$$J[u] = \int_0^{t_f} W\left(u(t) - \max\{0, \sin(\alpha t)\}\right)^2\, dt - \exp\left(\frac{-\left(y(t_f) - \frac{(P_l+P_u)}{2}\right)^2}{\sigma^2}\right)$$

$$\mathbf{x}(t) = \begin{bmatrix} y(t) \\ y'(t) \end{bmatrix} \quad \mathbf{x}'(t) = \begin{bmatrix} y(t) \\ y'(t) \end{bmatrix}' = \begin{bmatrix} y'(t) \\ u(t) - 9.8 \end{bmatrix} \quad \mathbf{x}(0) = \begin{bmatrix} y_0 \\ 0 \end{bmatrix} \quad \mathbf{x}(t_f) = \begin{bmatrix} \text{free} \\ 0 \end{bmatrix}$$

$$H = \mathbf{p} \cdot \mathbf{f} - L = p_0 y' + p_1(u - 9.8) - (u(t) - \max\{0, \sin(\alpha t)\})^2 \tag{1}$$

$$\begin{aligned} p_0' &= -\frac{\partial H}{\partial y} = 0 \\ p_1' &= -\frac{\partial H}{\partial y'} = -p_0 \end{aligned}$$

$$p_0(t_f) = -\frac{\partial \phi}{\partial y(t_f)} = \left(\frac{-1}{\sigma^2}\right)(2y(t_f) - (L+U)) \exp\left(\frac{-\left(y(t_f) - \frac{(P_l+P_u)}{2}\right)^2}{\sigma^2}\right)$$

$$p_1(t_f) = \text{free}$$

$$\frac{\partial H}{\partial u} = 0 = p_1 + 2(u(t) - \max\{0, \sin(\alpha t)\} \implies \tilde{u} = W\left(\max\{0, \sin(\alpha t)\} - \frac{p_1}{2}\right)$$