MATH3411 INFORMATION, CODES & CIPHERS

Test 1 2015 S2 SOLUTIONS

Version A

Multiple Choice: a, b, b, d, c, c, d, e, c, a

1. **(a)**:

2. **(b)**: One or two errors.

3. **(b)**: There are $2^4 = 16$ codewords, namely the 16 linear combinations of the rows of G.

4. **(d)**: There is just one codeword ending with 1011, namely the sum of rows 1, 3 and 4 of G. That is, the codeword is $\mathbf{m}G$ (which works because the information bit columns in G form the identity matrix).

5. **(c)**:

6. **(c)**:

7. **(d)**: The Kraft-McMillan number $K = \sum \frac{1}{r^{\ell_i}}$ must be at most 1 for UD codes. Testing values of $r = 2, 3 \ldots$ gives us that $r = 5$ is the minimum radix that satifies this. (You could also draw a decision tree.)

8. **(e)**: Draw a decision tree.

9. **(c)**: One dummy symbol is needed.

10. **(a)**:

11. (i) Since H is in standard form $(I\; B)$, we can find a generator matrix easily:
$$G = (-B^T\; I) = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

    (ii) $C = \{00000, 01110, 10101, 11011\}$.

    (iii) $\mathrm{w}(00000) = 0$, $\mathrm{w}(01110) = 3$, $\mathrm{w}(10101) = 3$, $\mathrm{w}(11011) = 4$.

    (iv) $\mathrm{d}(00000, 01110) = 3$, $\mathrm{d}(00000, 10101) = 3$, $\mathrm{d}(00000, 11011) = 4$, $\mathrm{d}(01110, 10101) = 4$, $\mathrm{d}(01110, 11011) = 3$, $\mathrm{d}(10101, 11011) = 3$.

    (v) The minimum distance of C is $d = 3 = 2t + 1$ where $t = 1$, so C is 1-error correcting and detecting. This is using the usual minimal distance decoding strategy. If we allow any strategy, then the code can correct up to 1 error and detect up to 2 errors.

Version B

Multiple choice: d, d, c/d, a, b, c, c, b, e, b

1. **(d)**:

2. **(d)**:

3. **(c)**: Four errors. **Update**: The problem had been written so as to imply that a detection-only decoding strategy was being used, so if the receiver receives a message with three errors, then the receiver will detect the presence of errors. However, the problem could also be interpreted such that a minimal distance strategy was being used, in which case three errors would be seen as just a single error, and be (incorrectly) corrected. Under this interpretation, (d) is correct.

4. **(a)**: There are $2^3 = 8$ codewords, namely the 8 linear combinations of the rows of G.

5. **(b)**: There is just one codeword ending with 011, namely the sum of rows 2 and 3 of G. That is, the codeword is $\mathbf{m}G$ (which works because the information bit columns in G form the identity matrix).

6. **(c)**: 001 could either be $\mathbf{c}_3$ or $\mathbf{c}_4\mathbf{c}_1$.

7. **(c)**: The Kraft-McMillan number $K = \sum \frac{1}{r^{\ell_i}}$ must be at most 1 for UD codes. Testing values of $r = 2, 3, 4, \ldots$ gives us that $r = 4$ is the minimum radix that satisfies this. (You could also draw a decision tree.)

8. **(b)**: Draw a decision tree.

9. **(e)**: One dummy symbol is needed.

10. **(b)**:

11. (i) Since H is in standard form $(I\,B)$, we can find a generator matrix easily:
$$G = (-B^T\,I) = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$
    (ii) $C = \{00000, 10110, 11101, 01011\}$.
    (iii) $\mathrm{w}(00000) = 0$, $\mathrm{w}(10110) = 3$, $\mathrm{w}(11101) = 4$, $\mathrm{w}(01011) = 3$.
    (iv) $\mathrm{d}(00000, 10110) = 3$, $\mathrm{d}(00000, 11101) = 4$, $\mathrm{d}(00000, 01011) = 3$, $\mathrm{d}(10110, 11101) = 3$, $\mathrm{d}(10110, 01011) = 4$, $\mathrm{d}(11101, 01011) = 3$.
    (v) The minimum distance of C is $d = 3 = 2t + 1$ where $t = 1$, so C is 1-error correcting and detecting. This is using the usual minimal distance decoding strategy. If we allow any strategy, then the code can correct up to 1 error and detect up to 2 errors.