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Accuracy Assessment of Jason-1 and TOPEX/POSEIDON Along-Track Sea Surface Slope

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Sea surface slope computed from along-track Jason-1 and TOPEX/POSEIDON (T/P) altimeter data at ocean mesoscale wavelengths are compared to determine the equivalent 1 Hz instrument height noise of the Poseidon-2 and TOPEX altimeters. This geophysical evaluation shows that the Ku-band 1-Hz range noise for both instruments is better than 1.7 cm at 2 m significant wave heights ($H_{1/3}$), exceeding error budget requirements for both missions. Furthermore, we show that the quality of these instruments allows optimal filtering of the 1-Hz along-track sea surface height data for sea surface slopes that can be used to calculate cross track geostrophic velocity anomalies at the baroclinic Rossby radius of deformation to better than 5 cm/sec precision along 87.5% of the satellite ground track between 2 and 60 degrees absolute latitude over the deep abyssal ocean (depths greater than 1000 m). This level of precision will facilitate scientific studies of surface geostrophic velocity variability using data from the Jason-1 and T/P Tandem Mission.

Keywords altimetry, geostrophic velocity, mesoscale

Weeks after launch on 7 December 2001, Jason-1 was positioned in the operational orbit to sample the ocean surface within the 1 km nominal mission ground track 72 s ahead of the T/P satellite. The two satellites remained in this configuration during the first eight months of 2002 until T/P was moved into an interleaved ground track to begin the Tandem Mission. Direct comparison of Jason-1 and T/P data collected during the verification phase permits geophysical evaluation of the on-orbit performance of the two altimeter systems. The oceanographic signal measured by the two satellites is nearly identical because the ocean state changes very little over the short time between sampling, allowing direct, on-orbit assessment of the engineering systems and subsequent scientific analyses.

In this article, we evaluate the on-orbit performance of the Topex and Poseidon-2 altimeters onboard T/P and Jason-1, respectively, by comparing sea surface slopes at ocean mesoscale wavelengths computed from the 1 Hz height measurements. The white noise levels of the altimeter instruments are quantified by the statistical differences between the

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tandem measurements of the along-track sea surface slopes. Because the measured slope is used to calculate geostrophic velocity variations at the ocean surface, the quality of these instruments will directly affect the accuracy of geophysical studies using Tandem Mission data where Jason-1 and T/P ground tracks are interleaved to improve ocean mesoscale sampling (Fu et al. 2003).

All radar altimeters are affected by internally generated white noise that affects the accuracy of slope estimates of the ocean mesoscale such as eddies, meanders, squirts, and jets. This instrument noise dominates errors at mesoscale wavelengths, because the finite difference operation on the along-track height measurements to compute the derivative acts as a high pass filter, enhancing the high wavenumber component of the noise. This artifact of differentiation is obvious if we consider the propagation of an uncorrelated error in height, σ_h, through a finite difference formula for the derivative. A general centered finite difference formula for computing the cross-track geostrophic velocity, v, from 1 Hz sea surface height (SSH) measurements, h_i, at sample location, i, along the altimeter ground track is

$$v_i = \frac{g}{fL}\frac{(h_{i+N} - h_{i-N})}{2N}, \tag{1}$$

where g is the acceleration due to gravity, f is the Coriolis parameter, and L is the distance between 1 Hz measurements (L $\approx$ 5.75 km for T/P and Jason-1); so that $2NL$ is the span over which the finite difference is calculated. If we consider the propagation of σ_h, through (1), the geostrophic velocity error, σ_v, is given by (Strub et al. 1997)

$$\sigma_v = \frac{\sqrt{2}g}{f}\frac{1}{2NL}\sigma_h. \tag{2}$$

The altimeter-derived geostrophic velocity error associated with instrument noise increases as a function of the "wavenumber,"$1/(2NL)$, associated with the finite difference span. This is a direct consequence of the derivative theorem of Fourier analysis which states that taking a derivative of a function multiplies its transform by the wavenumber (Bracewell 2000), a result which holds for any spectrum whether colored or white. For example, evaluating Equation (2) with values of f at mid-latitudes (30 to 40 degrees) over three sample points ($N = 2; 2NL \sim 11.5$ km) gives velocity noise from 20 to 30 cm/s for the budgeted instrument range noise equal to 1.7 cm at 2 m $H_{1/3}$ for 1 Hz sampling by the Ku-band instruments onboard the T/P and Jason-1 satellites. This instrument noise, however, can be reduced to manageable levels by differencing over larger intervals. Finite differencing over larger point-to-point intervals is equivalent to running average smoothing of the geostrophic velocities calculated at the midpoints between successive SSH samples (Strub et al. 1997). A variety of along-track smoothing and estimation techniques have been used to mitigate the range noise contribution to velocity errors for the study mesoscale geostrophic velocity signals (Morrow et al. 1994; Ohlmann et al. 2001; Strub et al. 1997). Recently, we have developed optimal filters with error estimates to calculate along-track slope in the presence of white noise (Powell and Leben submitted) that will be utilized in this study to examine the noise floor the Topex and Jason-1 altimeters.

For absolute height accuracy, path length and environmental corrections (i.e., ionosphere, troposphere, sea state bias, inverted barometer, and tides) are typically applied to the range measurements. These corrections are unnecessary if one is interested in slopes over short-length scales. This has been well known in the marine gravity community (Yale et al. 1995), where sea surface slopes are used to calculate short wavelength variations in the gravity field for evaluating anomalies and estimating bottom topography. In particular, Yale (1997) shows that no corrections are needed in deepwater (depths greater than 200 m) by carefully examining the impact of each altimetric correction on the accuracy of slope estimates for marine gravity studies. We came to a similar conclusion in our study of the impact of slope corrections on accuracy of altimeter-derived geostrophic velocity estimates for mesoscale ocean studies (Powell and Leben, submitted) based on global comparisons of the RMS power associated with each correction. Slope databases prepared for mesoscale studies, therefore, can be based on simple orbit minus range estimates for the height, eliminating any errors arising from the differences between the path length corrections for the two satellites. These slope data sets are also ideally suited for study of the errors attributed to the range precision of the altimeter instrument.

In this article, we use Jason-1 and T/P sea surface slope databases, computed with an optimal filter designed to resolve the ocean mesoscale, to evaluate the noise floor of the respective altimeter instruments. Data and methods are covered in the section below followed by results and discussion concerning the performance of the instruments in the next section. In the final section, we conclude with a summary of the performance of the altimeter instruments and give some general recommendations with regards to the study of surface geostrophic circulation variability using precision altimetry.

Data and Methods

A brief overview of the data sets, data processing, and statistical analysis we have used to investigate the altimeter instrument noise is given in this section. We note that careful editing and robust filtering of the data in the presence of data outages and bathymetric editing was necessary to ensure accurate noise estimates.

Jason and T/P Data Archives

Full stack file archives, in which all of the respective fields in the geophysical data records (GDRs) are available online for analysis, were created from the Jason Interim GDRs, the merged T/P GDRs (Benada 1997), and the T/P GDR correction product (Callahan 2002). The T/P GDR correction product (GCP) is based on a waveform retracking of the Topex altimeter waveform returns using the method of Rodrìguez and Martin (1994) to provide a data set of corrections to the GDR range measurements and new environmental corrections for comparison with the coincident Jason-1 verification data. The T/P GCP data is often referred to as “retracked” Topex data.

Altimeter data at valid ocean points were extracted from the stack archive, and have Ku-band range RMS less than 0.15 m and pointing angles less than 0.04 degrees. A valid range measurement also required at least 75% of the original high rate data (10 Hz for TP and 20 Hz for Jason-1). After this minimal editing, SSH with respect to the mean sea surface was computed as:

$$h_i = H - R - h_{mss}, \tag{3}$$

where H is the orbit height, R is the range to the sea surface measured by the Ku-band altimeter, and h_{mss} is the Goddard Space Flight Center (GSFC) mean sea surface height at the subsatellite point (Wang 2001).

The validation phase Jason cycles (3–21) and the corresponding T/P cycles (346–364) were used for analysis. The T/P Poseidon-1 altimeter was turned on during T/P cycle 361, so that cycle was not used. Also, the first two cycles of Jason-1 were not reliable and were eliminated from this study.

Along-Track Data Processing

SSH anomaly values were interpolated to a 1 Hz reference gound track based on a ground track computed for T/P cycle 18. Simple linear interpolation/extrapolation was used to determine the value at a reference ground track point from the two nearest subsatellite points within 12 km, producing collinear databases minimally affected by along-track interpolation and/or smoothing.

An optimal difference operator (Powell and Leben submitted) was used to compute along-track slopes (in units of mm/s, the vertical velocity of the surface as viewed from the satellite) from the collinear along track SSH anomaly values. This filter is optimal in the sense that it minimizes the white noise contribution to the slope error for a given window of data points. An interesting feature of this differencing technique is that an equivalent smoothing kernel for the along-track height data before application of a consecutive point difference equation can be derived for the optimal solution. The kernel for a 15-point ($\approx$80 km) smoothing window is shown in Figure 1, along with the corresponding filter gain as a function of frequency and the half-power point (wavelength $\approx$107 km). The 15-point filter has a corresponding slope error, σ_{slope}, of 0.06 s^{-1} σ_h. A sample derivation of the filter coefficients for a 5-point filter is given in the Appendix.

Data sets were generated using a variety of filter lengths for centered windows with no missing data to ensure zero phase response of the filter over varying ocean length scales. Before filtering, edited or missing data values in each dataset were flagged at the corresponding locations in the other datasets. Bathymetric editing also checked ocean depths over the full filter windows not just at center points of windows. This further ensures that statistics analysis can be performed without filtering artifacts affecting the results.

Statistical Analysis

We performed several exploratory studies to find a suitable method for editing outlying data points that could adversely affect the noise calculations. Accurate estimation of the noise requires a robust estimate of both the standard deviation and mean of the distributions of satellite measurements and their differences. A robust estimate of standard deviation (Iglewicz 1983), the Median Absolute Deviation (MAD), was used to check outlier editing of the original data and its impact on computed RMS values. We found that MAD values

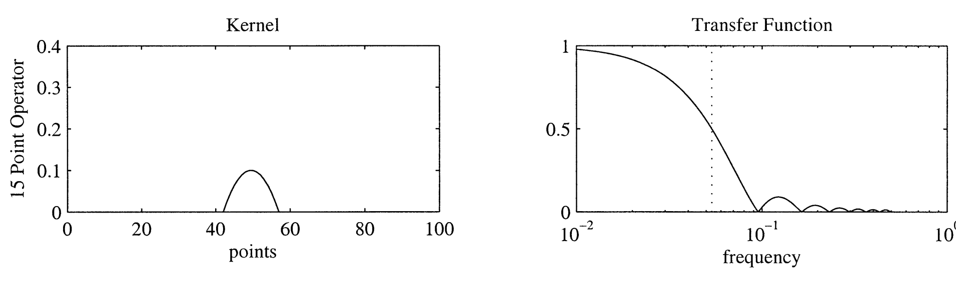

FIGURE 1 Characteristics of the 15-point optimal derivative filter. Shown on the left is the equivalent filter weightings on the 1 Hz height measurements. The length of the 15-point operator for T/P and Jason-1 is about 80 km. On the right is the transfer function with the half-power location at 107.5 km.

computed from the unedited distributions closely matched RMS values computed from distributions with the extreme 10% tails of the distributions removed. The edited distributions statistics were preferred because the calculated values were not affected by quantization error as were the robust estimates based on median values. We did not require a robust estimate of the mean because the anomalies were calculated relative to the GSFC mean sea surface which appropriately centered the distribution. Any systematic errors in the mean sea surface were shared "signal" observed by the two satellites and did not affect the noise estimate. Some short wavelength geolocation errors remain because of the cross-track offset between the two satellite orbits within the 1 km nominal ground track. These differences are not removed by the implicit mean sea surface gradient correction due to fine-scale errors in the mean sea surface.

Results and Discussion

The data sets and along-track processing described above were used to calculate along-track slope data sets for ocean depths greater than 1000 m. RMS statistics were used to calculate the slope noise and the equivalent 1-Hz range noise for the Poseidon-2 and Topex instruments. We also examined the variation of the noise floors with $H_{1/3}$. The precision of cross-track velocities at mesoscale wavelengths associated with the on-orbit range precision were assessed to provide a lower bound on the global accuracy attainable with these precision instruments. The results are described and discussed in the following sections.

Noise Calculations

The variance of a population is given by $\sigma^2 = \langle (x - \mu)^2 \rangle$, where x is the variate, μ the known population mean, and $\langle\ \rangle$ the expected value. For a measured signal, the standard deviation, σ, is a measure of the precision of the instrument. An unbiased estimator of the variance of the discrete distribution is the sample variance:

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2. \tag{4}$$

Assuming that the measurements collected are a combination of only signal variance and measurement noise variance, the sample variance for the two satellites may be written as

$$\sigma^2(\text{Topex}) = \sigma^2(\text{signal}) + \sigma^2(T_{\text{noise}}), \tag{5}$$

$$\sigma^2(\text{Jason}) = \sigma^2(\text{signal}) + \sigma^2(J_{\text{noise}}), \tag{6}$$

where $\sigma(T_{\text{noise}})$ and $\sigma(J_{\text{noise}})$ are the respective noises in the Topex and Poseidon-2 instruments. Since the oceanographic signals are nearly identical in the coincident measurements this is equivalent to

$$\sigma^2(\text{Topex}) - \sigma^2(\text{Jason}) = \sigma^2(T_{\text{noise}}) - \sigma^2(J_{\text{noise}}). \tag{7}$$

We can also take the expected value of the square of the differences of the two coincident measurements, which gives

$$\sigma^2(\text{Topex} - \text{Jason}) = \sigma^2(T_{\text{noise}}) + \sigma^2(J_{\text{noise}}). \tag{8}$$

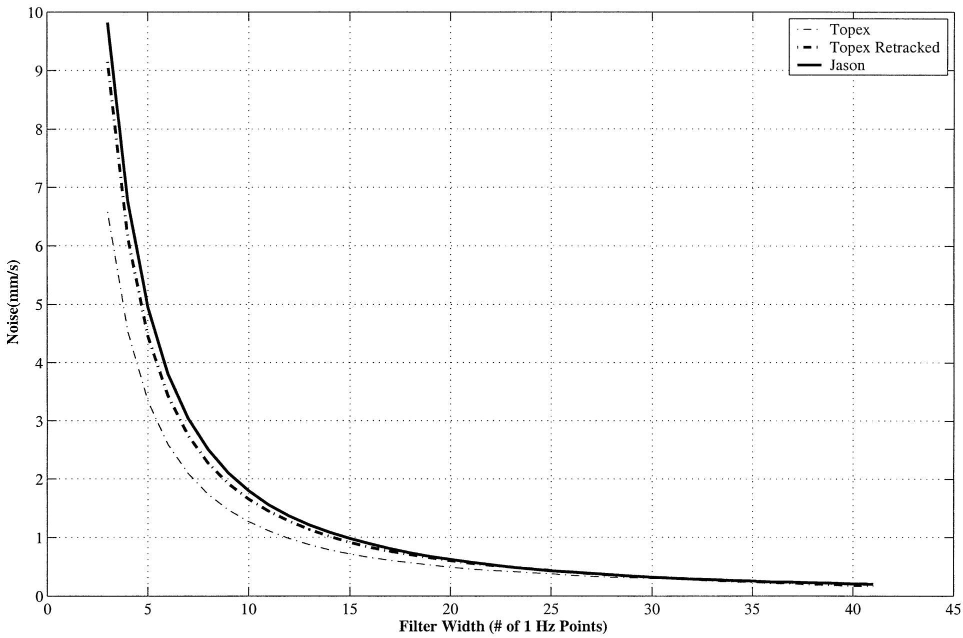

FIGURE 2 Using a single cycle of T/P (355) and Jason-1 (12) data, the alongtrack slope noise of varying filter lengths are computed. The reduction in the white, along-track slope noise is consistent with the error estimate provided by the optimal solution.

Using Equations (7) and (8), we can solve for $\sigma(T_{\text{noise}})$ and $\sigma(J_{\text{noise}})$, the errors in the respective measurements. The variance needed for these error estimates is computed from the along-track slope (using trimmed distributions as described previously) from the Jason IGDR, T/P GDR, and T/P GCP data sets filtered with window widths ranging from 3 through 42 pts (11.5 km to 236 km windows; half-power points from 17.3 km to 301.2 km). The estimated slope errors plotted for each data set as a function of filter width is shown in Figure 2 in units of mm/s (along-track height change between 1-Hz sampling points). Given the latitude of a measurement, these values can be converted to cross-track geostrophic velocity errors by multiplying by $g/(fL)$. The error falls off rapidly as a function of increasing points in agreement with the error analysis provided during the design of the optimal filter. More details concerning the performance of the instruments can be gleaned by using the computed slope errors and optimal filtering error analysis to back out an estimate of the equivalent 1-Hz SSH noise.

Equivalent 1-Hz SSH Noise

We estimate the equivalent 1-Hz noise corresponding to the altimeter instrument's range precision from the slope error using the optimal derivative filter error estimate (see the Appendix for more details). A scaling factor, which is a function of window size, allows conversion between slope error and height error, and vice versa. Scale factor values are tabulated in Powell and Leben (submitted) for selected filter widths.

In Figure 3, we show the estimates for data from Jason cycle 12 and T/P cycle 355 with T/P measured $H_{1/3}$ values between 1.5 m and 2.5 m. In the idealized case of only white noise, the equivalent 1-Hz estimate should match spectral estimates using high rate data and be relatively constant across window width. This is the result we find for our estimates

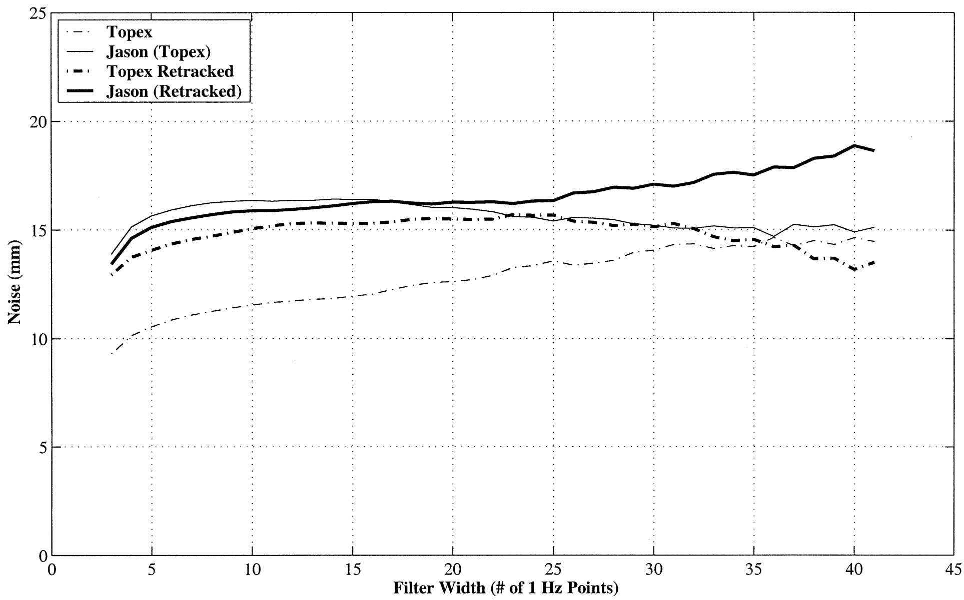

FIGURE 3 Using a single cycle of T/P (355) and Jason-1 (12) data, the equivalent 1-Hz noise from the slopes of varying filter lengths is computed. Jason noise is computed relative to TOPEX and relative to the Retracked dataset (as signified in the legend). The effect of T/P GDR ground processing to correct for on-board tracking errors is evident by the significant variation with filter width as shown in the Topex data. The equivalent 1-Hz values are consistent between filter lengths of 5 to 25 for T/P Retracked and Jason estimates.

from the Poseidon-2 and retracked Topex measurements over the midrange of filter widths. The original Topex GDR data, however, gave estimates that varied substantially over the entire range of filter widths examined, which is consistent with the "coloring" of the range noise associated with problems with the on-board tracking algorithm (Rodríguez and Martin 1994) and additional processing over 3-s frames (Callahan 1991) in ground processing. In the rest of this article, we will focus on the results derived from the Poseidon-2 and retracked Topex data.

Our 1-Hz noise floor estimates compare well with estimates based on spectral analysis of Topex and Poseidon-2 data. The values at fixed filter widths were also very stable across cycles (Figure 4). The mean and standard deviation of the mean over all validation phase cycles was 1.67 $\pm$ 0.08 cm for Poseidon-2 and 1.52 $\pm$ 0.09 cm for Topex (15 point filter; 1.5 to 2.5 m $H_{1/3}$). These results compare favorably with the spectral estimates of 1.7 cm (Carayon et al. this issue) and 1.5 cm (from Figure 5 in Fu et al. 1994) at 2 m $H_{1/3}$ reported for the Poseidon-2 and Topex instruments, respectively.

The equivalent 1-Hz noise estimated for windows less than 10 points and greater than 25 points merits further discussion. At less than 10-points, the unrealistic decrease in the noise floor estimate may be an artifact of georeferencing, collinear interpolation, and/or the decreasing signal to noise (Figure 5). Shared signal between the two instruments decreases below the noise level at around 9-point filtering, which corresponds to 46-km filter width with a 64-km half-power point. Since spectral estimates of the noise floor do not show a decrease at shorter wavelength, the most likely source of the decreasing error estimate with shorter window widths is the smoothing of the data that occurs when interpolating and/or extrapolating the original subsatellite data to the reference ground track. We hope

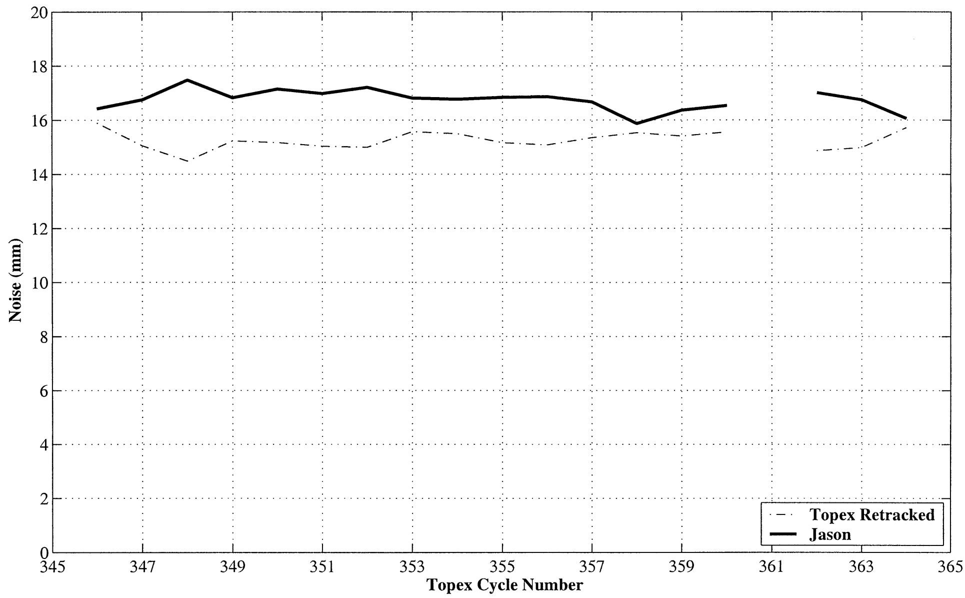

FIGURE 4 Utilizing the 15-point optimal slope, the equivalent 1-Hz noise is computed as a function of the cycles studied (T/P: 346–360, 362–364 and Jason-1: 3–17, 19–21; skipping the Poseidon-1 cycle 361). The mean and standard deviation of the mean is 1.52 ± 0.09 cm for T/P and 1.67 ± 0.08 cm for Jason-1. These results are consistent with the spectral analysis of the high-rate data from each satellite.

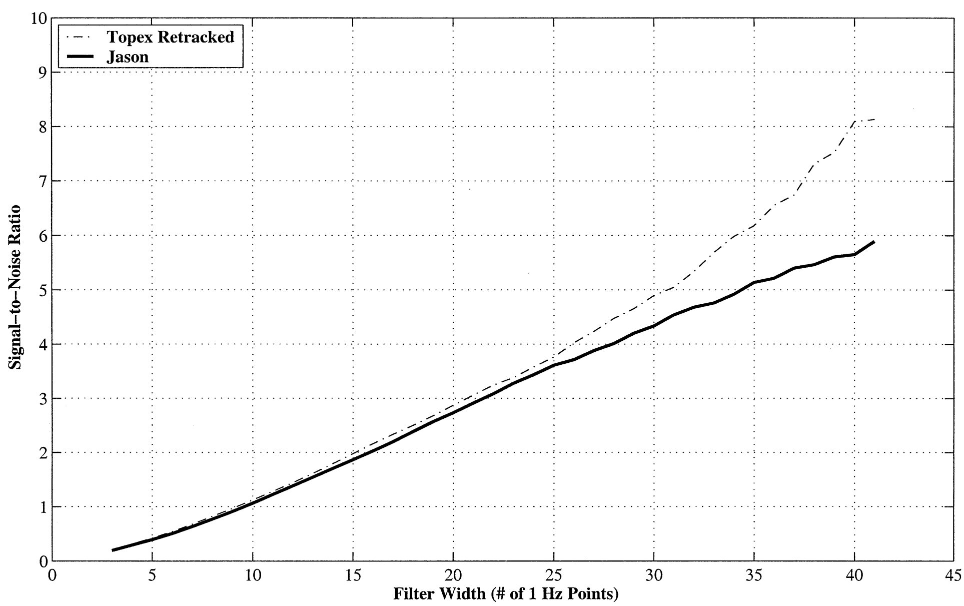

FIGURE 5 The equivalent 1-Hz signal-to-noise ratio from a single cycle of T/P (355) and Jason-1 (12) is shown for varying filter lengths. Although the amount of noise removed by the optimal slope is not linear, the signal-to-noise ratio is. A filter length greater than 10 points is required to resolve at least as much signal as noise.

to clarify this discrepancy in studies with collinear 10 Hz databases being developed. The diverging estimates of the 1-Hz noise floor for greater than 25-point windows indicate that there may be correlated differences between the two measurements systems data at longer wavelengths. The source of these differences should be investigated when retracked Jason-1 GDR data are available to determine if there are systematic orbit or instrument errors affecting the slope measurements. It is also possible that this is an artifact of our filtering since the divergence was much more pronounced before we ensured zero phase response of the filters by consistent editing of the T/P and Jason-1 data.

Equivalent 1-Hz Noise Floor as a Function of Significant Wave Height ($H_{1/3}$)

Only about one-third of all Jason and T/P altimeter data are collected over ocean points where $H_{1/3}$ is between 1.5 and 2.5 m. Increasing $H_{1/3}$ increases the noise floor on the altimeter range through the effect of the random wave field on the returned power and the tracking algorithms. To investigate this instrument artifact, the equivalent 1-Hz noise floor as a function of $H_{1/3}$ for the two instruments was calculated from Jason cycle 12 and (T/P cycle 355) as a function of measured Topex GDR $H_{1/3}$ values using databases based on the 15-point window filter (Figure 6). The trends and specific values are once again in good agreement with spectral estimates for Poseidon-2 (2.3 cm for $H_{1/3}$ of 4 m; Menard et al. this issue) and Topex (2.0 cm for $H_{1/3}$ of 4 m; from Figure 5 in Fu et al. 1994).

Resolving the Mesoscale Signal and the Associated Errors

A characteristic length scale associated with mesoscale circulation in the ocean is the baroclinic Rossby radius of deformation (Chelton et al. 1998). This length scale is associated with baroclinic instability, which is the dominant eddy generation mechanism in the global ocean (Stammer 1997). The pronounced decrease of this length scale with increasing latitude

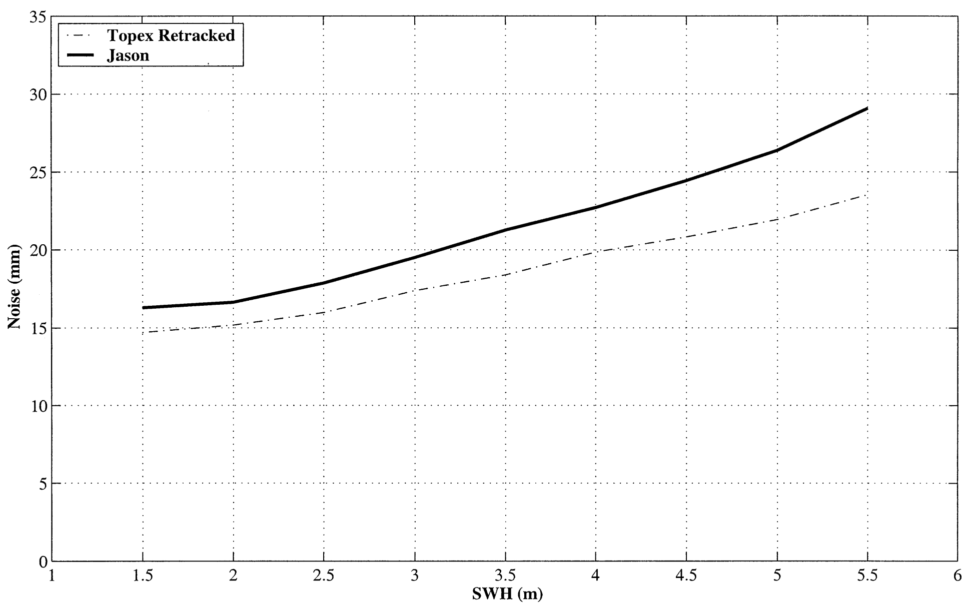

FIGURE 6 Utilizing the 15-point optimal slope, the equivalent 1-Hz noise as a function of Significant Wave Height ($H_{1/3}$) is shown over all cycles studied.

requires significant range precision from a satellite altimeter for study of mesoscale surface currents because of the commensurate increase in the noise floor associated with derivatives across these scales. Also, the variation of the Coriolis parameter with latitude causes the geostrophic velocity noise floor to vary as a function of $1/f$ (for fixed filter widths), which increases the noise floor at lower latitudes. The impact of these variations on velocity accuracy was a motivating factor behind our derivation of our derivative filter, a filter that is both compact and optimal.

The altimeter's range precision directly affects the accuracy of cross-track geostrophic velocity, especially at the ocean mesoscale. We examined this error source by converting the budgeted Ku-band range precision (1.7 cm at 2 m $H_{1/3}$) to equivalent geostrophic velocity error for the optimal derivative filter with a half-power length scales equal to twice the Rossby radius of deformation. The Rossby radius values were interpolated along the altimeter ground tracks from the 1 gridded field prepared by Chelton et al. (1998). We find the cross-track velocity error estimates to be less than 5 cm/s along 87.5% of the satellite ground track between 2 and 60 degrees absolute latitude over the deep abyssal ocean (depths greater than 1000 m) where the dynamics and scales of motion considered are dominant. This precision is a lower bound on the attainable accuracy since the absolute accuracy will depend on the magnitude of the slope error arising from neglected pathlength corrections and/or errors in the applied pathlength corrections, which were not considered in the present study. This level of precision exhibited by these state-of-the-art altimeter instruments, however, will facilitate scientific studies of surface geostrophic velocity variability using data from the Jason-1 and T/P Tandem Mission.

Conclusions

We have shown that the 1-Hz instrument noise of the Poseidon-2 and TOPEX altimeters is better than 1.7 cm at 2 m $H_{1/3}$, exceeding error budget requirements for both missions. The mean and standard deviation of the mean overall validation phase cycles was 1.67 ± 0.08 cm for Poseidon-2 and 1.52 ± 0.09 cm for Topex, based on our analysis using a 15-point filter and coincident $H_{1/3}$ values between 1.5 m and 2.5 m. These results are in good agreement with spectral estimates for T/P and Jason based on short arc analysis of high rate data.

All radar altimeters are affected by internally generated white noise that must be mitigated during processing of the data for oceanographic studies. This is especially true for studies of geostrophic velocities at mesoscale wavelengths, where the derivative of the height measurements enhances both the range and pathlength correction noise floors. Altimeter data processed for absolute height accuracy, therefore, may not be appropriate for studies of geostrophic velocity variability over the significant variations of length scales associated with mesoscale features in the global ocean. Detailed evaluation of the temporal and spatial variation of individual pathlength corrections and their associated errors and correlations are needed to determine what corrections and/or processing should be applied for mesoscale studies. Detailed evaluation of the instrument noise floor and slope filtering is an important first step in this evaluation process. We have shown in this article that it is theoretically possible to compute cross-track geostrophic velocities from both Jason and TOPEX along-track data to better than 5 cm/s RMS at the local baroclinic Rossby radius of deformation for over 87.5% of the points between 2 and 60 degrees absolute latitude at ocean depths greater than 1000 m. More research is needed to determine corrections and processing for along-track slope to approach this level of accuracy; nevertheless, our assessment of the on-orbit performance of the two satellites bodes well for studies of geostrophic velocity variability from the Jason-1 and T/P Tandem Mission.

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Appendix: Optimal Difference Filter

In this article, we utilize a specific, central difference case of the optimal difference operator as derived by Powell and Leben (submitted). Our approach was based upon the derivation of finite-difference equations for the first derivative using Taylor series expansions. In typical finite difference applications, the coefficients found by the Taylor expansions around successive points cancel higher order terms in the series, which minimizes the truncation error in the calculation to provide higher order accuracy. Rather than minimizing truncation error, we solve for a set of coefficients that minimize the white noise propagated through a calculation of along-track slopes. For brevity, the optimal, five point, central difference operator is presented here.

The standard central difference operator for a five-point case (two points on either side of the point of interest) is given by

$$\frac{\Delta h}{\Delta t} = w_1\left(\frac{h_{i+1} - h_{i-1}}{2\Delta t}\right) + w_2\left(\frac{h_{i+2} - h_{i-2}}{4\Delta t}\right), \tag{A1}$$

where h is the height at a given location, Δt is the time difference between each point (in this case, 1 s), and w is the coefficient term for each difference. A constraint that all coefficients must sum to one is placed on the system. As mentioned, a standard finite difference case minimizes the truncation by solving for coefficients that eliminate the neglected higher order terms in the Taylor series expansion. For the given case, the weights are: $w_1 = 4/3$ and $w_2 = -1/3$, providing fourth-order accuracy.

We wish to solve (A1) such that the instrument white noise, σ_h, associated with the height measurements (h_i) is minimized. Because this noise is uncorrelated, the total white noise of the derivative is computed by taking the root of the sum of the squares of the error in each measurement:

$$\sigma_v = \sqrt{\frac{w_1^2}{4\Delta t^2}\left(2\sigma_h^2\right) + \frac{w_2^2}{16\Delta t^2}\left(2\sigma_h^2\right)} = \frac{\sqrt{2}\sigma_h}{4\Delta t}\sqrt{4w_1^2 + w_2^2}. \tag{A2}$$

Using the method of Lagrange Multipliers with the given constraint, a solution of coefficients, w_i, can be found that minimizes this error. In the five-point case, as shown in Equation (A2), the coefficients are found to be $w_1 = 1/5$ and $w_2 = 4/5$.

Substituting the coefficients back into Equation (A2) gives us the noise in the velocities as a function of the noise in the heights. For the five-point case, $\sigma_v = 0.3162(\mathrm{s}^{-1})\sigma_h$. Thus, with our estimate of σ_v, we can convert to the equivalent 1-Hz height noise, σ_h, or vice versa, by the scale factor given.