PHIL1012 Lecture 10: Functional Completeness, Pt. 2

Question

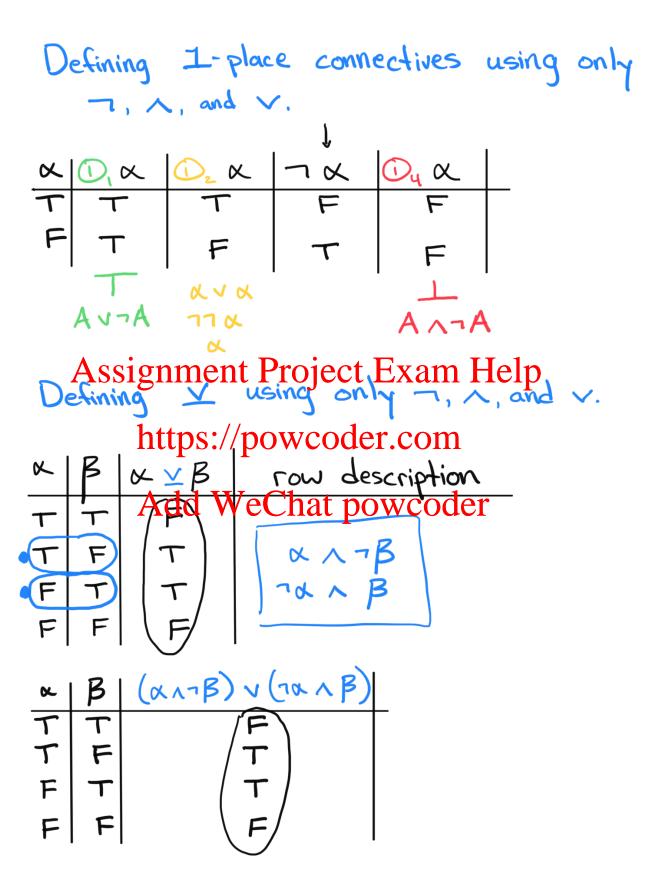
- · Is $\{7, \land, \lor, \rightarrow, \longleftrightarrow\}$ functionally complete?
- . Assignment Project Examilial pa formul
 of Plensith that truth table?
 https://powcoder.com
- · Can the connectives of Pl be used to define all possible connectives?

We will start by showing that the set $\{2-1, 1, 1, 1, 2\}$ is functionally complete.

This of course implies that the set $\{7, \wedge, \vee, \rightarrow, \rightarrow\}$ is functionally complete as well. (If we can desire then we can define https://pow.coder.com every connective using 7, \wedge , \vee , \vee , \rightarrow , Add WeChat pow.coder.

Defining O-place connectives using only 7, 1, and v.

A	1		AV¬A	A ^¬A
T	T	F	+	F
F	T	F	一	F



Defining any connective using only 7, 1, and v.

X	B	8	*(x, B, x)	row description
7	+	T	F	
τ	T	F	F	
OT	F	T	丁	KABAY E
T	F	F	F	
F	T	T	F	
F	T	F	F	
• F	F	T	T	7dA7BA8 =
•F	As	sig	nment Pro	ject Exam Help

(a https://powcoder.com/ (7217817)

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We have shown that $\{27, 1, 1, 1\}$ is a functionally complete set of connectives!

Can we find other, smaller functionaller complete sets of connectives?

X	B	x VB	7(7d 17B)	NNB	7(7XV7B)	
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	'	'	\	—		
F	F	https	s://powc	oder.co	m F	

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So, both 27, 13 and 27, 2 are functionally complete as well!

① 27, 1, v 3

② 27, 1, v 3

③ 37, v 3

Important Fact

Suppose X is a functionally complete set of connectives and Y is another set of connectives. Then, if every connective in X can be defined using only connectives in Y, then Assignmental arbject lighter Helpwell.

Example https://poxycoder.com/, \}

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Can we find even smaller functionally comple sets? For instance ...

Is {v3 functionally complete?

No! But how to show it?

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In general, to show that a set of connectives is not functionally comple you must find a connective that cannot be defined using only the connectives in the set.

Jo, let's snow triat -1 carrier be defined using only v.

Start by observing that - has the following property:

For any basic proposition P, ¬P

Assignment Project Exam Help its

truth table. PIP

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Does anydownwell hat powedodelnat contain only a have the property & ?

Of course not! No matter how complex x is, e.g.

 $x = ((P \vee Q) \vee (P \vee P)) \vee R$, if it contains only \vee , it will be true in the first row of its truth

table, unlike $\neg P$.

So, it is impossible to define - using only

That is, $3 \vee 3$ is not functionally Assignment Project Exam Help complete.

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Is there any functionally complete set that contains only one connective?

Yes! Consider...

\[
\lambda \beta \beta \beta \beta \cdot \beta \

Summary

1) We showed that $\{7, 1, 1, 1\}$ (and hence

using the "row description method".

- ② We showed that $\{27, 13, \{7, 13, 17, 17, 18, 18\}$ and $\{1, 1, 18\}$ are truth functionally complete using the Important Fact above.
- 3) We showed that Ev 3 is not truth functionally complete.

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