Lecture 9: Birth - Death Chain

$$\Gamma_{3} = \frac{g_{1} \cdots g_{4}}{g_{1} \cdots g_{5}}, \quad 5 \ge 1 = \frac{g_{1}}{g_{1}} \Gamma_{3}$$

$$u(x) = P_{x}(T_{a} < T_{b}) = \frac{g_{1}}{g_{1}} \Gamma_{4}$$

$$\frac{g_{2}}{g_{1}} \Gamma_{4}$$

Let X=1, a=0, b= n for N>1

Assignment Project=Exam Help

https://powcoder.com

Add Warner Then

Add WeChat powcoder

Noting that the Markov chain moves up one unit at a time, it will visit in before not. Hence

1 < T2 < T3 < T4 < - ..

=> Ty 1 increasing and Ty ≥ N-1 Thus I To < Tul is increasing in n. By the continuity of probability, we obtain

$$P_{1}(T_{0} < T_{0}) \rightarrow P_{1}(V_{0} < T_{0} < T_{0})$$

$$= P_{1}(T_{0} < \infty)$$

$$= P_{1}(T_{0} < \infty)$$

$$= P_{1}(T_{0} < \infty)$$

Assignment Project Exam Help

Theorem The imeducible birth — death Markov

https://powcoder.com

chair weducible birth — death Markov

chair beautoper.com

is readd. WeChat powcodor of

Proof: Assume that
$$\frac{2}{3-0}$$
 = ∞ . Then

 $P_{10} = 1 - \frac{1}{\frac{3}{2}} \frac{1}{1} = 1 - 0 = 1$
 $P_{10} = P(T_0 < \alpha) = P(0,0) + P(0,1) P_{10}$
 $P_{10} = P(0,0) + P(0,1) = 1$

By i weducibility, the chain is

recurrent.

Next assume that chain is recurrent. Then we have

Assignment Project Exam Help

thes https://powcoder.com

Add WeChat powcoder

Example 1: Consider the birth-decth chain with S={0,1,2,--} and

 $P_{x} = \frac{x+2}{2(x+1)}$, $e_{x} = \frac{x}{2(x+1)}$ x20

Determine whether the chair is transient or recement.

$$\frac{g_{\chi}}{p_{\chi}} = \frac{\chi}{\frac{\chi+2}{2(\chi+1)}} = \frac{\chi}{\chi+2}$$

$$\frac{1}{x} = \frac{g_1}{p_1} \frac{g_2}{p_2} \dots \frac{g_x}{p_x}$$

$$= \frac{1}{p_1} \frac{g_2}{p_2} \dots \frac{g_x}{p_x}$$

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder

$$= \sum_{y=0}^{\infty} \frac{1}{y} = 1 + 2 \frac{2}{2} \left[\frac{1}{2+1} - \frac{1}{2+2} \right]$$

$$= 1 + 2 \cdot \frac{1}{2} = 2 < \infty$$

Hence the chain is transient

Branching Markou Chain

consider ce population et individuds

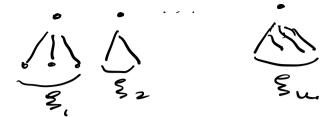
- O Initially at generation of theme one individuals
- ② Each individued at generation of has a randon hunter of offspring at generation 1
- Assignment Project Exam Helpt

 Let 3, denote le Exam Helpt

 offlitips://powcoder.com/
- A Add Weehat powcoder cas
- (5) the Port of 3, 12, f: p(3,=x)=f(x) x=0,1,2,---
- 6 Generation n+1 is generated from generation n in Similar manner.

Generation O

Generation 1









Assignment Project Exam Help

Add WeChat powcoder

Set & = 0

Xu = population 5:30 of generation n

Then

P(Xn+1=9 | Xn=x) = P(3,+ ...+ 3= = y)

is independent of n.

Hence SXn: u=0,12, is a consumer with stationary

Assignment Project Exam Help

Then Sites to the stationary

we https://powcoder.com

Brandd We Chat powcoder

clearly, o is an absenting state.

when $X_n = 0$, the population becomes extinct.

Definition 1: the extinction probability
of the branching chair is $P = P(T_0 < \infty) = P_0$

Sassignment Project Exam Help it
off spring https://powcoder.com

Add We Chat powcoder

Question: What is the value of

$$E[3,] = \frac{\infty}{x} \times f(x) = \frac{\infty}{x=1} \times f(x)$$

$$= \frac{\infty}{x=1} \times p(3,=x)$$

$$= \mu$$

$$\underline{\mathbf{E}}(\mathbf{t}) = \sum_{\mathbf{x}=0}^{\infty} f(\mathbf{x}) e^{\mathbf{x}} \qquad 0 \leq t \leq 1$$

= f(0) + f(1) + f(2) + +

Assignment Project Exam Help The Problem Ley

and the second server of several server serv

Add WeChat powcoder

Theorem 2: Etc)=t has a unigue Solution t=1 if M≤1. If M>1, then B(t)=t has two Solutions octol, and I and to so of from so.

In the seguel, we assume that

$$\Xi(t) = f(x) + f(x)t + f(x)t^{2}t^{2}$$

$$\Rightarrow \Phi(x) = f(x), \quad \Xi(x) = f(x)t^{2}t^{2}$$

$$= 1$$

In addition, But, is

non-decreasing in t. In other

word Fit, 1 & Date, of t, st.

Assignment Project Exam Help

phttps://powleoder.tom Probability
geneadt WeCharpoweoder be cause

$$E(0) = f(0)$$

$$E(t) |_{t=0} = f(1)$$

$$E(t) |_{t=0} = 2! f(2)$$

$$E(t) |_{t=0} = n! f(n)$$

Proof: First we show that P = \P(P)

By definition,

Assignment Project Exam Help. $\tau > 0$ https://power.com/ (To 200)

Add WeChat ppwcoder

Noting that

p(1, y)= P(8,=y)=f2y)

it follows that

P = fco, + = fey, py = E(P)

The if MEI, then P=1
is the unique solution cat

But 1=t.

Now we assure that u>1.

For any nzo, we have

 $P(T_{0} \leq n+1) = P(X_{0} = 0, T_{0} \leq n+1)$ $Assignment Project Exam Help (X_{0} \neq 0, T_{0} \leq n+1)$

https://powcoder.com

= Add WeChat powcoder

= $P(1,0) + \frac{20}{3}P(1,y)(P_1(T_0 \leq 4))^{\frac{1}{3}}$

= from + \frac{\infty}{2} fey, (P, (To \infty))

= \(\mathbb{P} \left(\mathbb{P}_0 \left(\mathbb{T}_0 \left(\mathbb{n} \right) \right)

Noting that

P(To so)=0

it follows that

Since Detris hou-decreasing int it follows that

$$P(T_0 \leq 1+1) = \bigoplus (P(T_0 \leq 1))$$

$$\leq \bigoplus (+_0) = +_0$$

Assignment Project Exam Help

https://powcoder.com

Company

Add WeChat powcoder

Letting u sa, we obtain

Assignment Project Exam Help

https://powcoder.com

Add WeChat powcoder