

Power-Efficient Neural Networks Using Low-Precision Data Types and Quantization

Part 2: Quantization Algorithms Fundamentals



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Google Scholar

- Only small amount of calibration data needed
- Requires little compute and scales well
- Optimization is often on local loss
- Less user effort and little hyper-parameter tuning

- Training on end-to-end task loss
- Requires significant amount of training data and compute
- Higher user effort and can require domain knowledge
- Achieves best accuracy-efficiency trade-off

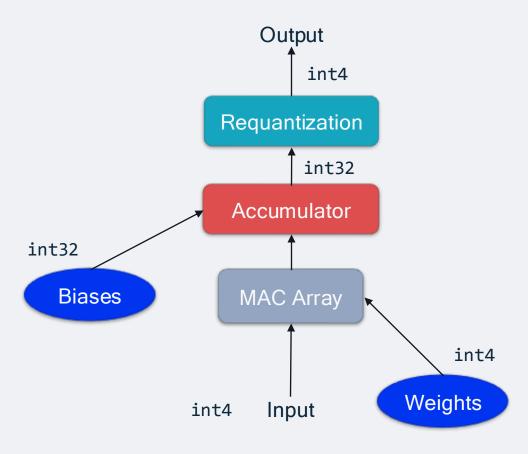
Spectrum

Efficient training

training

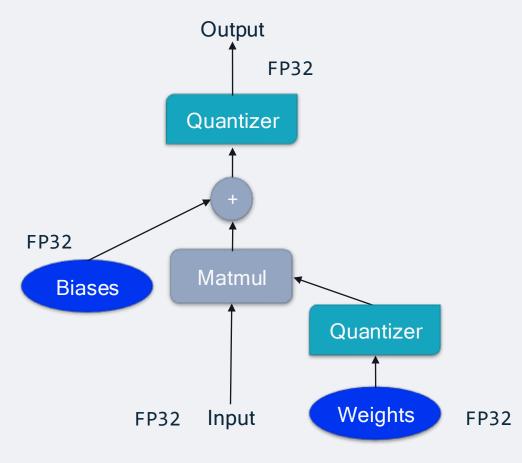
Recap: fixed point quantization

Fixed-point inference



$$Y = WX \approx \frac{S_W S_X}{S_Y} (W_{int} X_{int})$$

Simulated quantization

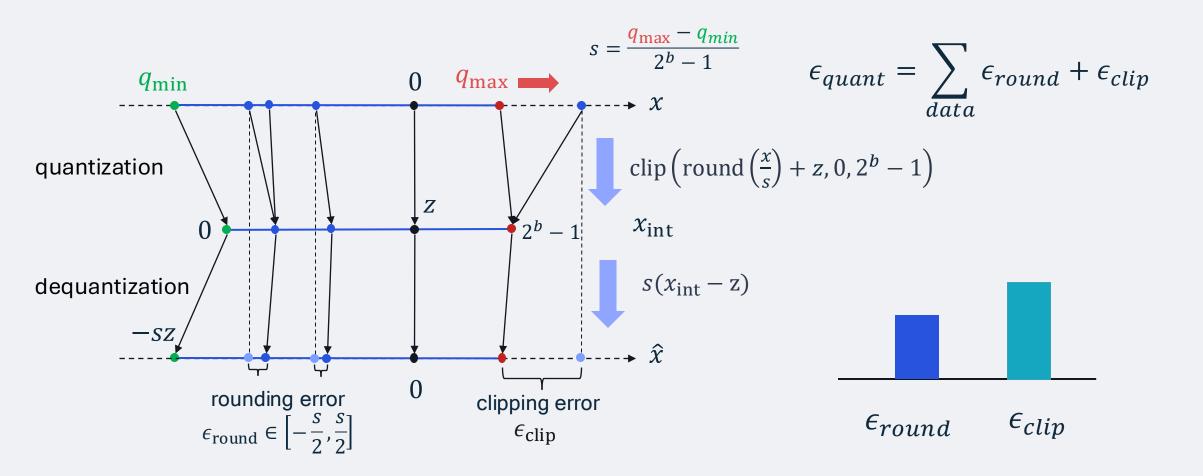


$$Y = WX \approx q(q(W, s_w)q(X, s_x), s_y)$$

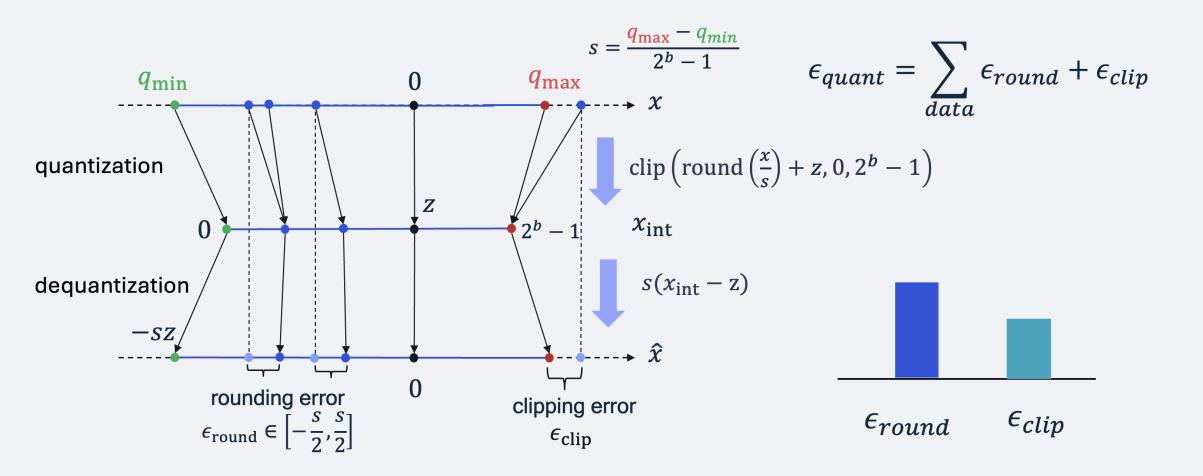
Part 1: Post training quantization



Sources of quantization error



Sources of quantization error



Quantization range setting

• Min-max range:

$$q_{\min} = \min X$$

 $q_{\max} = \max X$

• Optimization-based methods:

$$\operatorname{argmin}_{q_{\min},q_{\max}} \ell\left(X, \widehat{X}(q_{\min}, q_{\max})\right)$$



- Line/grid search
- Golden section search
- OMSE

Common loss functions

- MSE
- KL divergence
- Cross-entropy
- LP-norm (e.g. p={2.4, 3.0, 3.5})

Optimizing weight assignment



Optimizing weight assignment using local loss

Traditionally, in PTQ we use rounding-to-nearest (RTN)

$$W_{\text{int}} = \text{clip}\left(\text{round}\left(\frac{W}{s}\right), n, p\right)$$

• RTN is a natural choice as it is optimal for the local MSE

$$argmin_{W_{int}} ||W - W_{int}s||_F^2$$

• However, is RTN optimal for the task loss?

Rounding-to-nearest is not optimal for task loss

Toy example

• Taylor series expansion of task loss $\mathcal{L}(\mathbf{x}, \mathbf{y}, \mathbf{w})$

$$\mathcal{L}\left(\mathbf{w} + \Delta \mathbf{w}\right) - \mathcal{L}\left(\mathbf{w}\right) \approx \frac{1}{2} \Delta \mathbf{w}^T \mathbf{H}^{(\mathbf{w})} \Delta \mathbf{w}$$

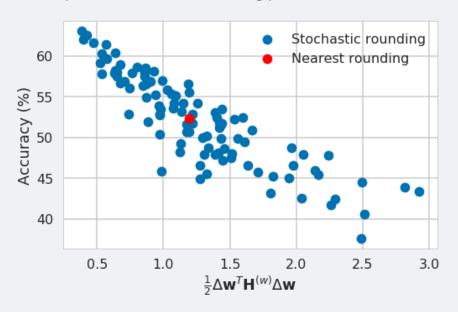
• Assume $\mathbf{H^{(w)}} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$

then
$$\Delta \mathbf{w}^T \cdot \mathbf{H}^{(\mathbf{w})} \cdot \Delta \mathbf{w} = \Delta \mathbf{w}_1^2 + \Delta \mathbf{w}_2^2 + \Delta \mathbf{w}_1 \Delta \mathbf{w}_2$$

- Rounding-to-nearest:
 - Only considers the magnitude of ΔW
 - Ignores all off-diagonal terms of $H^{(w)}$

Empirical example

- We draw 100 stochastic rounding samples
 - Out of space of 2^9408 rounding possibilities



Rounding-to-nearest far from optimal!



Task loss-based weight assignment

$$\underset{\Delta w}{\operatorname{arg\,min}} \quad \mathsf{E}\left[L\left(\mathsf{x},\mathsf{y},\mathsf{w}+\Delta\mathsf{w}\right)-L\left(\mathsf{x},\mathsf{y},\mathsf{w}\right)\right]$$

How can we solve this efficiently?

Taylor series approximation

Problem formulation:

$$\underset{\Delta w}{\operatorname{arg min}} \quad \mathbb{E}\left[L\left(x,y,w+\Delta w\right)-L\left(x,y,w\right)\right]$$

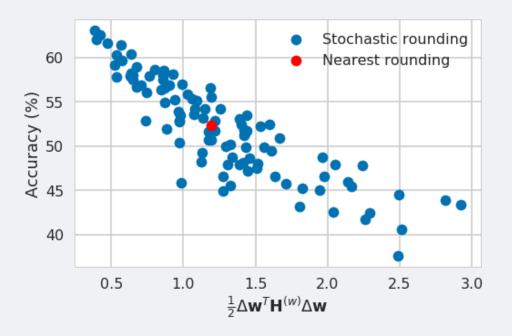
Taylor series expansion of loss

$$\mathbb{E}\left[\mathcal{L}\left(\mathbf{x}, \mathbf{y}, \mathbf{w} + \Delta \mathbf{w}\right) - \mathcal{L}\left(\mathbf{x}, \mathbf{y}, \mathbf{w}\right)\right]$$

$$= \Delta \mathbf{w}^T \mathbf{g}^{(\mathbf{w})} + \frac{1}{2} \Delta \mathbf{w}^T \mathbf{H}^{(\mathbf{w})} \Delta \mathbf{w} + \mathbf{w}^T \mathbf{g}^{(\mathbf{w})} + \frac{1}{2} \Delta \mathbf{w}^T \mathbf{g}^{(\mathbf{w})} + \mathbf{w}^T \mathbf{g$$

- Assumptions:
 - Converged model (ignore gradient term)
 - Small enough $\Delta \mathbf{w}$ (ignore third and higher order terms)
- Final optimization problem

$$\underset{\Delta \mathbf{w}}{\operatorname{arg\,min}} \quad \Delta \mathbf{w}^T \mathbf{H}^{(\mathbf{w})} \Delta \mathbf{w}$$



Rounding Method	First Layer (%)
Nearest	52.29
Task loss Hessian	68.62±0.17

4-bit weight quantization of 1st layer of Resnet 18.

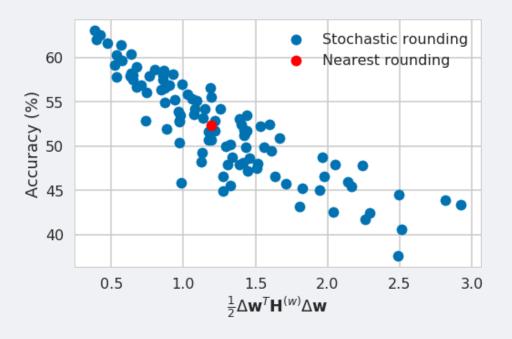
Task loss-based weight assignment

arg min
$$E[L(x,y,w+\Delta w)-L(x,y,w)]$$

Taylor approximation

$$\arg\min_{\Delta w} \Delta w^T \mathbf{H}^{(\mathbf{w})} \Delta w$$

Intractable for full network!



Rounding Method	First Layer (%)	All Layers (%)	
Baseline (FP32)	69.68		
Nearest	52.29	23.99	
Full Hessian	68.62±0.17	N/A	

Impact of rounding method for 4-bit weight quantization of Resnet18 in ImageNet.

Layer-wise Hessian approximation

• For a fully-connected layer:
$$\mathbf{z} = \mathbf{W}\mathbf{x}$$
 $\mathbf{H^{(w)}} = \mathbb{E}\left[\mathbf{x}\mathbf{x}^T\otimes\nabla^2_{\mathbf{z}}\mathcal{L}\right]$

• Assuming diagonal $abla_{\mathbf{z}}^2 \mathcal{L}$ and constant $abla_{\mathbf{z}}^2 \mathcal{L}_{i,i}$

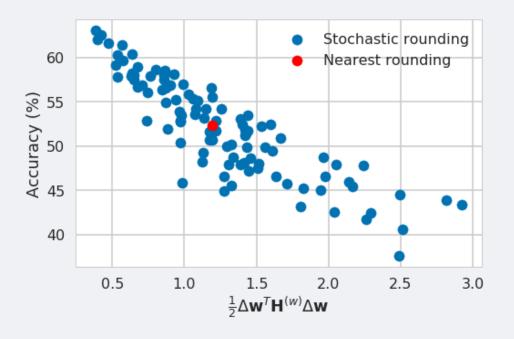
$$\underset{\Delta \mathbf{w}}{\operatorname{arg \, min}} \quad \Delta \mathbf{w}^{T} \mathbf{H}^{(\mathbf{w})} \Delta \mathbf{w} = \underset{\Delta \mathbf{W}_{k,:}}{\operatorname{arg \, min}} \quad \Delta \mathbf{W}_{k,:} \mathbb{E} \left[\mathbf{x} \mathbf{x}^{T} \right] \Delta \mathbf{W}_{k,:}^{T}$$

$$= \underset{\Delta \mathbf{W}_{k,:}}{\operatorname{arg \, min}} \quad \mathbb{E} \left[(\Delta \mathbf{W}_{k,:} \mathbf{x})^{2} \right]$$

- Approximation is applicable to other model efficiency tasks, e.g., pruning
 - Strong relation with layer-wise knowledge distillation

Task loss-based weight assignment

Solving this is NP hard!



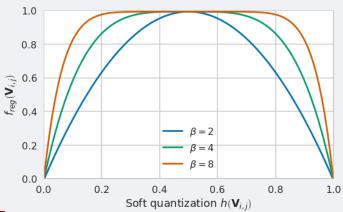
Rounding Method	First Layer (%)	All Layers (%)
Baseline (FP32)	69.	68
Nearest	52.29	23.99
Full Hessian	68.62±0.17	N/A
Layer-wise loss	69.39±0.04	65.83±0.14

Impact of rounding method for 4-bit weight quantization of Resnet18 in ImageNet.

AdaRound

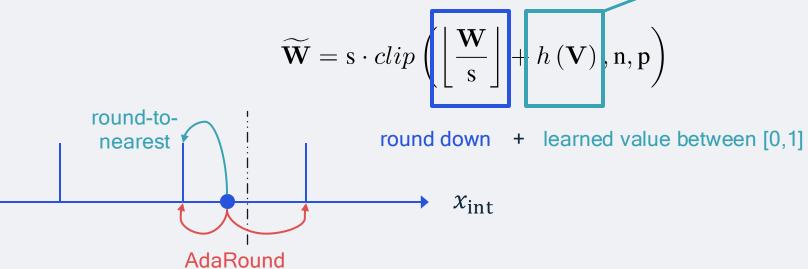
• Solve rounding efficiently using continuous relaxation

$$\underset{\mathbf{V}}{\operatorname{arg\,min}} \quad \left\| \mathbf{W}\mathbf{x} - \widetilde{\mathbf{W}}\mathbf{x} \right\|_{F}^{2} + \lambda f_{reg}\left(\mathbf{V}\right)$$



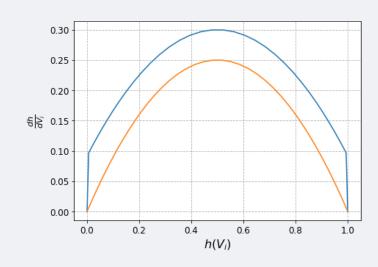
regularizer forces h(V) to be 0 or 1

• Where $\widetilde{\boldsymbol{W}}$ are soft-quantized weights:

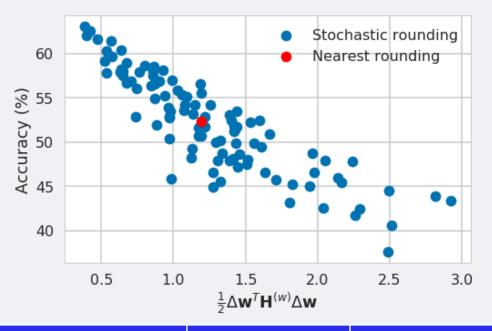


$$h(\mathbf{V}) = \text{clip}\left(\sigma(\mathbf{V})(\zeta - \gamma) + \gamma, 0, 1\right)$$

rectified sigmoid



Task loss-based weight assignment



Rounding Method	First Layer (%)	All Layers (%)	
Baseline (FP32)	69.68		
Nearest	52.29	23.99	
Full Hessian	68.62±0.17	N/A	
Local MSE loss	69.39±0.04	65.83±0.14	
AdaRound	69.58±0.03	68.71±0.06	

Impact of rounding method for 4-bit weight quantization of Resnet 18 in ImageNet.

Zooming out a bit

arg min
$$\sum_{\Delta \mathbf{w}} \mathbb{E}[L(\mathbf{x}, \mathbf{y}, \mathbf{w} + \Delta \mathbf{w}) - L(\mathbf{x}, \mathbf{y}, \mathbf{w})]$$

Taylor approximation

$$\operatorname{arg min}_{\Delta \mathbf{w}} \Delta \mathbf{w}^T \mathbf{H}^{(\mathbf{w})} \Delta \mathbf{w}$$
Hessian approximation

$$\operatorname{arg min}_{\Delta \mathbf{W}_{k,:}} \mathbb{E}\left[(\Delta \mathbf{W}_{k,:} \mathbf{x})^2\right]$$
Continuous relaxation

$$\operatorname{arg min}_{\Delta \mathbf{W}_{k,:}} \|f_a(\mathbf{W} \mathbf{x}) - f_a\| \mathbb{W} \hat{\mathbf{x}}\|_F^2 + \lambda f_{reg}(\mathbf{V})$$

In most literature this is same, even dates to 1990s (OBD/OBS)

Mostly similar but sometimes different notation/derivation

- $H \approx 2XX^T \rightarrow \operatorname{argmin}_{w_{\Delta}} w_{\Delta} XX^T w_{\Delta}$
- BRECQ uses block-wise Hessian
- Local loss often used elsewhere (e.g. quant params, pruning)

Many different approaches on solving efficiently the weight assignment

BRECQ

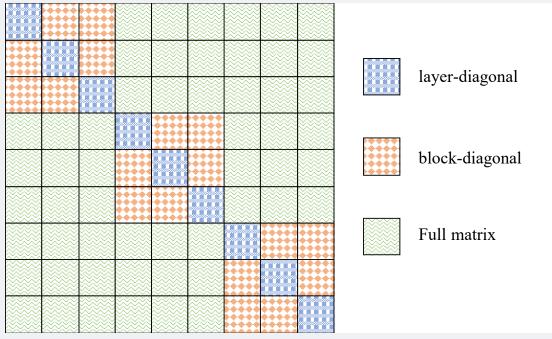


Image courtesy BRECQ paper (fig 1b).

$$\min_{\hat{\mathbf{w}}} \mathbb{E}\left[\Delta \mathbf{z}^{(\ell),\mathsf{T}} \mathbf{H}^{(\mathbf{z}^{(\ell)})} \Delta \mathbf{z}^{(\ell)}\right] = \min_{\hat{\mathbf{w}}} \mathbb{E}\left[\Delta \mathbf{z}^{(\ell),\mathsf{T}} \mathrm{diag}\left((\frac{\partial L}{\partial \mathbf{z}_{1}^{(\ell)}})^{2}, \ldots, (\frac{\partial L}{\partial \mathbf{z}_{a}^{(\ell)}})^{2}\right) \Delta \mathbf{z}^{(\ell)}\right]$$

- Use Fisher Information Matrix (FIM) to approximate cross-layer dependencies.
- ullet Relaxes diagonal $abla_{\mathbf{z}}^2 \mathcal{L}$ and constant $abla_{\mathbf{z}}^2 \mathcal{L}_{i,i}$
- Algorithm details:
 - Uses AdaRound continuous relaxation
 - Use LSQ to jointly learn activation ranges
 - Introduced additional mixed precision
- Shows clear improvement, SOTA at the time

Model	Layer	Block	Stage	Net
ResNet-18	65.19	66.39	66.01	54.15
MobileNetV2	52.13	59.67	54.23	40.76

OBQ and **GPTQ**

Optimal Brain Quantization (OBQ)

- Generalizes Optimal Brain Surgeon (OBS)
- Layer-wise greedy and iterative procedure
 - Quantizes weights row by row with RTN
 - Updates remaining weights to compensate for error

$$w_q = \operatorname{argmin}_{w_q} \frac{(\operatorname{quant}(w_q) - w_q)^2}{[H_F^{-1}]_{qq}}, \quad \delta_F = -\frac{w_q - \operatorname{quant}(w_q)}{[H_F^{-1}]_{qq}} \cdot (H_F^{-1})_{:,q}$$

GPTQ: scaling OBQ to LLMs

- Independent rows \rightarrow update H_F once per column
- Batched updates for multiple columns
- Cholesky reformulation for stability and efficiency

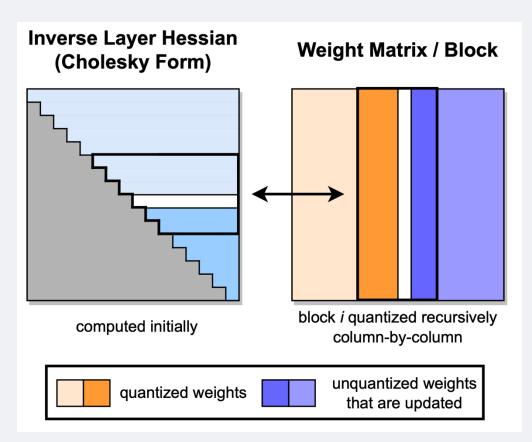


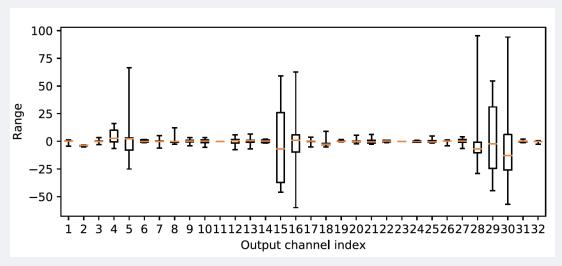
Image courtesy GPTQ paper (fig 2).

Function preserving transformations



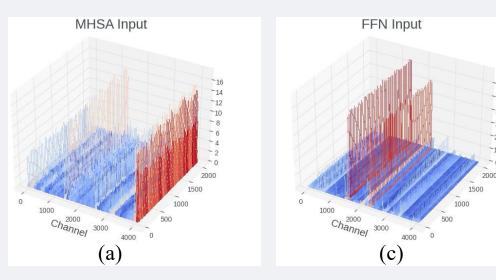
Outliers are common in neural networks

Weight outliers



Weight distribution in MobileNetV2. Image courtesy DFQ (fig 2).

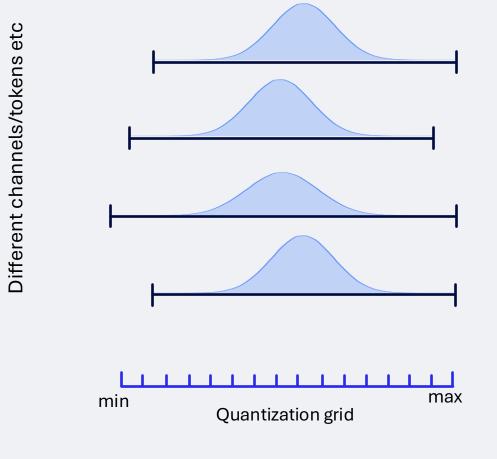
Activation outliers



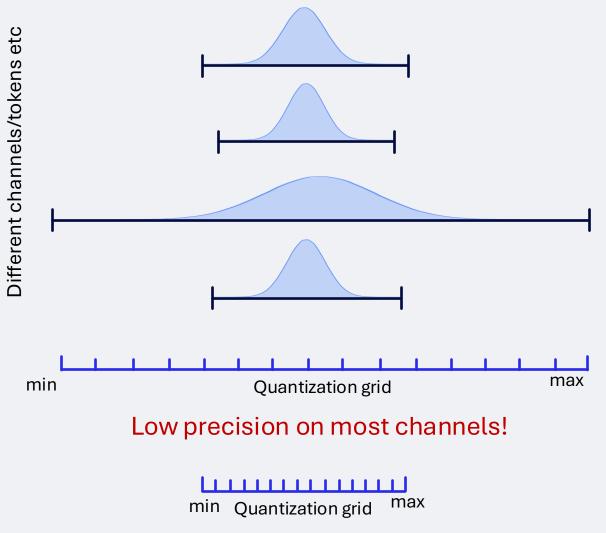
Activation distribution in Llama v2 7B. Image courtesy SpinQuant (fig 2).

NB: you will hear in the next session more on the structure and origin of outliers in LLMs.

Why are outlier problematic for uniform quantization

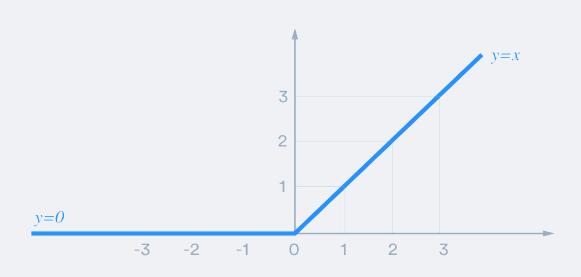


Easy to quantize! Sufficient precision for all channels/tokens



High clipping error for outlier channel/tokens! 25

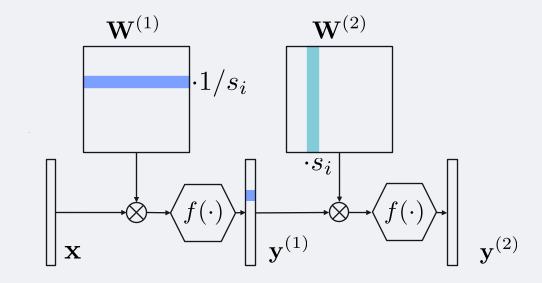
DFQ: Cross-layer equalization



$$ReLU(x) = \max(0, x)$$

ReLU is scale-equivariant

$$ReLU(sx) = s \cdot ReLU(x)$$



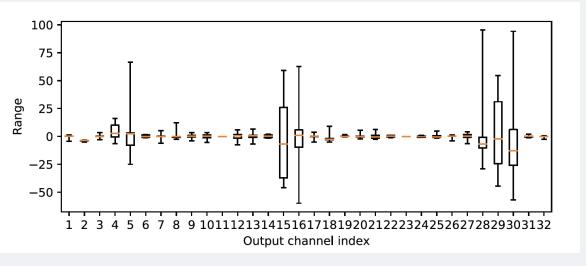
We can scale two neighboring layers together to optimize it for quantization

$$y = W_2 \cdot ReLU(W_1 x) = W_2 S \cdot ReLU(S^{-1} W_1 x)$$

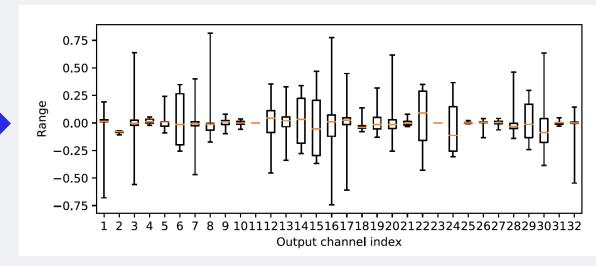
$$\widehat{W}_1 = q(S^{-1} W_1) \quad \widehat{W}_2 = q(W_2 S)$$
 With $S = diag(S)$

Cross-layer equalization significantly improves accuracy





After cross-layer equalization



Model	FP32	INT8	
Original Model	71.72	0.12	TEO (
CLE	71.70	69.91	+69.
CLE + absorbing bias	71.57	70.92	

ImageNet validation accuracy (%) for MobileNetV2

SmoothQuant

- Activation in LLM have very strong outliers
- Use scaling to migrate difficulty to weights
- Enables INT8 for weights and activations

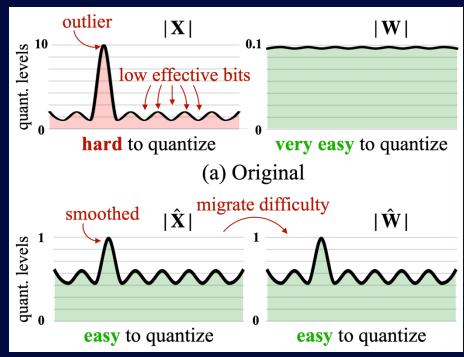


Image courtesy SmoothQuant paper (fig 2).

AWQ

- Salient weights are important to protect
- Find salient weight channels based on activation distribution
- Scale salient weights to minimize quantization error

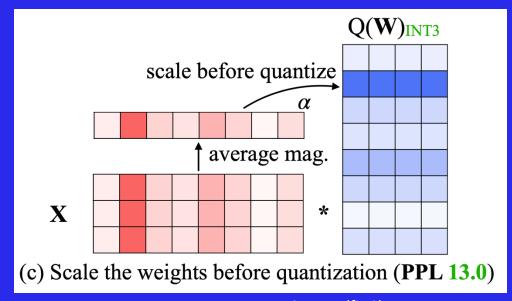


Image courtesy AWQ paper (fig 2).

AWQ: Activation-aware Weight Quantization for On-Device LLM Compression and

Acceleration (Lin et al., MLSys 2024)

OmniQuant

Learnable equivalent transform (LET)

$$\mathbf{Y} = \mathbf{X}\mathbf{W} + \mathbf{B} = \underbrace{[(\mathbf{X} - \delta) \oslash s]}_{\tilde{\mathbf{X}}} \cdot \underbrace{[s \odot \mathbf{W}]}_{\tilde{\mathbf{W}}} + \underbrace{[\mathbf{B} + \delta \mathbf{W}]}_{\tilde{\mathbf{B}}}$$
$$\mathbf{Y} = Q_a(\tilde{\mathbf{X}})Q_w(\tilde{\mathbf{W}}) + \tilde{\mathbf{B}},$$

Learnable weight clipping (LWC)

$$\mathbf{W_q} = \operatorname{clamp}(\lfloor \frac{\mathbf{W}}{h} \rceil + z, 0, 2^N - 1)$$

$$h = \frac{\gamma \max(\mathbf{W}) - \beta \min(\mathbf{W})}{2^N - 1}, z = -\lfloor \frac{\beta \min(\mathbf{W})}{h} \rfloor$$

 Trained on block-wise quantization error (cf. AdaRound, BRECQ)

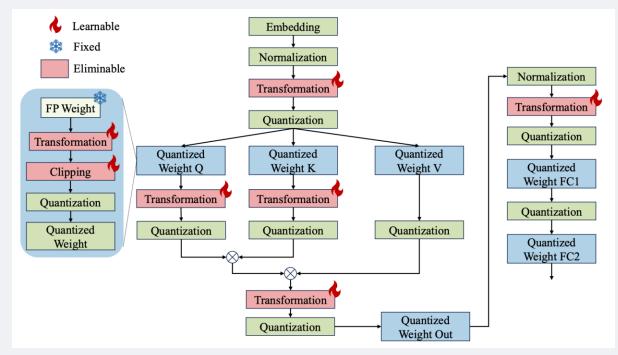


Image courtesy OmniQuant paper (fig 3).

Note, all transformations are mergeable!

Function preserving transformations (FPTs)

Generalize channel-wise scaling to any function preserving transformations (FPTs)

$$Q(X)Q(W) \rightarrow Q(XT)Q(T^{-1}W)$$

- Key properties of function preserving transformations (FPTs):
 - 1. Function-preservation: Should not change network output. In practice this means the FPT needs to have an inverse operation and operations between needs to commute under the FPT.
 - 2. Expressivity: Transforms with a continuous parameterization and more degrees of freedom are desirable. They allow optimization with SGD and have more flexibility to reduce quantization error.
 - 3. Compute overhead: Ideally FPTs are mergeable to adjacent operations. If not, they injure inference time overhead which should be minimal.
- Rotations are commonly used reduce outliers through channel mixing
 - Walsh-Hadamard transformations is commonly used as cheap online transform ($O(d \log_2(d))$
 - First introduced as incoherence processing in QuIP and QuIP# for extreme weight compression
 - Rotation commutes with RMSNorm (SliceGPT): RMSNorm(X)T = RMSNorm(<math>XT)

Transformations applied for LLM quantization

QuaRot

- Shows that Hadamard makes activation quantization easy
- Online Hadamard in attention to remove outliers from keys and values
- Enables full INT4 LLM inference (weight, activations, KV cache)

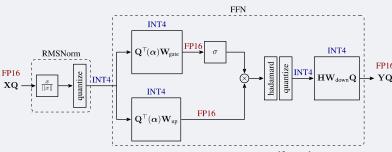


Image courtesy QuaRot paper (fig 3).

SpinQuant

- Learned rotations end-to-end
- Random and Hadamard rotation show substantial variance
- Uses Cayley SGD to optimize rotations on Stiefel manifold

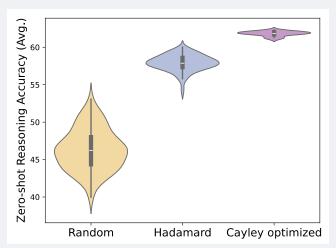


Image courtesy SpinQuant paper (fig 4).

FlatQuant

- Significance of flat distributions
- Fast affine transformation using Kronecker decomposition
- Fused transformation in joint quantization kernel

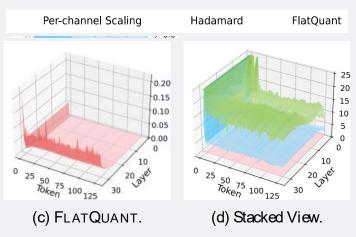
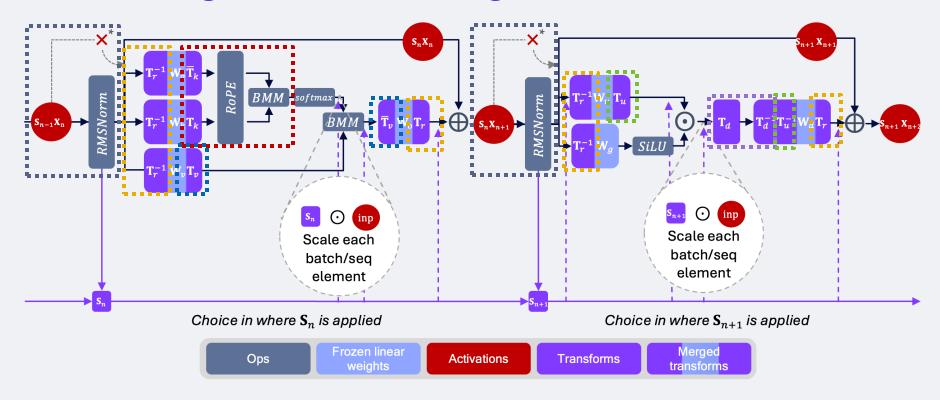


Image courtesy FlatQuant paper (fig 2).

FPTQuant: maximizing the amount of mergeable transforms



Introduce novel and fully mergeable transforms:

- PreRoPE transform: scaled 2x2 rotation
- Value transform: linear transform (full matrix per-head)
- Up/down scaling: per-channel scaling
- Pseudo-dynamic residual scaling: normalize residual

Reuse some existing transforms:

- Residual transform: joint over all layers (same as R1 in SpinQuant)
- Down project transform: online Hadamard (same as R4 in SpinQuant)

HadaNorm: mean-centered Hadamard transform

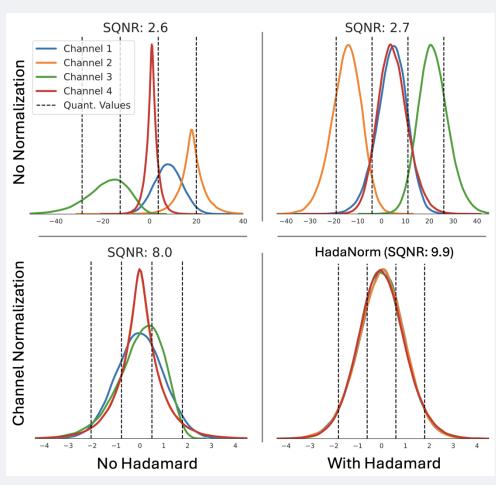


Image courtesy HadaNorm paper (fig 1).

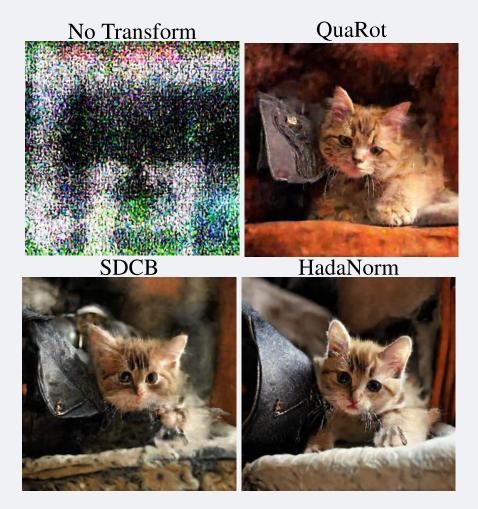


Image courtesy HadaNorm paper (fig 4).

Summary PTQ

- Range setting is important to trade-off between rounding and clipping error
- Optimizing weight assignment based on Hessian approximation significantly improves low bit weight quantization

 Outliers are a common issue and function preserving transformations (FTPs) can make distributions more quantization friendly

- NB: there are many other effective algorithms that do not fall in these 3 main buckets
 - E.g. bias correction, channel splitting, high precision outliers and many more

Part 2: Quantization-aware training



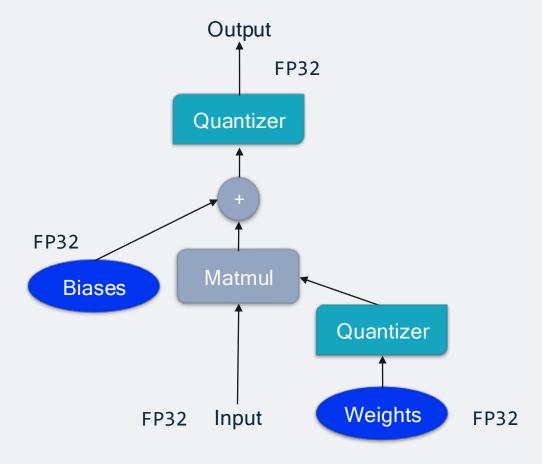
Quantization-Aware Training

- Train with *simulated quantization* to achieve best inference time accuracy
- Quantizers discretize the input data

Key challenges:

- Discretization is non-differentiable
 - E.g. rounding, codebook assignment etc
- Learning quantization parameters
 - E.g. scale of quantization grid, codebook entries etc

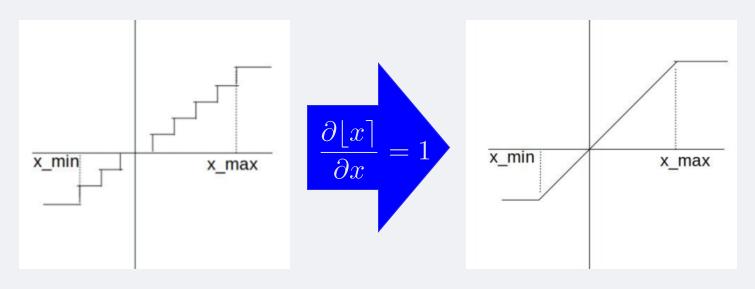
Simulated quantization



Straight-through estimator (STE)

Actual forward pass

- Approximated non-differentiable operation with straight-through estimator (STE)
- Most commonly used approach and highly effective



Simulated forward pass

Known drawbacks of STE:

- STE gradient is biased
- Weights don't converge, they can oscillate

Oscillating weights in QAT

Example regression problem:

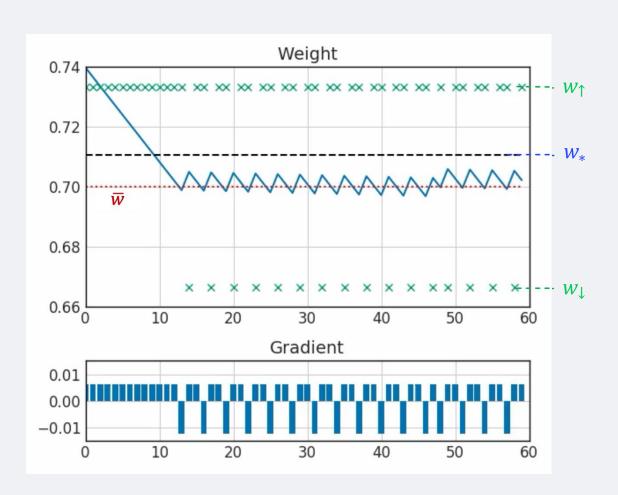
• Latent weight: w

• Quantized weight: $q(w) = s_w \cdot \text{round}(w/s_w)$

• Objective: $\min_{w} \mathcal{L}(w) = (w_* - q(w))^2$

• Rounding is approximated by STE^[1]:

$$\frac{\partial \mathcal{L}}{\partial w} = \frac{\partial \mathcal{L}}{\partial q(w)} = \begin{cases} w_* - w_{\uparrow}, & \text{if } w \ge \overline{w} \\ w_* - w_{\downarrow} & \text{if } w < \overline{w} \end{cases}$$



Alternatives to STE

Stochastic rounding

Round proportional to distance

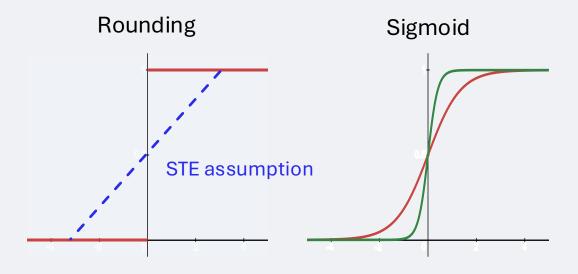
$$round(x) = \begin{cases} [x] & \text{w.p. } x - [x] \\ [x] & \text{w.p. } x - [x] \end{cases}$$

Unbiased estimator:

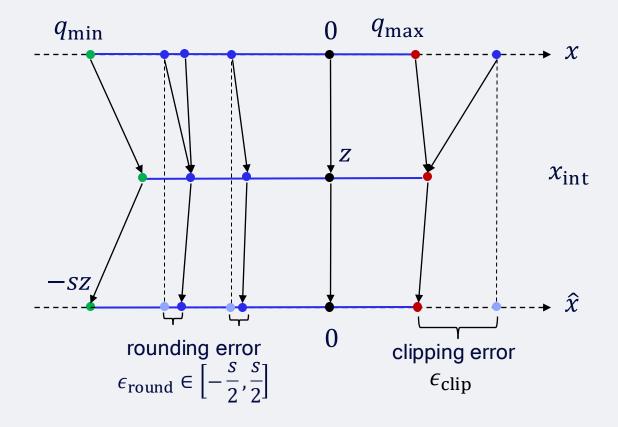
$$\mathbb{E}[\operatorname{round}(x)] = x$$

Approximate rounding function

- Approximate quantization operation with differentiable non-linear mapping
- Stacked sigmoid with temperature



Learning the quantization parameters



Quantization parameters are learnable when STE is applied to rounding only

$$X_{\text{int}} = \text{clamp}\left(\text{round}\left(\frac{X}{S}\right) + Z, \text{min} = 0, \text{max} = 2^b - 1\right)$$

$$\widehat{X} = s(X_{\text{int}} - z)$$

Through task loss gradients, we find the optimal trade-off between ϵ_{clip} & ϵ_{round}

Note, various parameterization might lead to different gradients and learning behaviour

Summary QAT

• Discretization is non-differentiable and requires gradient approximation

• Straight-through estimator (STE) works extremely well in practice

• Quantization parameters can be learned end-to-end using auto-grad

FastForward

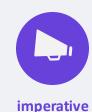




pytorch







workflow



Neural Network quantization library for PyTorch

 Focus on prototyping and experimentation; embracing PyTorch's imperative workflow

```
01 import fastforward as ff
    [...]
     ff.quantize_model(model)
     initialize_quantizers(model) # user provided
     with ff.estimate_ranges(model, ff.range_estimation.mse):
       for batch in data[:20]:
         model(batch)
10
```

Example: PTQ using FastForward

 Find more at on GitHub: github.com/qualcomm-ai-research/fastforward





Thanks to all my amazing collaborators



Mart van Baalen



Tijmen Blankevoort



Marios Fournarakis



Rana Ali Amjad



Yelysei Bondarenko



Andrey Kuzmin



Christos Louizous



Ties van Rozendaal



Andrii Skliar



Max Welling



Paul Whatmough



Boris van Breugel



Marco Federici



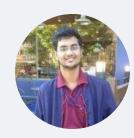
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