PHIL1012 Lecture 13: Semantics of MPL, Pt. 1

Goals

① give a precise account of what a "scenario in which a proposition is true or false" is;

② give a precise account of how to determine the truth value of each proposition in MPL in such a scenario.

Guiding idea

The values of nonlogical symbols are unconstrained:

for any way of assigning values to nonlogical symbols, there is a "scenario" in which the symbols have those values.

Example PL

Nonlogical symbols: $A, B, C, \ldots, A_1, A_2, \ldots$
$B_1, B_2, \ldots$

Values of nonlogical symbols: truth values (T or

"Scenarios": a row of truth table

Problem case

All kelpies are dogs	K	T
Maisie is a kelpie	M	T
Maisie is a dog	D	F

MPL solution

- The propositions in the above argument are complex.
- Their values are constrained by the values of their nonlogical (nonpropositional) components 1 and the laws of truth that govern their logical components 2

I Values of nonlogical symbols of MPL; truth conditions for atomic propositions

Nonlogical symbols:

- Names: $a, b, c, \ldots, s, t, a_1, a_2, a_3, \ldots, b_2, b$

- Predicates: $A, B, C, \ldots, A_1, B_1, \ldots$

Consider an atomic proposition Km ("Maisie is a kelpie").

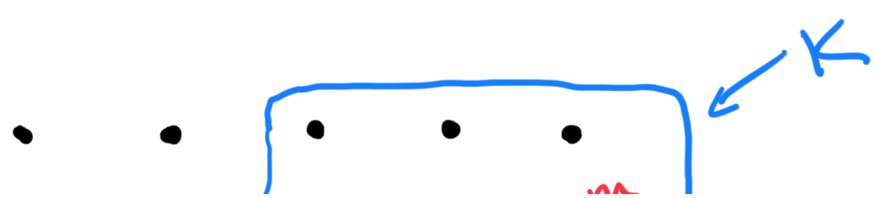
Intuition: Km is true iff the object picked out by m has the property picked out by K

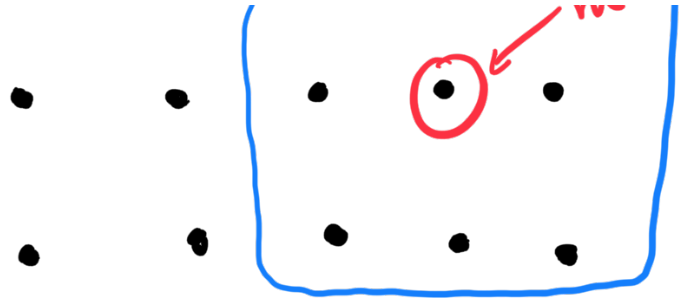
It's more convenient to work with a set of objects that has a given property rather than the property itself.

(Sets are extensional — identity is determined by members, e.g. $\{1,2\} = \{2,1\}$; the identity condition for properties is not clear.

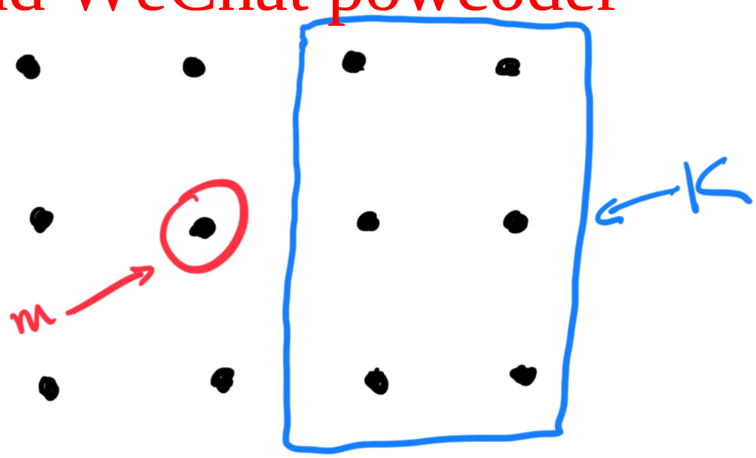
Refined intuition Km is true iff the object picked out by m is in the set of objects that have the property picked out by K.

Examples Km is true





Km is false



So, the value of a name is an object (called its referent).

The value of a predicate is a set of objects (called its extension).

An atomic proposition is true iff the referent of its name is in the extension of its predicate.

II Simple quantified propositions; models

Intuition $\forall x Px$ is true iff "everything" is in the extension of P.

[illegible] specifying referents of names

So, in addition to specifying referents of names and extensions of predicates, a "scenario" specifies what counts as "everything."

More precisely...

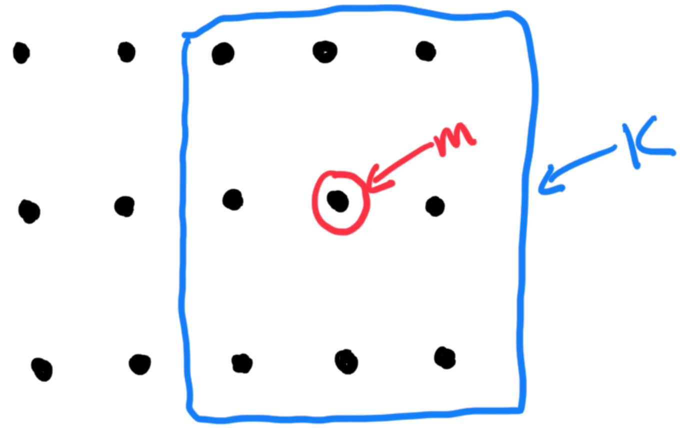
Models A model has three components:

① A domain — a nonempty set of objects (intuitivel this specifies what counts as "everything" in the model);

② For each name, a specification of a referent (an object in the domain);

③ For each predicate, a specification of an extension (a subset of the domain).

Example Km is true



- $\forall \underline{x}\, \underline{Px}$ is true in a model M iff every object in the domain of M is in the extension of $\underline{P}$

- $\exists \underline{x}\, \underline{Px}$ is true in a model M iff some object in the domain of M is in the extension of $\underline{P}$.

- $\underline{Pa}$ is true in a model M iff the referent of $\underline{a}$ in M is in the extension of $\underline{P}$ in M.

Note The domain of a model must contain at least one object (it must be nonempty).

However, the extension of a predicate can be the empty set $\emptyset$.

(What is $\emptyset$? The set containing no members!

Example Consider predicate "is a dog":

- Obviously there are scenarios in which the extension is nonempty;

- But there are also scenarios in which the extension is empty, i.e. in which there are no dogs.

III Fragment and signature

We have said that a model specifies a referent for each name and an extension for each predicate.

But MPL has infinitely many names and infinitely many predicates!

From now on we will talk about models of fragments of MPL.

- A fragment of MPL is the set of wffs that can be generated from a nonempty set of predicates and a possibly empty set of nam
  $\forall x Px$
- The set of nonlogical symbols (names / predicat used to generate a fragment is called the fragment's signature.

Example When translating, we use fragments.

Maisie is a kelpie and all kelpies are dogs.

$Km \wedge \forall x (Kx \rightarrow Dx)$

K, D, m signature

is in the fragment

IV Connectives

The truth value of a proposition in MPL whose main operator is a connective is determined by the truth values of its component propositions just like in PL.

Example $\alpha \wedge \beta$

α	β	$\alpha \wedge \beta$

$\alpha \wedge \beta$ is true in a

T	T	T
T	F	F
F	T	F
F	F	F

model $\mathcal{M}$ iff both α and β are true in $\mathcal{M}$.

(V) True or false in the given model?

Domain: {Alice, Bill, Charlie, Dan} (1)

Extension of P: {Alice, Bill, Dan} (3)

Referent of c: Charlie (2)

(1) $\forall x Px$ F

(2) $\exists x Px$ T

(3) $\underset{F}{Pc} \vee \neg \underset{T}{\forall x Px}$ T

(4) $\underset{F}{\forall x Px} \rightarrow \neg Pc$ T

Domain: {1, 2, 3, 4, 5}

Domain: $\{1, 2, 3, 4, 5\}$

P: $\emptyset$, Q: $\{1,3,5\}$

a: 1 , b: 2 , c: 5

① $\underbrace{\exists x Qx}_{T} \longleftrightarrow \underbrace{Pb}_{F}$ F

④ Py

Trick question

② $(\exists x Px \land \exists x Qx) \rightarrow \underbrace{Qc}_{T}$ T

③ $\underbrace{\neg \exists x Px}_{T} \land \underbrace{\forall x Qx}_{F}$ F

Ⓥ Make a model in which it's true

① $\neg \forall x Rx \land \exists x Bx$

② $(Ra \land Rb) \rightarrow \forall x Rx$