" or "false".

i) If C is the code consisting of all vectors $\mathbf{x} = x_1x_2x_3x_4 \in {\mathbb{Z}_5}^4$ satisfying the check equations

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &\equiv 0 \pmod 5, \\ 2x_1 + 3x_2 + 4x_3 + x_4 &\equiv 0 \pmod 5 \end{aligned}$$

then 1234 is a valid code word in C.

ii) The Hamming (7,4) code with the usual parity check matrix contains the codeword 0010110.

iii) The binary code $\mathbf{c}_1 = 0$, $\mathbf{c}_2 = 100$, $\mathbf{c}_3 = 101$, $\mathbf{c}_4 = 1101$ is a UD-code.

iv) In the standard binary I-code with codeword lengths $\ell_1 = 2$, $\ell_2 = 3$, $\ell_3 = 3$, $\ell_4 = 3$, $\ell_5 = 3$, the codeword $\mathbf{c}_5$ corresponding to symbol s_5 is 111.

v) The Markov matrix $M = \frac{1}{10}\begin{pmatrix} 6 & 1 & 1 \\ 1 & 7 & 8 \\ 3 & 2 & 1 \end{pmatrix}$ has equilibrium vector $\mathbf{p} = \frac{1}{5}\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$.

7. **[10 marks]** Consider the source $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ with probabilities

$$p_1 = 3/10, \quad p_2 = 1/5, \quad p_3 = 3/20, \quad p_4 = 3/20, \quad p_5 = 1/10, \quad p_6 = 1/20, \quad p_7 = 1/20.$$

(i) Find the standard binary Huffman code for the source S. Show your working.

(ii) Calculate the average length L for this code. Show your working. (Leave your answer as a fraction.)