Name: .... Student ID: ....

UNSW SCHOOL OF MATHEMATICS AND STATISTICS

MATH3411 INFORMATION CODES AND CIPHERS

2013 S2 **TEST 1** **VERSION A**

- Time Allowed: **45 minutes**

For multiple choice questions, **circle the correct answer**; each multiple choice question is worth **1 mark**. For written answer questions, **use extra paper**. Staple all papers together when finished.

1. You are given the following 7-bit ASCII codewords:

L	1001100	o	1101111	v	1110110	e	1100101
$	0100100	0	0110000	h	1101000	d	1011011

Define a 5-character 8-bit ASCII burst code by encoding characters in blocks of four together with a 5th character which is used as a check codeword. (This is similar to the 9-character 8-bit and 8-character 8-bit ASCII code studied in lectures.) The message "Love" together with its check character is given by:

(a) Love$ (b) Love0 (c) Loveh (d) Lovet (e) Loved

For the next 3 questions, let C_1 be a binary linear code with check matrix

$$H = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

Assume that the check bits correspond to columns 1, 2, and 4.

2. The codeword encoding the message 1010 in code C_1 is

(a) 1101110 (b) 1110010 (c) 1101100 (d) 0111110 (e) 0110100

3. A generator matrix G corresponding to the check matrix H for the code C_1 has size

(a) 4×7 (b) 3×7 (c) 4×3 (d) 3×4 (e) none of these.

4. Supposing that the word 1110011 is received using code C_1, what is the minimum number of errors that could have occurred?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. Consider a binary 2-error correcting code C with $k = 3$ information bits and $m = 5$ check bits. What is the largest possible number of codewords in C?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

6. A binary code C has minimum distance $d = 7$. Suppose this is used to correct a errors and detect b errors. Which of the following pairs (a, b) **does not** give a valid strategy for decoding C?

(a) $(0, 6)$ (b) $(1, 5)$ (c) $(2, 4)$ (d) $(3, 3)$ (e) $(4, 2)$

7. The message $s_3 s_2 s_4 s_2$ was encoded using a comma code of length 4. The encoded message is

(a) 10011111110 (b) 1100111110 (c) 11010111110
(d) 01111100110 (e) 11010111010

8. A binary UD-code has codewords lengths (not necessarily in order) 1, 3, 3, 4, 4, ℓ. What is the minimum value must ℓ take in order for the code to exist?

(a) $\ell = 1$ (b) $\ell = 2$ (c) $\ell = 3$ (d) $\ell = 4$ (e) $\ell = 5$.

9. Consider a binary Huffman code for a source with 6 symbols, where the source symbols are given in non-increasing probability order. Suppose that the codeword for symbol s_6 is $c_6 = 1011$. Then the codeword for symbol s_5 is

(a) 101 (b) 1010 (c) 1111 (d) 0011 (e) 011

10. Let $S = \{s_1, s_2\}$ be a source with probabilities $p_1 = \frac{3}{5}$, $p_2 = \frac{2}{5}$. The average length per original symbol of a **radix 3** Huffman code for the **second extension** $S^{(2)}$ of this source (constructed with the usual strategies) is

(a) $\dfrac{7}{10}$ (b) 1 (c) $\dfrac{41}{25}$ (d) $\dfrac{41}{50}$ (e) $\dfrac{7}{5}$

11. [**5 marks**]

(a) Show that there is no uniquely decodable **ternary** (i.e. radix 3) code for the source

$$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$$

with codeword lengths $1, 1, 2, 2, 3, 3, 3, 4$, respectively.

(b) The symbol s_1 of the source $S = \{s_1, s_2\}$ occurs with probability $5/8$ and s_2 occurs with probability $3/8$. Find a uniquely decodable binary code of minimal average length for S^2, assuming that successive symbols occur independently, and state the average length per original source symbol of the code.

Name: .... Student ID: ....

UNSW SCHOOL OF MATHEMATICS AND STATISTICS

MATH3411 INFORMATION CODES AND CIPHERS

2013 S2 **TEST 1** **VERSION 1B**

- Time Allowed: **45 minutes**

For multiple choice questions, **circle the correct answer**;
each multiple choice question is worth **1 mark**.
For written answer questions, **use extra paper**.
Staple all papers together when finished.

1. A message is sent using a 5-character 8-bit ASCII burst code, which encodes characters in blocks of four together with a 5th character which is used as a check codeword for even parity in rows and columns. (This is similar to the 9-character 8-bit and 8-character 8-bit ASCII codes studied in lectures.)

The message 00001001 11111111 10101011 11001011 11010100 is received and has been corrupted by a burst of noise that has converted all 0s to 1 but left all 1s as 1. What is the minimum length of the noise burst?

(a) 3 (b) 4 (c) 5 (d) 6 (e) none of these.

For the next 3 questions, let C_1 be a binary linear code with check matrix

$$H = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Assume that the check bits correspond to columns 1, 3, 4 and 6.

2. The codeword encoding the message 110 in code C_1 is

(a) 1001110 (b) 0010100 (c) 0011100 (d) 1110100 (e) 1101100

3. A generator matrix G corresponding to the check matrix H for the code C_1 has size

(a) 3×7 (b) 4×7 (c) 4×3 (d) 3×4 (e) none of these.

4. If the word 1101001 is received using code C_1, what is the minimum number of errors that could have occurred?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. Let C be a binary 1-error correcting code with k information bits, $m = 3$ check bits and 2^k codewords. The maximum possible value for k is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

6. A binary code C has minumum distance $d = 12$. Suppose this is used to correct a errors and detect b errors. Which of the following pairs (a, b) gives a valid strategy for decoding C?

(a) $(0, 12)$ (b) $(3, 9)$ (c) $(4, 8)$ (d) $(5, 6)$ (e) $(7, 4)$

7. The message $s_2 s_1 s_4 s_3$ was encoded using a comma code of length 4. The encoded message is

(a) 1001110110 (b) 1011011110 (c) 1101011110

(d) 0111110010 (e) 1001110100

8. A binary UD-code has codewords lengths (not necessarily in order) 2, 2, 3, 4, 4, ℓ. What is the smallest value must ℓ take in order for the code to exist?

(a) $\ell = 2$ (b) $\ell = 3$ (c) $\ell = 4$ (d) $\ell = 5$ (e) none of these.

9. Consider a binary Huffman code for a source with 7 symbols, where the source symbols are given in non-increasing probability order. Suppose that the codeword for symbol s_7 is $\mathbf{c}_7 = 01101$. Then the codeword for symbol s_6 is

(a) 010 (b) 0110 (c) 11011 (d) 01110 (e) 01100

10. Let $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ be a source with probabilities $p_1 = \frac{6}{17}$, $p_2 = \frac{4}{17}$, $p_3 = \frac{3}{17}$, $p_4 = \frac{2}{17}$, $p_5 = p_6 = \frac{1}{17}$. The average length of a **radix 4** Huffman code for this source (using the usual strategies) is

(a) $\frac{4}{17}$ (b) $\frac{12}{17}$ (c) $\frac{21}{17}$ (d) $\frac{24}{17}$ (e) $\frac{31}{17}$

11. [**5 marks**]

(a) Find an instantaneous **ternary** (i.e. radix 3) UD-code for the source

$$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$$

with codeword lengths $1, 2, 2, 2, 2, 4, 4, 4$, respectively.

(b) The symbol s_1 of the source $S = \{s_1, s_2\}$ occurs with probability $7/10$ and s_2 occurs with probability $3/10$. Find a binary UD-code of minimal average length for S^2, assuming that successive symbols occur independently, and state the average length per original source symbol of the code.