lec8_proofs_with_quantifiers

inductions

for the format of $\forall x, A(x)$

proof $\exists x, A(X)$

1. first solution, find a x that satisfy A(X)
   but this solution sometimes we cannot find an exact x that satisfy A(X).

for example, when we want to find a solution x that make a cube equation equal 0, we just prove that there exists a x that make the equation equal 0.

2. use indirect ways to prove -- proof by cases

Example

theorem: There are irrationals r,s such that r^s is rational

consider the $\sqrt{2}^{\sqrt{2}}$

if $\sqrt{2}^{\sqrt{2}}$ is rational, take r,s = $\sqrt{2}$

if $\sqrt{2}^{\sqrt{2}}$ is irrational, take r = $\sqrt{2}^{\sqrt{2}}$, s = $\sqrt{2}$, then

$r^s = \sqrt{2}^{\sqrt{2}^{\sqrt{2}}} = \sqrt{2}^{\sqrt{2}\cdot\sqrt{2}} = \sqrt{2}^2 = 2$

prove $\forall x, A(X)$

1. prove for all x, A(x) is correct

> to prove $\forall x \exists y, y > x^2$
> let n be an arbitrary number, set m = $n^2 + 1$, then
> $m = n^2 + 1$ always larger than n^2

2. assume $\neg \forall x, A(x)$, derive a controdictary

Induction

An example for induction

theorem: $1 + 2 + \cdots + n = \frac{1}{2}n(n+1)$
n = 1, n = 2, n = 3

> theorem: to prove $\forall x, A(x)$
>
> 1. A(1) satisfy, initial step
> 2. $(\forall n)$[A(n) $\implies$ A(n+1)] (induction step)

Example

theorem: if x > 0, then for any n, $(1+x)^{n+1} > 1 + (n+1)x$
proof: By mathematical induction, let A(n) be the statement
$(1+x)^{n+1} > 1 + (1+n)x$

1. A(1) is the statement $(1+x)^2 > 1 + 2x$, obviously, it's true
2. to prove $\forall n[A(n) \implies A(n+1)]$, assume that A(n)and deduce A(n+1).
   $(1+x)^{n+2} = (1+x)(1+x)^{n+1} > \cdots > 1 + (n+2)x$

Example

> theorem: every natural number grater than 1 is either prime or a product of prime.
> proof: by induction. the induction statement is :
> ∀ m,[$2 <= m <= n \implies$ m is either a prime or a product of prime.
>
> 1. for n = 2, A(2) is a prime
> 2. assume A(n), let m = n + 1