by 박정수

A) $(\theta_1 \sim \theta_3)$ 구하기

$$\underbrace{R_y(\theta_1)\, T_y(l_1)\, R_z(\theta_2)\, T_y(l_2)\, R_z(\theta_3)\, T_x(l_3)}_{M1} \cdot \underbrace{(T_y(-l_4)\, R_y(-\theta_4)\, R_z(\theta_5)\, R_x(\theta_6)\, T_x(l_5))}_{M2}$$
$$= T$$

$$M1 = T\, M2^{-1} = \begin{bmatrix} R & a \\ 0 & 1 \end{bmatrix}$$

$a = \cancel{T_y(l_4)\, T_x(l_5)}$

$T \cdot T_y(l_4)\, T_x(l_5)$ 의 $\begin{bmatrix} & x \\ & x \\ & x \end{bmatrix}$ 성분

$R_y(\theta_1) C_1 = a$

$R_z(\theta_2) C_2 = C_1 - T_y(l_1)$

$R_z(\theta_3) T_x(l_3) = C_2 - T_y(l_2)$

$$\begin{bmatrix} c\theta_1 & 0 & s\theta_1 \\ 0 & 1 & 0 \\ -s\theta_1 & 0 & c\theta_1 \end{bmatrix} \begin{bmatrix} C_{1x} \\ C_{1y} \\ C_{1z} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \rightarrow C_{1y} = a_2,$$

$$c\theta_1 C_{1x} + s\theta_1 C_{1z} = a_1$$
$$-s\theta_1 C_{1x} + c\theta_1 C_{1z} = a_3$$
$$\rightarrow \begin{bmatrix} C_{1x} & C_{1z} \\ C_{1z} & -C_{1x} \end{bmatrix} \begin{bmatrix} c\theta_1 \\ s\theta_1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_3 \end{bmatrix}$$
$$\rightarrow \begin{bmatrix} c\theta_1 \\ s\theta_1 \end{bmatrix} = \frac{1}{+C_{1x}^2 + C_{1z}^2} \begin{bmatrix} +C_{1x} & +C_{1z} \\ +C_{1z} & -C_{1x} \end{bmatrix} \begin{bmatrix} a_1 \\ a_3 \end{bmatrix} \quad ①$$

$$\begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{2x} \\ C_{2y} \\ C_{2z} \end{bmatrix} = \begin{bmatrix} C_{1x} \\ C_{1y} - l_1 \\ C_{1z} \end{bmatrix} \rightarrow$$

$$c\theta_2 C_{2x} - s\theta_2 C_{2y} = C_{1x}$$
$$s\theta_2 C_{2x} + c\theta_2 C_{2y} = C_{1y} - l_1$$
$$C_{2z} = C_{1z} = 0$$
$$\rightarrow \begin{bmatrix} C_{2x} & -C_{2y} \\ C_{2y} & C_{2x} \end{bmatrix} \begin{bmatrix} c\theta_2 \\ s\theta_2 \end{bmatrix} = \begin{bmatrix} C_{1x} \\ C_{1y} - l_1 \end{bmatrix}$$
$$\begin{bmatrix} c\theta_2 \\ s\theta_2 \end{bmatrix} = \frac{1}{C_{2x}^2 + C_{2y}^2} \begin{bmatrix} C_{2x} & +C_{2y} \\ -C_{2y} & C_{2x} \end{bmatrix} \begin{bmatrix} C_{1x} \\ C_{1y} - l_1 \end{bmatrix} \quad ②$$

$$\begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{2x} \\ C_{2y} - l_2 \\ C_{2z} \end{bmatrix} \rightarrow$$

$$c\theta_3 l_3 = C_{2x} \quad ③$$
$$s\theta_3 l_3 = C_{2y} - l_2$$
$$0 = C_{2z}$$
$$l_3^2 = C_{2x}^2 + (C_{2y} - l_2)^2$$
$$= C_{2x}^2 + C_{2y}^2 + l_2^2 - 2C_{2y} l_2$$

$$\frac{(C_{2x}C_{1x} + C_{2y}(C_{1y} - l_1))^2 + (-C_{2y}C_{1x} + C_{2x}(C_{1y} - l_1))^2}{(C_{2x}^2 + C_{2y}^2)^2} = 1$$

$$\cancel{(C_{2x}^2 + C_{2y}^2)}(C_{1x}^2 + (C_{1y} - l_1)^2) = (C_{2x}^2 + C_{2y}^2)^{\cancel{2}}$$

$$C_{1x}^2 + (C_{1y} - l_1)^2 = C_{2x}^2 + C_{2y}^2$$

$$a_1^2 + a_3^2 + (a_2 - l_1)^2 = \text{''}$$

$$l_3^2 = a_1^2 + a_3^2 + (a_2 - l_1)^2 + l_2^2 - 2C_{2y} l_2$$

$$2C_{2y} l_2 = a_1^2 + (a_2 - l_1)^2 + a_3^2 + l_2^2 - l_3^2$$

$$C_{2y} = \frac{a_1^2 + (a_2 - l_1)^2 + a_3^2 + l_2^2 - l_3^2}{2l_2}$$

$$\frac{(C_{1x}a_1 + C_{1z}a_3)^2}{(C_{1x}^2 + C_{1z}^2)^2} + \frac{(C_{1z}a_1 - C_{1x}a_3)^2}{(\text{''})^2} = 1$$

$$C_{1x}^2 a_1^2 + C_{1z}^2 a_3^2 + C_{1z}^2 a_1^2 + C_{1x}^2 a_3^2 = (C_{1x}^2 + C_{1z}^2)^{\cancel{2}}$$
$$\cancel{(C_{1x}^2 + C_{1z}^2)}(a_1^2 + a_3^2)$$

$$C_{1x} = \pm\sqrt{a_1^2 + a_3^2 - C_{1z}^2}$$
$$= \pm\sqrt{a_1^2 + a_3^2}$$

$$\therefore \begin{cases} C_{1x} = \pm\sqrt{a_1^2 + a_3^2} \\ C_{1y} = a_2 \\ C_{1z} = 0 \\ C_{2x} = \pm\sqrt{a_1^2 + a_3^2 + (a_2 - l_1)^2 - C_{2y}^2} \\ C_{2y} = \dfrac{a_1^2 + (a_2 - l_1)^2 + a_3^2 + l_2^2 - l_3^2}{2l_2} \\ C_{2z} = 0 \end{cases}$$

①, ② ③ 과 위의 결과로 $\theta_1 \sim \theta_3$ 구한다.

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$$\left(R_y(\theta_1)T_y(l_1)R_z(\theta_2)T_y(l_2)R_z(\theta_3)T_x(l_3)\right)^{-1}T$$

$$= T_y(-l_4)R_y(-\theta_4)R_z(\theta_5)R_x(\theta_6)T_x(l_5)$$

$T_y(-l_4)^{-1} = T_y(l_4)$

B) $\theta_4, \theta_5, \theta_6$ 을 구해야 함.

$$R_y(-\theta_4)R_z(\theta_5)R_x(\theta_6) = T_y(l_4)\left(R_y(\theta_1)T_y(l_1)R_z(\theta_2)T_y(l_2)R_z(\theta_3)T_x(l_3)\right)^{-1}\cdot T\cdot T_x(-l_5)$$

$$= \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 \\ b_{21} & b_{22} & b_{23} & 0 \\ b_{31} & b_{32} & b_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = B$$

회전행렬의 역행렬: $-\theta$ 대입.

$$R_z(\theta_5)R_x(\theta_6) = R_y(\theta_4)B$$

$$= \begin{bmatrix} c\theta_4 & 0 & s\theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 \\ b_{21} & b_{22} & b_{23} & 0 \\ b_{31} & b_{32} & b_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \cdots$$

$$\begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta_6 & -s\theta_6 & 0 \\ 0 & s\theta_6 & c\theta_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_4 b_{11}+s\theta_4 b_{31} & c\theta_4 b_{12}+s\theta_4 b_{32} & c\theta_4 b_{13}+s\theta_4 b_{33} & 0 \\ b_{21} & b_{22} & b_{23} & 0 \\ -s\theta_4 b_{11}+c\theta_4 b_{31} & -s\theta_4 b_{12}+c\theta_4 b_{32} & -s\theta_4 b_{13}+c\theta_4 b_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_5 & -s\theta_5 c\theta_6 & s\theta_5 s\theta_6 & 0 \\ s\theta_5 & c\theta_5 c\theta_6 & -c\theta_5 s\theta_6 & 0 \\ 0 & s\theta_6 & c\theta_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$s\theta_4 b_{11} = c\theta_4 b_{31}$

$\tan\theta_4 = \dfrac{b_{31}}{b_{11}}$

$$\therefore \begin{cases} \theta_4 = \tan^{-1}\left(\dfrac{b_{31}}{b_{11}}\right) \\ s\theta_5 = b_{21} \\ s\theta_6 = -s\theta_4 b_{12} + c\theta_4 b_{32} \end{cases}$$

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