Name: .... Student ID: ....

UNSW SCHOOL OF MATHEMATICS AND STATISTICS

MATH3411 INFORMATION CODES AND CIPHERS

2018 S2 **TEST 2** **VERSION A**

- Time Allowed: **45 minutes**

For multiple choice questions, **circle the correct answer**; each question is worth **1 mark**.

1. Let C be the ternary linear code with parity check matrix

$$H = \begin{pmatrix} 2 & 0 & 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Here, the first three bits are check bits. The codeword that encodes $\mathbf{m} = 1221$ is

(a) 1221011 (b) 2021221 (c) 1221221 (d) 1221000 (e) None of these

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2. A message $\mathbf{m}'$ has been encoded using the code C in Question 1 and has been corrupted by a single error to give the received word $\mathbf{y} = 0021210$. The message $\mathbf{m}'$ is

(a) 0020 (b) 0021 (c) 1010 (d) 1110 (e) None of these

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3. Let C be the code of all vectors $\mathbf{x} = x_1x_2x_3x_4 \in \mathbb{Z}_3^4$ satisfying the check equations

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$$\begin{array}{rcccccl} x_1 & & & + & 2x_3 & + & 2x_4 \equiv 0 \pmod 3 \\ x_1 & + & x_2 & + & & + & 2x_4 \equiv 0 \pmod 3 \end{array}$$

There are two information bits but you are not told in which positions they lie. Which of the following codewords could possibly encode the message 10?

(a) 1001 (b) 1100 (c) 1201 (d) 0121 (e) None of these

4. Consider a compression code with codewords $\mathbf{c}_1 = 0$, $\mathbf{c}_2 = 11$, $\mathbf{c}_3 = 101$, $\mathbf{c}_4 = ?$ where $\mathbf{c}_4$ is to be chosen from the list of four possibilities below.
Which choice, if any, of $\mathbf{c}_4$ makes the resulting code uniquely decodable?

(a) $\mathbf{c}_4 = 1$ (b) $\mathbf{c}_4 = 011$ (c) $\mathbf{c}_4 = 000$ (d) $\mathbf{c}_4 = 010$ (e) None of these

5. The minimum radix that would be needed to create a UD-code for the source

$$S = \{s_1, s_2, \ldots, s_7\}$$

with codeword lengths $1, 1, 2, 2, 2, 2, 3$, respectively, is

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

6. If arithmetic coding with source symbols a, b and stop symbol $\bullet$ with probabilities 0.4, 0.3 and 0.3 is used, then what is the message $ba\bullet$ encoded as?

(a) 0.25 (b) 0.3 (c) 0.5 (d) 0.55 (e) None of these

7. Using the LZ78 algorithm a message is encoded as $(0, b)(0, a)(2, a)(3, b)(3, a)$. What is the last dictionary entry after decoding?

(a) a (b) aa (c) ba (d) aaa (e) None of these

8. Let $S = \{s_1, s_2, s_3, s_4\}$ be a source with probabilities $p_1 = 0.4$, $p_2 = 0.3$, $p_3 = 0.2$, $p_4 = 0.1$. The average length of a **radix 3** Huffman code for S is

(a) 1 (b) 1.3 (c) 1.6 (d) 1.9 (e) None of these

9. Let S be the source in Question 8. The average length per symbol of a radix 3 Huffman code for $S^{(n)}$ converges, as $n \to \infty$, to approximately

(a) 1.06 (b) 1.16 (c) 1.43 (d) 1.85 (e) 2.12

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10. Let S be the source in Question 8. Using the binary Shannon-Fano code for S, the message $\mathbf{m} = s_1 s_4 s_2$ is encoded as

(a) 00101001 (b) 00110001 (c) 010101001 (d) 0111010 (e) None of these

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11. **[5 marks]**
A Markov source S has transition matrix and equilibrium probability distribution

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$$M = \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.4 \\ 0.1 & 0.2 & 0.5 \end{pmatrix} \qquad \text{and} \qquad \mathbf{p} = \frac{1}{17}\begin{pmatrix} 6 \\ 7 \\ 4 \end{pmatrix}.$$

(a) Find appropriate binary Huffman codes to encode S as a Markov source.

(b) Determine the average codeword length L_M for this encoding.

(c) Using these Huffman codes, encode the string $s_1 s_3 s_2 s_1$.

Name: .... Student ID: ....

UNSW School of Mathematics and Statistics

MATH3411 Information Codes and Ciphers

2018 S2 **TEST 2** **VERSION B**

- Time Allowed: **45 minutes**

For multiple choice questions, **circle the correct answer**;
each question is worth **1 mark**.

1. Let C be the ternary linear code with parity check matrix

$$H = \begin{pmatrix} 2 & 0 & 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Here, the last three bits are check bits. The codeword that encodes $\mathbf{m} = 1221$ is

(a) 1221011 (b) 2021221 (c) 1221221 (d) 1221000 (e) None of these

Assignment Project Exam Help

2. A message $\mathbf{m}'$ has been encoded using the code C in Question 1 and has been corrupted by a single error to give the received word $\mathbf{y} = 1111111$. The message $\mathbf{m}'$ is

(a) 1111 (b) 1121 (c) 1211 (d) 1101 (e) None of these

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3. Let C be the code of all vectors $\mathbf{x} = x_1x_2x_3x_4 \in \mathbb{Z}_3^4$ satisfying the check equations

$$\begin{array}{ccccccccl} x_1 & & & + & x_3 & + & 2x_4 & \equiv & 0 \pmod 3 \\ x_1 & + & 2x_2 & + & & + & x_4 & \equiv & 0 \pmod 3 \end{array}$$

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There are two information bits but you are not told in which positions they lie. Which of the following codewords could possibly encode the message 10?

(a) 1001 (b) 1100 (c) 0122 (d) 1201 (e) None of these

4. Consider a compression code with codewords $\mathbf{c}_1 = 0$, $\mathbf{c}_2 = 11$, $\mathbf{c}_3 = 100$, $\mathbf{c}_4 = ?$ where $\mathbf{c}_4$ is to be chosen from the list of four possibilities below.
Which choice, if any, of $\mathbf{c}_4$ makes the resulting code uniquely decodable?

(a) $\mathbf{c}_4 = 00$ (b) $\mathbf{c}_4 = 011$ (c) $\mathbf{c}_4 = 000$ (d) $\mathbf{c}_4 = 101$ (e) None of these

5. The minimum radix that would be needed to create a UD-code for the source

$$S = \{s_1, s_2, \ldots, s_7\}$$

with codeword lengths $1, 2, 2, 2, 2, 3, 4$, respectively, is

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

6. If arithmetic coding with source symbols a, b and stop symbol $\bullet$ with probabilities 0.4, 0.4 and 0.2 is used, then what is the message $ab\bullet$ encoded as?

(a) 0.25 (b) 0.3 (c) 0.5 (d) 0.55 (e) None of these

7. Using the LZ78 algorithm a message is encoded as $(0, b)(1, a)(2, a)(3, b)(3, a)$. What is the last dictionary entry after decoding?

(a) a (b) aa (c) ba (d) baa (e) $baaa$

8. Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ be a source with probabilities $p_1 = 0.4$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.1$, $p_5 = 0.1$. The average length of a **radix 4** Huffman code for S is

(a) 1.2 (b) 1.4 (c) 1.6 (d) 2.2 (e) None of these

9. Let S be the source in Question 8. The average length per symbol of a radix 4 Huffman code for $S^{(n)}$ converges, as $n \to \infty$, to approximately

(a) 1.06 (b) 1.16 (c) 1.43 (d) 1.85 (e) 2.12

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10. Let S be the source in Question 8. Using the binary Shannon-Fano code for S, the message $\mathbf{m} = s_1 s_4 s_2$ is encoded as

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(a) 01100100 (b) 001001010 (c) 001000010 (d) 01100100 (e) None of these

11. **[5 marks]**
A Markov source S has transition matrix and equilibrium probability distribution

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$$M = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{12} & \frac{1}{6} & \frac{1}{4} \end{pmatrix} \qquad \text{and} \qquad \mathbf{p} = \frac{1}{27}\begin{pmatrix} 10 \\ 13 \\ 4 \end{pmatrix}.$$

(a) Find appropriate binary Huffman codes to encode S as a Markov source.

(b) Determine the average codeword length L_M for this encoding.

(c) Using these Huffman codes, encode the string $s_1 s_3 s_2 s_1$.