

## PHIL1012 Lecture 12: The Language of MPL, Pt. 2

### Plan for today

- ① Syntax of MPL
- ② Constructing wffs
- ③ Translation tip: think syntactically!
- ④ Quantifier scope
- ⑤ Free / bound variables; vacuous quantifiers
- ⑥ Open / closed wffs

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### ① Syntax of MPL

#### Symbols

- Nonlogical: **names** ( $a, b, c, \dots, s, t$ ), **predicates** ( $A, B, C, \dots$ )
- Logical: **variables** ( $u, v, w, x, y, z$ ), **connectives** ( $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ ), **quantifier symbols** ( $\forall, \exists$ )

- Auxiliary: parentheses

Terminology A **term** of MPL is either a name or a variable

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Well-formed formulas (wffs)

- **Atomic wffs:** If  $P$  is any predicate, and  $t$  is any term, then  $Pt$  is a wff.
- If  $x$  is any variable, and  $\alpha$  and  $\beta$  are wffs, then the following are wffs as well  
 $(\neg \alpha)$      $(\forall x. \alpha)$      $(\exists x. \alpha)$      $(\alpha \rightarrow \beta)$

- (i)  $\neg \alpha$       (ii)  $(\alpha \wedge \beta)$       (iii)  $(\alpha \vee \beta)$   
 (iv)  $(\alpha \rightarrow \beta)$       (v)  $(\alpha \leftrightarrow \beta)$       (vi)  $\forall x \alpha$   
 (vii)  $\exists x \alpha$

## Terminology

Assignment: quantifier symbol & variable,

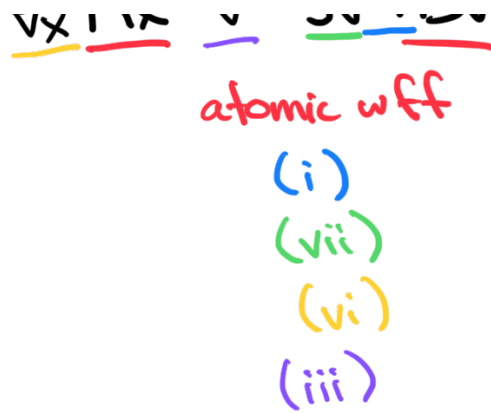
eg.  $\forall x, \exists z, \forall y$   
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## ② Constructing wffs

Example       $\forall x R_x \vee \exists v \neg B_v$

$\neg R_v \vee \neg \neg R_v$



## Terminology

- **(Logical) operator**: connective or quantifier, e.g.  $\forall x, \rightarrow, \neg, \exists z$
- **Main operator**: the last operator added in a wff construction  
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- **Sub formulas**: all of the wffs obtained during the construction

Example  $\forall x (Kx \rightarrow Dx) \wedge Km$

- What is the main operator?  $\wedge$

- What are the subformulas?

$K_m, K_x \rightarrow D_x, K_x, D_x, \forall x (K_x \rightarrow D_x)$

$\forall x (K_x \rightarrow D_x) \wedge K_m$

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Translation tip: think syntactically!

If Gary is lying, then  
not (everyone in this room  
is telling the truth.)

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(i) What is the main operator?

(ii) Look at subformulas

(iii) Repeat

$L_g \rightarrow \neg \forall x ((P_x \wedge R_x) \rightarrow T_x)$

#### Glossary

$g$ : Gary     $P_x$ :  $x$  is a person

$L_x$ :  $x$  is lying

$T_x$ :  $x$  is telling the truth

$R_x$ :  $x$  is in this room

#### ④ Quantifier scope

- If a wff has a quantifier in it, it got there by attaching to some wff  $\alpha$  in the construction process. We call this  $\alpha$  the **scope** of the quantifier.

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Examples Identify each quantifier's scope.

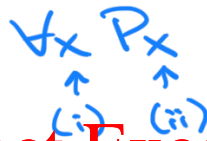
•  $\forall x$  ( $Px \rightarrow Qx$ ) ( $Px \rightarrow Qx$ )

•  $\forall x$   $Px \rightarrow$  ( $Qx$ )  $Px$

•  $\exists y$   $\exists z$   $Lz$   $\exists z Lz$   $Lz$

## ⑤ Free / bound variables ; vacuous quantifiers

- An occurrence of a variable in a wff is **bound** if either (i) it is in a quantifier or (ii) it is in the scope of a quantifier containing the same variable.



- An occurrence of a variable that is not bound in a wff is called **free**.

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Examples Which variable occurrences are free/bound?

$$\cdot \exists x (C_x \wedge B_y)$$

Diagram showing the binding status of variables in the formula  $\exists x (C_x \wedge B_y)$ . Arrows point from the labels 'bound', 'bound', and 'free' to the occurrences of  $x$ ,  $C_x$ , and  $y$  respectively. The  $x$  in  $C_x$  is circled in yellow.

$$\cdot F_w \rightarrow \forall w G_w$$

Diagram showing the binding status of variables in the formula  $F_w \rightarrow \forall w G_w$ . Arrows point from the labels 'free', 'bound', and 'bound' to the occurrences of  $w$ ,  $w$ , and  $w$  respectively.

Note If a variable is in the scope of multiple quantifiers containing that variable, it is the first quantifier added in the construction process that binds it.

Example Example  $\forall x \exists x P x$

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$\forall x \exists x P x$

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- An occurrence of a quantifier is **vacuous** if the variable in the quantifier does not occur free in the scope of the quantifier.

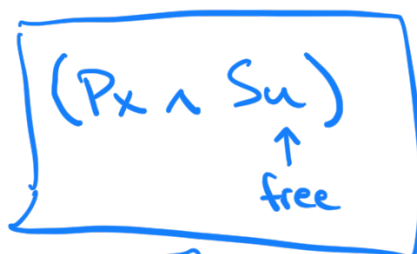


Examples      Vacuous or not?

•  $\forall y$   $Fa$       Vacuous

•  $\exists u$   $(Px \wedge Su)$

Not vacuous

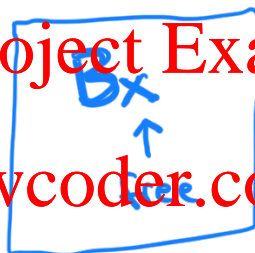


•  $\forall x$   $\forall x$   $Bx$

Vacuous

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Not vacuous



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⑥ Open / closed wffs

- A wff with no free occurrences of variables is called **closed**. A wff with one or more free occurrences of variables is **open**.

Note Both open and closed wffs are well-formed,  
but **open** wffs are **not propositions**. That is,  
open wffs are not truth-apt.

Example      $T_x$      "It is tall"

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