PHIL1012 Lecture 11: The Language of MPL, Pt. 1

All kelpies are dogs
Maisie is a kelpie

Maisie is a dog

K
M

D

All Ks are Ds
m is a K

---

m is a D

No kelpie is a cat
Maisie is not a cat

Maisie is a kelpie

$\neg K$
$\neg M$

A

invalid

- The validity/invalidity of these arguments depends on the internal structure of the propositions in them.

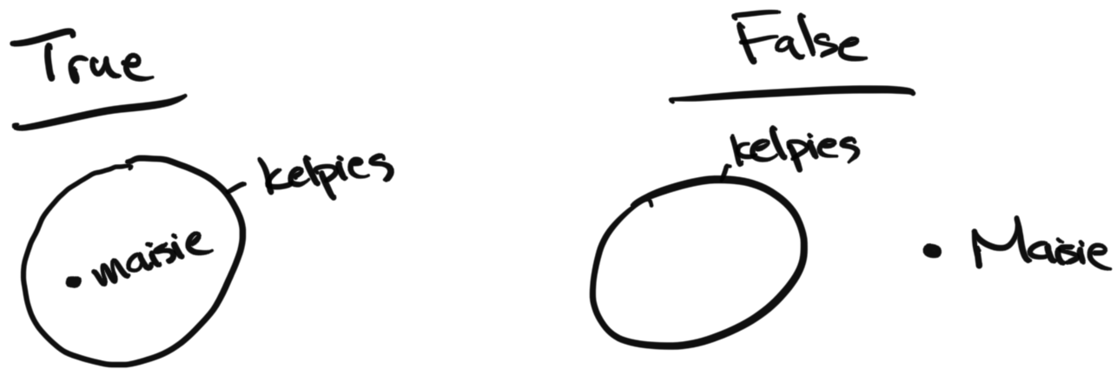
- We need a richer logical language that captures two parts of propositions like "Maisie is a kelpie."

(I) Names and Predicates

- Throw away symbols of PL for basic propositions

Consider "$\underbrace{\text{Maisie}}_{\text{name}}$ $\underbrace{\text{is a kelpie}}_{\text{predicate}}$"

- Two parts that are relevant to whether this proposition is true or false



- In MPL, names are represented by lowercase a, b, c, ..., t (not including u, v, w, x, y, z)
- Predicates are represented by uppercase letters A, B, C, ...
- One predicate followed by one name, e.g.

Ab, Cs, Nd, Yt, De

is an atomic proposition of MPL.

Glossaries

- When translating from English to MPL, we begin with a glossary linking names and predicates to their English counterparts. For example ...

Maisie is a kelpie

name predicate

Km

Glossary

m : Maisie

K : is a kelpie

Examples

1. Sophie is tall Ts
2. Victor is ill Ii
3. The Empire State Building is tall Te
4. 3 is even Et
5. 3 is odd Ot

Glossary

s : Sophie

i : Victor

e : ESB

t : 3

nam

T : is tall

I : is ill

E : is even

O : is odd

Connectives

- In MPL, we retain all of the propositional connectives of PL, i.e. $\neg, \wedge, \vee, \to, \leftrightarrow$
- Note that $\cancel{A \to B}$ is not well-formed in MPL (A and B are predicates not propositions.)

Examples

1. Both Sophie and the Empire State Building are tall $\quad Ts \wedge Te$
2. If Victor is tall, then so is Sophie $\quad Ti \to Ts$
3. 3 is even — unless it's odd $\quad \boxed{Et \vee Ot}$ $\quad \boxed{\neg Ot \to Et}$

Glossary

s : Sophie
e : Empire State Build
i : Victor
t : 3
T : is tall
E : is even
O : is odd

4. 3 is even only if it's not odd $\quad Et \rightarrow \neg Ot$

Indexicals

- Remember: we are representing in MPL the proposition expressed, not the English sentence used to express i[t]
- So be careful with words that pick out different things in different contexts, e.g.

Glossary
s: Saturday
t: Tuesday
W: is warm

1. Today is warm (said on Saturday) $\quad Ws \quad$ T

2. Today is warm (said on Tuesday) $\quad Wt \quad$ F

II Quantifiers and Variables

Compare ...

Compare ...

1. Georgie is a dog

2. Something is a dog

* "Something" is <u>not</u> a name

D_g ✓

$\cancel{D_s}$

Similarly for "everything": It is not a name!

<u>Something</u>: one or more. "Something is X" is true iff one or more individuals have the property X.

- Called an existential quantifier
- "some" "at least one" "there is a(n)"

Everything: "Everything is X" is true iff all individuals have the property X

- Called a universal quantifier
- "all," "every," "each"

In order to express quantifiers in MPL, we need symbols for variables.

Here is the idea:

Universal Quantifiers ∀

Everything is beautiful.

Every thing is such that it is beautiful.

Glossary

B: is beautif

• Variables: lowercase u, v, w, x, y, z

Every x is such that x is beautiful.

Every x is such that Bx

$\forall x\, Bx$ ("for all x, Bx")

Existential Quantifiers $\exists$

Something is beautiful

Some thing is such that it is beautiful

Some x is such that x is beautiful

Some x is such that Bx

$\exists x\, Bx$ "for some x, Bx"

Note

Now that we have introduced variables, we will write them next to predicates in glossaries, e.g.

Glossary

Bx: x is beautiful

Kx: x is a kelpie

Examples

Glossary

Gx: x is grey

Rx: x is round

Sx: x is speci

Ex: x is empt

Px: x is a pers

1. Something is both grey and empty.

   Some x is such that x is grey and x is empty.

   $\exists x (Gx \wedge Ex)$

2. Something is grey and something is empty.

   $\exists x\, Gx \wedge \exists y\, Ey$

3. Everything is either round or not round.

   Every x is such that x is round or x is not round. $\forall x (Rx \vee \neg Rx)$

4. Everything is round or everything is not round.

4. Everything is round or everything is not round.

$$\forall x\, Rx \vee \forall x \neg Rx$$

5. Nothing is special.

It's not the case that something is special

$$\neg \exists x\, Sx$$

Everything is not special. $\forall x \neg Sx$

6. Something isn't special.

$$\exists x \neg Sx$$

It's not the case that everything is special

$$\neg \forall x\, Sx$$

7. All round things are special.

For all x: if x is round, then x is special.

$$\forall x\,(Rx \longrightarrow Sx)$$

Not $\forall x\,(Rx \wedge Sx)$ "Everything is both round an special"

8. Everyone is special

Every x is such that if x is a person, th
x is special. $\forall x\,(Px \rightarrow Sx)$

Not $\forall x\, Sx$ "Everything is special"

9. If Maisie is special, then everything is.

$$Sm \rightarrow \forall x\, Sx$$

10. All kelpies are dogs

$$\forall x\,(Kx \rightarrow Dx)$$

Addition to Glossary

m: Maisie

Dx: x is a dog

Kx: x is a kelpie

Cx: x is a cat

Bx: x barks

11. No kelpie is a cat. "It's not the case that

some x is such that x is a kelpie and x is a cat." $\neg \exists x (Kx \wedge Cx)$

12. Only dogs bark.

- For all x, x barks only if x is a dog.

$\forall x (Bx \rightarrow Dx)$

13. If every grey dog barks, then either Maisie is a cat or she's a special kelpie.

$\forall x ((Gx \wedge Dx) \rightarrow Bx) \rightarrow (Cm \vee (Sm \wedge Km))$

$Cm \vee (Sm \rightarrow \exists x Kx)$

Maisie is a cat or (If Maisie is special, then something is a kelpie)