7.1 $R_1 \cap R_2 = A$

$A \to BC \Rightarrow A \to B$ and $A \to C$

$A \to B$ and $B \to D \Rightarrow A \to D$

$A \to C$ and $A \to D \Rightarrow A \to CD$ and $CD \to E \Rightarrow A \to E$

$A \to E$ and $E \to A \Rightarrow A \to A \Rightarrow A \to ABCDE$

$E \to A \Rightarrow E \to ABCDE$

$CD \to E$, $E \to ABCDE \Rightarrow CD \to ABCDE$

$B \to D \Rightarrow BC \to CD \Rightarrow BC \to ABCDE$

又 $\because$ $R_1 = (A, B, C)$ $R_1 \cap R_2 = A$

$R_2 = (A, D, E)$ and A is a candidate key

$\therefore$ $R_1 \cap R_2 \to R_1$

7.13 F_1 is F to (A, B, C)

F_2 is F to (C, D, E)

$B \to D$ to be preserved: FD $B \to \alpha$ in F_1^+ and $\alpha \to D$ in F_2^+

$\because$ $R_1 \cap R_2 = A$, $\alpha = A$ $B \to A$ is not in F_1^+

$\therefore$ not a dependency-preserving decomposition

7.21 $\{(A, B, C, E), (B, D)\}$ which fill $B \to D$ in 7.13

7.22 $\{(A, B, C), (C, D, E), (B, D), (E, A)\}$

(A, B, C) contains a candidate key. So R' is a third normal from dependency-preserving lossless-join decomposition

R is in third normal form. So the original schema is a lossless join, dependency-preserving decomposition.

7.30 a. $B \to BD$ $BD \to ABD$ $ABD \to ABCD$ $ABCD \to ABCDE$

$\therefore B^+ = ABCDE$

b. $A \to BCD \Rightarrow A \to ABCD$

$BC \to DE \Rightarrow ABCD \to ABCDE$ $\Rightarrow A \to ABCDE \Rightarrow AG \to ABCDEG$

c. with D a extra in dep. 1 and 2 because of 3.

we can get

$A \to BC$
$BC \to E$
$B \to D$
$D \to A$

and from problem a, we get $B^+ = ABCDE$ the FD $B \to E$ can be determined from this set. So C is extra in the third depedency:

$A \to BC$
$B \to DE$
$D \to A$

d every FD gives rise to its own relation, giving

$r_1(A, B, C)$
$r_2(B, D, E)$
$r_3(D, A)$

Because G is not dependent on any attribute. So G must be a part of every superkey. And none of above have G. So creating new relation $r_4(A, G)$

e. $r(A, B, C, D, E, G)$

$r_1(A, B, C, D)$ $r_2(A, E, G)$

because $A \to E$ is an FD in G^+

$\therefore$ $r_1(A, B, C, D)$, $r_2(A, G)$, $r_3(A, E)$