PHIL1012 Lecture 12: The Language of MPL, Pt. 2

Plan for today

1. Syntax of MPL
2. Constructing wffs
3. Translation tip: think syntactically!
4. Quantifier scope
5. Free / bound variables ; vacuous quantifiers
6. Open / closed wffs

1. Syntax of MPL

Symbols

- Nonlogical: names (a, b, c, ..., s, t), predicates (A, B, C, ...)

- Logical: variables (u, v, w, x, y, z), connectives (¬, ∧, ∨, →, ↔), quantifier symbols (∀, ∃

- Auxiliary: parentheses

Terminology: A term of MPL is either a name or a variable

Well-formed formulas (wffs)

- Atomic wffs: If $\underline{P}$ is any predicate, and $\underline{t}$ is any term, then $\underline{P}\,\underline{t}$ is a wff.

- If $\underline{x}$ is any variable, and α and β are wffs, then the following are wffs as well

(i) $\neg\alpha$ (ii) $(\alpha \wedge \beta)$ (iii) $(\alpha \vee \beta)$

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(iv) $(\alpha \rightarrow \beta)$ (v) $(\alpha \leftrightarrow \beta)$ (vi) $\forall \underline{x}\ \alpha$

(vii) $\exists \underline{x}\ \alpha$

Terminology

① Quantifier : quantifier symbol + variable,

e.g. $\forall x$, $\exists z$, $\forall y$

② Constructing wffs

Example $\forall x Rx \vee \exists v \neg Bv$

$\forall x Rx \vee \exists v \neg Bv$

$\forall x \neg Fx \vee \exists y \neg Dy$

atomic wff

(i)

(vii)

(vi)

(iii)

Terminology

- (Logical) operator : connective or quantifier,
  e.g. $\forall x$, $\rightarrow$, $\neg$, $\exists z$
- Main operator : the last operator added in a wff construction
- Subformulas : all of the wffs obtained during the construction

Example $\forall x (Kx \rightarrow Dx) \wedge Km$

- What is the main operator? $\wedge$

• What are the subformulas?

Km, $Kx \to Dx$, Kx, Dx, $\forall x(Kx \to Dx)$

$\forall x(Kx \to Dx) \wedge Km$

③ Translation tip: think syntactically!

If Gary is lying, then not (everyone in this room is telling the truth.)

(i) What is the main operator?
(ii) Look at subformula
(iii) Repeat

$Lg \to \neg \forall x((Px \wedge Rx) \to Tx)$

Glossary

g: Gary
Px: x is a person
Lx: x is lying
Tx: x is telling the truth
Rx: x is in this room

④ Quantifier scope

- If a wff has a quantifier in it, it got there by attaching to some wff α in the construction process. We call this α the scope of the quantifier.

Examples Identify each quantifier's scope.

- $\forall x (Px \to Qx)$ $\quad (Px \to Qx)$

- $\forall x Px \to Qx$ $\quad Px$

- $\exists y \exists z Lz$ $\quad \exists z Lz$ $\quad Lz$

⑤ Free / bound variables; vacuous quantifiers

- An occurence of a variable in a wff is bound if either (i) it is in a quantifier or (ii) it is in the scope of a quantifier containing the same variable.

$$\underset{(i)}{\forall x}\ P\underset{(ii)}{x}$$

- An occurence of a variable that is <u>not</u> bound in a wff is called free.

Examples Which variable occurences are free / bound?

- $\exists \underset{\text{bound}}{x} \, (C\underset{\text{bound}}{x} \wedge B\underset{\text{free}}{y})$

- $F\underset{\text{free}}{w} \rightarrow \forall \underset{\text{bound}}{w} \, G\underset{\text{bound}}{w}$

Note If a variable is in the scope of multiple quantifiers containing that variable, it is the first quantifier added in the construction process that binds it.

Example $\forall x \; \underline{\exists x} \; P x$

$\forall x \exists x P x$

- An occurence of a quantifier is vacuous if the variable in the quantifier does not occur free in the scope of the quantifier.

Examples Vacuous or not?

- $\underline{\forall y}\, Fa$ Vacuous

- $\underline{\exists u}(Px \wedge Su)$ $(Px \wedge Su)$ free

Not vacuous

- $\underline{\forall x}\, \underline{\forall x}\, Bx$ Bx free $\forall x\, Bx$ bound bound

Vacuous Not vacuous

⑥ Open/closed wffs

- A wff with no free occurences of variables is called closed. A wff with one or more free occurences of variables is open.

Note Both open and closed wffs are well-formed, but open wffs are not propositions. That is, open wffs are not truth-apt.

Example Tx "It is tall"

"_____ is tall"