PHIL1012 Lecture 16: Trees for MPL, Pt. 2

Plan for today

1. Using trees
2. Reading off models
3. Infinite trees
4. Looking ahead

1. Using trees

Is it a logical truth? Yes!

$\forall x Fx \rightarrow Fa$

$\neg(\forall x Fx \rightarrow Fa)$ ✓

$\forall x Fx$ \a

$\neg Fa$

Fa

×

Is the argument valid? Yes!

$\exists x\, Fx$

$\neg\exists x(Fx \wedge \neg Gx)$

$\exists x\, Gx$

$\exists x\, Fx$ ✓a

$\neg\exists x(Fx \wedge \neg Gx)$ ✓

$\neg\exists x\, Gx$ ✓

$\forall x \neg Gx$ \a

$\forall x \neg(Fx \wedge \neg Gx)$ \a

Not sat.

Fa

$\neg Ga$

$\neg(Fa \wedge \neg Ga)$ ✓

$\neg Fa$ ×

$\neg\neg Ga$ ×

Is it a logical truth? No!

$$\exists x Fx \rightarrow \forall x Fx$$

Domain: {1, 2}
a: 1 b: 2
F: {1}

$\neg(\exists x Fx \rightarrow \forall x Fx)$ ✓

$\exists x Fx$ ✓a

$\neg \forall x Fx$ ✓

$\exists x \neg Fx$ ✓b

Fa ←

$\neg Fb$ ←

↑

② Reading off models

Given an open, saturated path, we want to read off a model of the fragment whose

signature includes all names and predicates on the path.

We need to specify three things:

(i) Domain

(ii) Assign referents (objects in domain) to names

(iii) Assign extensions (subsets of domain) to predicates

(i) Domain: one object for each name on the path.

Example If path contains names a, b, c, then our domain is $\{1, 2, 3\}$.

(ii) Referents for names: one-to-one correspondenc

Example a:1 , b:2 , c:3 , ...

(for every name on the path)

(iii) Extensions for predicates:

The extension of F contains 1 iff

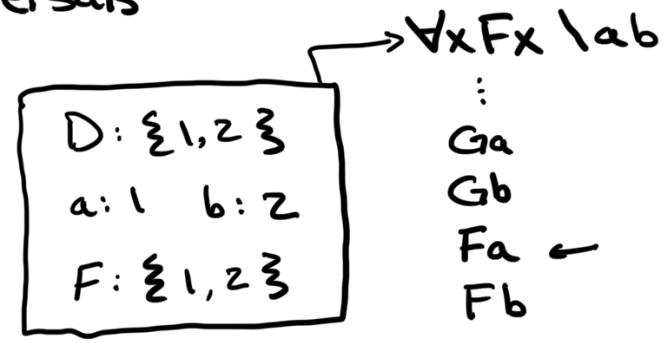
Fa appears (on its own) on the path.

Example See above.

Note

- Read off a model only once the path is saturated (see p. 227).

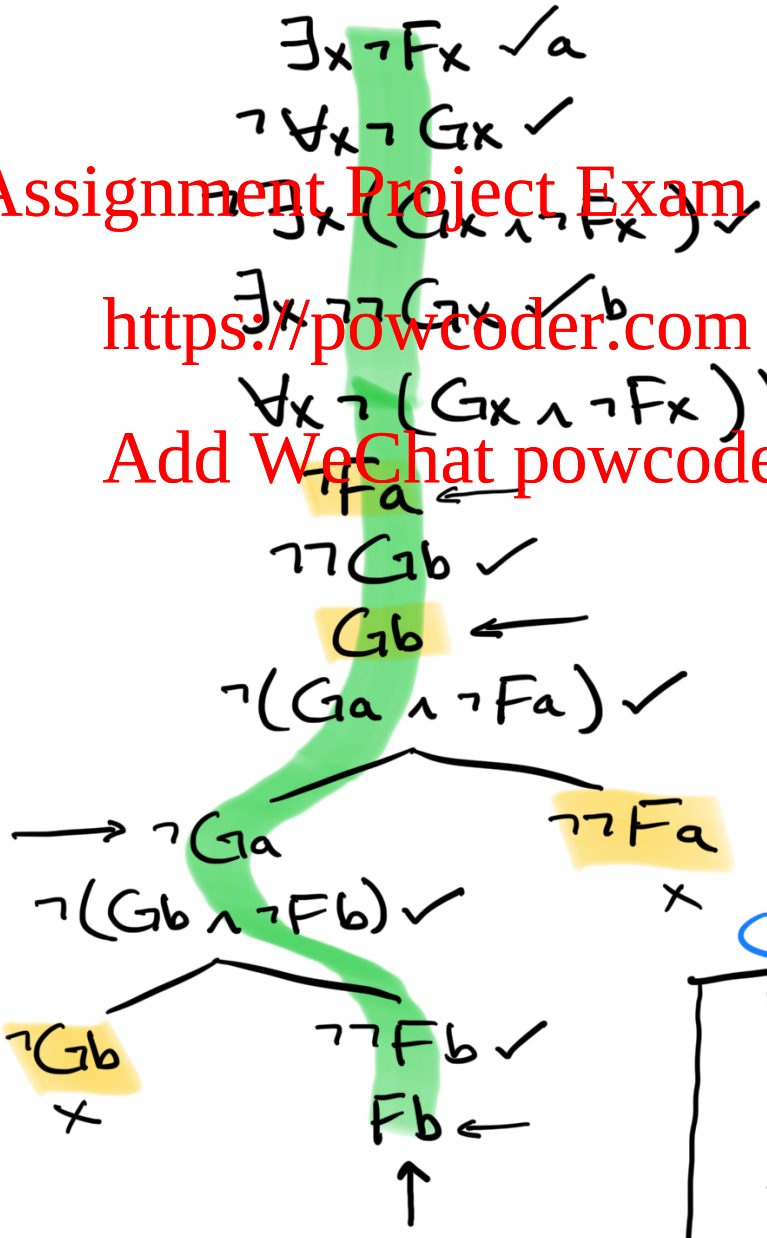
- Recall saturation requirements for paths with universals



Example Is it valid? No!

Counterexample is below.

$$\frac{\exists x \neg Fx \quad \neg \forall x \neg Gx}{\exists x (Gx \wedge \neg Fx)}$$



Counterexample

Domain: {1, 2}

a: 1, b: 2

F: {2}

③ Infinite trees

Example $\forall x(Fx \wedge \exists y Gy)$ \ a b c ...

$Fa \wedge \exists y Gy$ ✓

Fa

$\exists y Gy$ ✓ b

Gb

$Fb \wedge \exists y Gy$ ✓

Fb

$\exists y Gy$ ✓ c

Gc

⋮

The tree goes on forever. Why?

We can't write out an infinite path, but we can understand what it would look like if it were written out.

And we can read off a model from an infinite path provided it is saturated.

Note Not every infinite path is saturated, e.g.

$\forall x(Fx \wedge \exists y Gy) \wedge (Ga \wedge \neg Ga)$ ✓

$\forall x(Fx \wedge \exists y Gy)$ | ab

$Ga \wedge \neg Ga$

$Fa \wedge \exists y Gy$ ✓

Fa

$\exists y Gy$ ✓ b

Gb

$Fb \wedge \exists y Gy$ ✓

Fb

$\exists y \neg Gy$

⋮

How to read off a model from an infinite,

Saturated, open path:

The procedure is basically the same as for finite paths, but we must use our ingenuity to recognize a pattern in the path.

Example

Domain: $\{1, 2, 3, \dots\}$

a: 1, b: 2, c: 3, ...

F: $\{1, 2, 3, \dots\}$

G: $\{2, 3, 4, \dots\}$

~~$\forall x(Fx \to \exists y Gy)$~~ ... b

$Fa \wedge \exists y Gy$ ✓

Fa ←

$\exists y Gy$ ✓ b

Gb

$Fb \wedge \exists y Gy$ ✓

Fb ←

$\exists y Gy$ ✓ c

Gc

⋮

Note The lesson here is not that every model satisfying $\forall x(Fx \wedge \exists y Gy)$ has an infinite domain, only that some such model does.

(Exercise: show that $\forall x(Fx \wedge \exists y Gy)$ has a model with a finite domain.)

(4) Looking ahead

- Infinite trees require insight to see whether a saturated path will close — there's no mechanical test. This has very deep consequences. Take PHIL 3610 Logic and Computation!

- It is not obvious, and it requires verification, that our method for reading off models produces models that satisfy all the propositions on an open, saturated path. Take PHIL 2615 Logic and Proof!

- For more on infinite trees, see end of ch. 10. For philosophical discussion, read ch. 11.

- Next week, on to ch. 12 and GPL with Nick

Thank you!