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A Risk Constrained Control Approach for Adaptive Cruise Control

Dominik Moser, Luigi del Re and Stephen Jones

Abstract— **This work investigates the incorporation of a risk metric based on time to collision for Adaptive Cruise Control applications on multi-lane motorways. To improve the fuel-efficiency and safety, a stochastic model predictive control approach is suggested that limits the violation probability of the imposed risk metric. For this reason, a Bayesian network is used to predict the probability distributions of the surrounding vehicles' future motion based on actual measurements. Subsequently, these distribution functions are incorporated into a stochastic model predictive control algorithm. The novel strategy is evaluated in simulation using an artificial evaluation cycle and shows significant improvements in terms of fuel efficiency and safety in comparison to conventional Adaptive Cruise Control when sharp cut-off maneuvers occur.**

I. Introduction

Automated driving strategies are able to improve safety, efficiency and comfort in traffic. To help realize their full potential, better control algorithms are required to steer the vehicle in such a way that the aforementioned goals are reached. High uncertainty in terms of the assessment of the traffic situation and the non-deterministic behavior of human drivers implies that the risk of accidents cannot be zero when driving in public traffic. Therefore the incorporation of uncertainty and collision risk assessment would appear to be highly desirable.

Risk anticipation and risk perception by human drivers and their consequences on driving behavior are extensively studied, see e.g. [1]. However, literature provides a variety of definitions of risk for automated driving. The authors in [2] introduce risk in terms of probability of collision multiplied by its severity. They develop a foresighted driver model and assess how risky a certain behavior of the controlled vehicle is. On the other hand, many authors [3], [4] consider risk as the probability of the violation of a given distance policy to an obstacle. There exists a variety of distance policies or rather risk functions for automated driving which are introduced not only to avoid collisions with obstacles but also for other objectives as e.g. comfort, efficiency and string stability.

In many cases, conventional Adaptive Cruise Control (ACC) algorithms track a desired time headway to the prevailing preceding vehicle on the same lane [5]. However, observations of real traffic situations show that human drivers often approach (e.g. dense traffic) very close to the preceding vehicle with headways significantly lower than those typically found in ACC functions. As elaborated in [6], during merging scenarios, minimum safety distances are often not maintained by human drivers, i.e. the merging vehicle imposes in many cases risk to other vehicles. Therefore, merging scenarios represent an important use-case for ACC systems and a meaningful strategy to cope with unsafe entering vehicles is beneficial.

Dominik Moser and Luigi del Re are with the Institute for Design and Control of Mechatronical Systems at the Johannes Kepler University of Linz, Austria. (phone: +43-732-2468-6210; fax: +43-732-2468-6213; e-mail: {d.moser, luigi.delre}@jku.at Stephen Jones is with AVL List GmbH, Graz, Austria; e-mail: Stephen.Jones@avl.com

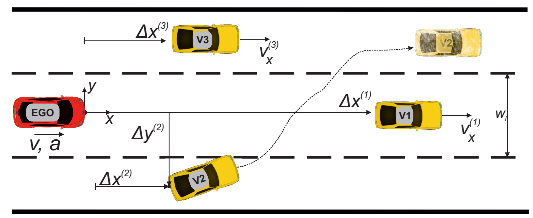

Fig. 1. Considered scenario: multi-lane traffic situation where vehicles on both the left and right adjacent lanes can enter the ego vehicle's lane.

Several studies [7]–[10] point out that it is beneficial to directly use time to collision (TTC) as risk function for decision making in automated driving applications because it is considered to meet drivers expectations with respect to a collision avoidance system and it is shown that drivers decisions as when to start braking in a critical situation may well be based on TTC. Similar to [11], an ACC algorithm is proposed that constrains the minimum TTC instead of a time headway or distance to other traffic participants in order to avoid rear end collisions. Differently to [11], this paper proposes a stochastic Model Predictive Control (MPC) for a multi-lane highway that incorporates the probability of merging vehicles and the uncertainty of the predicted velocity of the surrounding vehicles over the prediction horizon. The resulting control reacts in a risk-aware manner while the fuel consumption is minimized. It is shown that in such scenarios both fuel efficiency and safety can be improved significantly in comparison to non-predictive approaches. At the same time, the robustness in terms of the probability of violation of the imposed risk function to a traffic participant can be adjusted.

II. Problem Statement

A. Traffic scenario

The considered scenario focuses on multi-lane motorways where lane changing of vehicles occurs frequently. As illustrated in Fig. 1, the ego vehicle is driving with active ACC while it is following vehicle V1. Vehicles on the adjacent lane (V2, V3) can enter the lane of the ego vehicle. The ego vehicle's longitudinal velocity is denoted by v and its

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acceleration is a. The relative distances to the vehicles in the adjacent lanes and their velocities are marked by superscript, e.g. $v_x^{(1)}$ is the absolute longitudinal velocity of V1.

B. Conventional ACC

As mentioned in the introduction, many conventional ACC algorithms track a constant time headway policy to the leading vehicle

$$\Delta x_{\mathrm{des}}(t) = \Delta x_{\min} + t_{\mathrm{h}} v(t), \tag{1}$$

where t_{h} is the driver's selected time headway and the minimum distance is denoted $\Delta x_{\min}$. Typically, the driver can choose among several settings for t_{h}. A natural event that arises frequently on multi-lane roads with dense traffic is that vehicles from the adjacent lanes enter the ego vehicle's lane. This is more likely the larger the headway to the preceding vehicle is. It is shown in [12], that when sharp cut-off maneuvers occur, conventional ACC performance suffers because the sudden decrease of the inter-vehicle distance results in uncomfortable control actions.

Naturally, a predictive algorithm (P-ACC), that considers the probability of a vehicle to merge in front of the ego-vehicle over a prediction horizon could improve fuel-efficiency, safety and comfort. Therefore, in the following subsections, a stochastic MPC approach is developed based on the formulation in [3] but with consideration of multi-lane traffic and constraints on TTC.

Notice that this work focuses on fuel-efficient longitudinal control, i.e. the lateral position of the vehicle is not controlled. However, lateral movements of surrounding traffic participants are incorporated in order to react to potential lane changes of the surrounding traffic.

C. Cost function

As already introduced in [13], the explicit consideration of fuel consumption in the objective function can improve fuel-efficiency of the ACC. The objective includes therefore the vehicle's fuel consumption q_{f} as running costs which is summed over the prediction horizon h_{P}

$$Q^{\star}_{\mathrm{f},k} = \min_{\boldsymbol{u}_{k+i|k}} T_{\mathrm{S}} \sum_{i=0}^{h_{\mathrm{P}}-1} q_{\mathrm{f}}(\boldsymbol{x}_{k+i|k}, \boldsymbol{u}_{k+i|k}) + V_f(\boldsymbol{x}_{k+h_{\mathrm{P}}|k}). \tag{2}$$

Here, T_{S} is the sampling time, $\boldsymbol{x}_{k+i|k}$ denotes the system state at time k and i steps ahead of k. The input $\boldsymbol{u}_k$ is formed by the acceleration a_k and the engaged gear g_k of the vehicle. As the fuel consumption of a vehicle is a nonlinear map, it is approximated with piecewise affine functions for each gear. For further details and explanations see [14].

Naturally, exclusively minimizing the fuel consumption would lead to standstill of the vehicle. Therefore, terminal costs V_f are introduced. It is suggested to use a quadratic tracking error on the longitudinal distance to a vehicle which can be stated as

$$V_f = c_f \left(\Delta x_{k+h_{\mathrm{P}}|k} - \Delta x_{\mathrm{des}}\right)^2, \tag{3}$$

where Δx_{des} is a driver dependent preferred distance to the preceding vehicle, e.g. according to (1).

D. System dynamics

In order to consider multiple traffic participants, the system dynamics includes all inter-vehicle distances between the ego vehicle and the surrounding vehicles that can potentially enter the lane. The formulation for N surrounding vehicles is stated in (4). The manipulated variable is the acceleration a_k of the ego vehicle. The surrounding vehicles' future motion is considered with the disturbance input $\boldsymbol{d}_k$. For a real-time capable solution, those values must be predicted and are therefore uncertain, see section III.

$$\underbrace{\begin{bmatrix}\Delta x^{(1)}_{k+1}\\ \vdots \\ \Delta x^{(N)}_{k+1} \\ v_{k+1}\end{bmatrix}}_{\boldsymbol{x}_{k+1}} = \underbrace{\begin{bmatrix}1 & 0 & \cdots & -T_{\mathrm{S}}\\ 0 & \ddots & \cdots & -T_{\mathrm{S}}\\ \vdots & \vdots & 1 & -T_{\mathrm{S}} \\ 0 & 0 & 0 & 1\end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix}\Delta x^{(1)}_{k}\\ \vdots \\ \Delta x^{(N)}_{k} \\ v_{k}\end{bmatrix}}_{\boldsymbol{x}_{k}} + \underbrace{\begin{bmatrix}0\\ \vdots \\ 0 \\ T_{\mathrm{S}}\end{bmatrix}}_{\boldsymbol{B}} a_k + \underbrace{\begin{bmatrix}T_{\mathrm{S}} & \cdots & 0\\ 0 & \ddots & 0\\ \vdots & \vdots & T_{\mathrm{S}} \\ 0 & 0 & 0\end{bmatrix}}_{\boldsymbol{B_d}} \underbrace{\begin{bmatrix}v^{(1)}_{x,k}\\ \vdots \\ v^{(N)}_{x,k}\end{bmatrix}}_{\boldsymbol{d}_k} \tag{4}$$

E. Constraints

In order to ensure safety, the cost function (2) is minimized subject to constraints on the risk R that is represented by obstacles and surrounding traffic participants. In general, the risk is a function of the system state $\boldsymbol{x}$ and the disturbance input $\boldsymbol{d}$.

Notice, that in the case of the presented multi-lane ACC strategy, only the longitudinal motion is controlled. The controller is required to steer the vehicle in a way that the probability that the headway to each surrounding traffic participant ensures that $R_{\max}$ is not exceeded. Naturally, a surrounding traffic participant imposes only a risk, if the path of the ego vehicle and the traffic participant intersect. A vehicle is considered to be in the same lane as the ego vehicle, if its lateral position Δy (relative to the ego-vehicle's lane) belongs to the set $\mathscr{Y}$ where w_l denotes the lane width of the road, see Fig. 1.

$$\mathscr{Y} = \{\Delta y | -w_l/2 \leq \Delta y \leq w_l/2\} \tag{5}$$

As reported in [9], the inverse of TTC, i.e. $R = \frac{1}{t_{\mathrm{c}}}$ is a meaningful risk measure. Therefore constraints on the minimum TTC are used to ensure safety of the presented P-ACC. In contrast to constraints on time headway, time to collision allows in general lower headways between vehicles as long as the relative velocity is low. Note that $R_k^{(n)}$ represents the risk with respect to vehicle n and is computed using the states $[\Delta x_k^{(n)}, v_k]^{\top}$ and the disturbance input $v_{x,k}^{(n)}$. The maximum acceptable risk is $R_{\max}$ and can be chosen e.g. according to a driver's preference.

By incorporation of the predicted lane of the surrounding vehicles, i.e., $p^{(n)}_{y,k+i|k} = \Pr\left(\Delta y^{(n)}_{k+i|k} \in \mathscr{Y}\right)$, the following individual chance constraints of the surrounding vehicles can be stated

$$\Pr\left(R^{(n)}_{k+i|k}(\boldsymbol{x}_{k+i|k}, \boldsymbol{d}_{k+i|k}) \geq R_{\max} \cap \Delta y^{(n)}_{k+i|k} \in \mathscr{Y}\right) \leq \alpha \tag{6}$$
$$\forall i \in \{1, \ldots, h_{\mathrm{P}}-1\} \quad \forall n = \{1, \ldots, N\}.$$

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where α is the risk probability which can be chosen accordingly. In fact, (6) requires that the joint probability for violation of the risk related to each surrounding vehicle which is on the path of the ego vehicle is constrained to be smaller than α. In order to solve the chance constrained program using efficient numerical solvers, (6) will be approximated by equivalent deterministic constraints, see section IV.

III. Prediction Model

As it can be seen in (4), the sequence of the disturbance input $\boldsymbol{d}_{k+i|k}$ must be estimated at time k for all vehicles to solve the optimization problem in real-time. Analogously, the probability of all surrounding vehicles to enter the ego-vehicle's lane $p_{y,k+i|k}^{(n)} = \Pr\left(\Delta y_{k+i|k}^{(n)} \in \mathscr{Y}\right)$ must be predicted for the upcoming sampling intervals to formulate (6).

In the sense of [15], [16], a Bayesian network approach is applied to model the probability distribution of the future longitudinal velocity $v_{x,k+i|k}^{(n)}$ and the lane probability $p_{y,k+i|k}^{(n)}$ of a surrounding traffic participant. The basic structure of the approach is briefly explained in the next sections. For further information on the prediction model, the reader referred to [17].

A. Layout

To model the joint probability of the input variables and the predicted trajectories, a graphical model [18] is suggested. In the sense of [16], a Conditional Linear Gauss (CLG) model is used to predict the distribution function of the surrounding vehicle's future velocity and their lateral position on the road along the prediction horizon. It is assumed that the predicted velocity v_x and the lateral position y can be described by Gauss distributions.

For prediction of the future motion of a surrounding vehicle n, the following measurements are assumed to be assessed by appropriate sensor equipment (radar, lidar and camera) and are subsequently used as inputs:

- longitudinal velocity $v_{x,k}^{(n)} \in \mathbb{R}$
- lateral position on lane $y_k^{(n)} \in \mathbb{R}$
- indicator signal $I_k^{(n)} \in \{\text{left}, \text{off}, \text{right}\}$

The upcoming velocity is modeled using CLG distributions which are defined as follows.

$$\mathscr{L}(X|Z=z, I=i) = \mathscr{N}\left\{\mu(i) + B(i)^\top z, \Sigma(i)\right\} \tag{7}$$

Here, $\mathscr{N}$ denotes a Gaussian distribution where μ is a vector of mean values, B denotes the matrix containing the regression vectors for the continuous parents z and Σ is a table of variances. The probability for the lane $p_{y,k+i|k}$ is modeled using discrete probability tables that are conditioned on the indicator signal. The joint probability distribution over all random variables of a CG model represented by the graph is a multivariate Gaussian distribution, conditioned on the values of the discrete parents. The parameters of the model, i.e. $\mu(i), B(i), \Sigma(i)$ of the continuous nodes conditional and the probability tables of the discrete nodes are fitted using maximum likelihood estimation and a training data set based on real driving measurements. After incorporation of evidence, i.e. the measured variables, one can compute the marginal distribution, i.e. the future velocity and lateral position. In general $p_{y,k+i|k}$ may be also conditioned on the vehicle's actual inter-vehicle distance, e.g. the probability of merging increases with longer gaps. However, in a first approach it is assumed that the probability $p_{y,k+i|k}$ of a lane-change of a vehicle on the adjacent lane is independent on the gap.

IV. Stochastic Model Predictive Control

Although, in general the prediction uncertainty of the future motion of surrounding vehicles is bounded, it is difficult to determine meaningful tight bounds. To overcome these issues, a stochastic MPC is applied where probabilistic constraints on TTC are imposed. As elaborated in [3], [4], stochastic model predictive control strategies are well suited for automated driving application since they enable a meaningful treatment of uncertainty.

A. Chance constrained MPC (CC-MPC)

The chance constraints in (6) can be reformulated by using the predicted probability $p_{y,k+i|k}^{(n)}$ as follows:

$$\begin{aligned}\left[1-\Pr\left(R_{k+i|k}^{(n)}(\boldsymbol{x}_{k+i|k}, \boldsymbol{d}_{k+i|k}) \le R_{\max}\right)\right]\Pr\left(\Delta y_{k+i|k}^{(n)} \in \mathscr{Y}\right) \le \alpha \\ \Leftrightarrow \Pr\left(R_{k+i|k}^{(n)}(\boldsymbol{x}_{k+i|k}, \boldsymbol{d}_{k+i|k}) \le R_{\max}\right) \ge 1 - \frac{\alpha}{p_{y,k+i|k}^{(n)}}\end{aligned} \tag{8}$$

As a result, (8) can be used for tightening of the original constraints on the risk to each traffic participant. In other words, by imposing (8) to the optimization, the joint probability of a vehicle to be in the feasible set while it is in the ego vehicle's lane is constrained, i.e.

$$\Pr\left[R^{(n)}(\boldsymbol{x}_{k+i|k}, \boldsymbol{d}_{k+i|k}) \le R_{\max}\right] \ge 1 - \frac{\alpha}{p_{y,k+i|k}^{(n)}}. \tag{9}$$

As mentioned in the introduction, this work suggests to use the TTC to each traffic participant to constrain the risk and ensure safety. In the next steps, the deterministic equivalent constraints for the chance constraints on the TTC (9) are briefly explained. Further information on chance constrained programming can be found in [19].

B. Approximation of the chance constrained program

The deterministic equivalent objective function is formed by the expectation value $\overline{(\cdot)}$ of the state. The tightened constraints for vehicle n are calculated such that the probability of violating the original constraint equals the imposed risk probability α. The reformulation of the individual chance constraints (8) into deterministic equivalents can be examined for Gauss distributed state variables using the quantile function $\Phi^{-1}(\cdot)$. In case of constraints on TTC, (9) can be written as

$$\Pr\left[t_{c\min}\Delta v_{k+i+1|k}^{(n)} \le \Delta x_{k+i+1|k}^{(n)}\right] \ge 1 - \frac{\alpha}{p_{y,k+i|k}^{(n)}} \tag{10}$$

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where the relative velocity is given by

$$\Delta v^{(n)}_{k+i+1|k} = v_{k+i+1|k} - v^{(n)}_{x,k+i+1|k}. \tag{11}$$

Obviously, the prediction uncertainty of the velocity of vehicle n appears in both sides of the inequality (10) but with a difference of one sampling instant. However, it is assumed that the prediction model provides an estimation of the cumulative uncertainty of the multi-step ahead prediction, i.e. the uncertainty of $v^{(n)}_{k+i|k}$ is already considered in $v^{(n)}_{k+i+1|k}$. In order to avoid overestimation of the uncertainty, the constraints are tightened using the variance of the velocity prediction of vehicle (n).

$$\Pr\left[\frac{\Delta x^{(n)}_{k+i+1|k}}{t_{\text{cmin}}} - v_{k+i+1|k} + v^{(n)}_{x,k+i+1|k} \geq 0\right] \geq 1 - \frac{\alpha}{p^{(n)}_{y,k+i|k}} \tag{12}$$

$$\Phi\left(\frac{\frac{\Delta x^{(n)}_{k+i+1|k}}{t_{\text{cmin}}} - v_{k+i+1|k} + \bar{v}^{(n)}_{x,k+i+1|k}}{\sqrt{\Sigma^{(n)}_{v_{x,k+i+1|k}}}}\right) \geq 1 - \frac{\alpha}{p^{(n)}_{y,k+i+1|k}}. \tag{13}$$

where $\Sigma^{(n)}_{v_{x,k+i|k}}$ is the prediction variance of the velocity of vehicle n. In order to avoid close distances for low relative velocities, a static minimum distance Δx_{min} is added to the minimum headway due to TTC. Finally, the equivalent deterministic constraints for the allowed minimum distance of the flexible spacing corridor can be stated as follows.

$$\Delta x^{(n)}_{k+i+1|k} \geq \tag{14}$$
$$t_c\left[\Delta\bar{v}_{k+i+1|k} + \Phi^{-1}\left(1 - \frac{\alpha}{p^{(n)}_{y,k+i+1|k}}\right)\sqrt{\Sigma^{(n)}_{v_{x,k+i+1|k}}}\right] + \Delta x_{\text{min}}$$

with the expectation value of the relative velocity $\Delta\bar{v}_{k+i+1|k} = v_{k+i+1|k} - \bar{v}^{(n)}_{k+i+1|k}$ to vehicle n. The resulting constraint tightening margin $m = \Phi^{-1}\left(1 - \frac{\alpha}{p_y}\right)$ is illustrated in Fig. 2. It can be seen that for low p_y, the margin is negative which basically shifts the constraint away, i.e. the constraint is not supportive. The higher the probability of a vehicle to join the ego-vehicle's lane is, the tighter becomes the prevailing constraint. It has to be mentioned, that predictions with $p_y \leq \alpha$ are set to $m \to -\infty$, since the integral of the Gauss function is always positive.

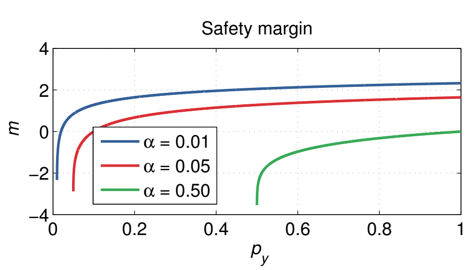

Fig. 2. Safety margin m as a function of p_y for different α.

C. Summary

As elaborated in [3], the approximated fuel consumption map $\hat{q}_{\text{f}}$ can be incorporated into optimization by imposing logic constraints. Using Big-M reformulation, the logic implications are reformulated in order to apply numeric MILP solvers, see [20]. For (15c) meaningful values for the tire force saturation and maximum acceleration are imposed.

As the leading vehicle can change, the question arises to which vehicle the terminal costs (3) are referred. Therefore, the terminal costs (3) are defined for each surrounding vehicle. Subsequently, $V^{(n)}_f$ is incorporated into the cost function by multiplication with the corresponding predicted probability of the vehicle to be the leading vehicle, denoted $p^{(n)}_{L,k+h_{\text{P}}|k}$. This ensures that the inter-vehicle distance to the most likely future leader is tracked. Note that $\hat{V}^{(n)}_f$ is a piecewise affine approximation of the quadratic function with $j = 12$ segments. The approximation enables to apply standard MILP solvers.

Subsequently, the deterministic equivalent receding horizon optimization problem for multi-lane P-ACC can be stated as follows.

$$\overline{Q}^{\star}_{\text{f},k} = \min_{u_k} T_{\text{S}} \sum_{i=0}^{h_{\text{P}}-1} \hat{q}_{\text{f},k+i|k} + \sum_{n=0}^{N} \hat{V}^{(n)}_f p^{(n)}_{L,k+h_{\text{P}}|k} \quad \text{s.t.} \tag{15a}$$
$$\forall i = \{0 \ldots h_{\text{P}} - 1\}:$$
$$\bar{\boldsymbol{x}}_{k+i+1|k} = \boldsymbol{A}\bar{\boldsymbol{x}}_{k+i|k} + \boldsymbol{B}a_{k+i|k} + \boldsymbol{B}_d\bar{\boldsymbol{d}}_{k+i|k} \tag{15b}$$
$$a_{\text{min}} \leq a_{k+i|k} \leq a_{\text{max}} \tag{15c}$$
$$\text{Eq. } (11), (14) \tag{15d}$$
$$\boldsymbol{x}_{k,k} = \hat{\boldsymbol{x}}_0 \tag{15e}$$
$$\hat{V}^{(n)}_f = f(\boldsymbol{x}^{(n)}_{k+h_{\text{P}}|k}, t_{\text{hdes}}) \tag{15f}$$
$$\hat{q}_{\text{f},k+i|k} = f(v_{x,k+i|k}, a_{x,k+i|k}, g_{k+i|k}) \tag{15g}$$

V. Evaluation

The control algorithm (15) is evaluated using the high fidelity simulation software IPG CarMaker® which provides an interface to Simulink®. The optimization problem has been formulated using YALMIP [21] employing the solver Gurobi [22]. A PI controller is used to set acceleration and brake pedals in order to track the desired acceleration determined by the MPC. The basic parameters are summarized in Table I.

A. Traffic scenario

For a first evaluation of the approaches, an artificial traffic scenario is constructed. As illustrated in Fig. 1, vehicle V1 is crossing the lane while the ego vehicle is following V1. Notice, that V2 is driving with lower velocity than V1, leading to a critical traffic scenario. The trajectories of the surrounding vehicles can be seen in the upper two subplots of Fig. 3.

B. Performance criteria

For the examined evaluations, a simple PD-ACC with a distance policy according to constant time headway [23] serves as basis of comparison. This means that the desired

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TABLE I

CONTROLLER PARAMETERS

	Parameter	Value
General	Lane width	$w_l = 3.75\,\mathrm{m}$
	Simulation time	$T_{\mathrm{Sim}} = 30\,\mathrm{s}$
P-ACC	Sampling time	$T_{\mathrm{S}} = 0.1\,\mathrm{s}$
	Prediction horizon	$h_{\mathrm{P}} = 6\,\mathrm{s}$
	Min. TTC	$t_{\mathrm{c\,min}} = 5\,\mathrm{s}$
	Max. risk	$R_{\mathrm{max}} = 0.2\,\mathrm{s}^{-1}$
	Terminal weight	$c_f = 1$
	Des. time headway	$t_{\mathrm{hdes}} = 2.7\,\mathrm{s}$
	Acceleration	$-5\,\mathrm{m/s^2} \leq a_k \leq 5\,\mathrm{m/s^2}$
	Minimum distance	$\Delta x_{\mathrm{min}} = 10\,\mathrm{m}$
	Considered gears	$g_k = [5,6,7]$
PD-ACC	Sampling time	$T_{\mathrm{S}} = 0.01\,\mathrm{s}$
	Time headway	$t_{\mathrm{hdes}} = 1.4\,\mathrm{s}$
	Parameters	$k_1 = 3,\ k_2 = 8$

inter-vehicle distance Δx_{des} consists of a minimum distance Δx_{min} and an additional time headway t_{h}, see (1). The desired distance to the preceding vehicle n_P is subsequently tracked with a PD controller, i.e.

$$a_k = k_1(\Delta x_k^{(n)} - \Delta x_{\mathrm{des},k}) + k_2(v_k^{(n)} - v_k) \quad \forall n = n_P \tag{16}$$

where n_P is the actual preceding vehicle. It is assumed, that this controller tracks the distance to the nearest vehicle n_P for which the lateral displacement belongs to the set $\mathscr{Y}$, see (5).

To assess the performance of the control approaches, the following criteria are evaluated and compared with the above explained standard PD-ACC algorithm. For assessing the safety of the approaches, the minimum TTC to the surrounding traffic participants on the path of the ego vehicle is evaluated over the simulation time T_{Sim}, i.e.

$$\hat{t}^*_{\mathrm{c\,min}} = \min_{T_{\mathrm{Sim}}} \left\{ t_{\mathrm{c},k} \right\} \tag{17}$$

In order to evaluate driving comfort, the root mean square value of the jerk is evaluated

$$\Delta a_{\mathrm{tot}} = \sqrt{\frac{T_{\mathrm{S}}}{T_{\mathrm{Sim}}} \sum_{k=1}^{T_{\mathrm{Sim}}/T_{\mathrm{S}}} (a_k - a_{k-1})^2}. \tag{18}$$

Moreover, the total root mean square value of the acceleration is computed

$$a_{\mathrm{tot}} = \sqrt{\frac{T_{\mathrm{S}}}{T_{\mathrm{Sim}}} \sum_{k=1}^{T_{\mathrm{Sim}}/T_{\mathrm{S}}} a_k^2}. \tag{19}$$

Lastly, the fuel consumption Q_{f}, calculated with the IPG CarMaker® multibody simulation model is evaluated.

C. Results

It can be seen in Fig. 3, subplot 6, that time-headway tracking of the classical PD-ACC results in excessive braking maneuvers and higher risk. This is caused mainly by the fact, that this controller does not incorporate any prediction of the traffic participants future movements.

In contrast, the stochastic MPC based P-ACC approach reacts aware of the potential cut-off of the surrounding vehicle. When the indicator of the car is activated (at $t = 5.5$s), the probability $p^{(2)}_{y,k+i|k}$ increases which leads to a predictive fall-back of the controlled vehicle. The controller automatically decreases the (relative) velocity to the cutting vehicle in order to maintain the constraint on the TTC. When V2 actually enters the lane, the controlled vehicle's risk is clearly below the maximum risk. Then, the probability $p_{y,k+i|k}$ decreases since V2 is going to leave the lane again and the desired distance to V1 is tracked.

Notice that for increasing α, the controller gets less conservative which can be seen in terms of the increased risk during the maneuver, see Fig. 3. The deterministic controller is represented by $\alpha = 0.5$, where the constraints are not tightened. However in the used simulation scenario, the MPC approaches are able to keep the risk lower than the imposed threshold, whereas the PD-ACC violates it when the cut-off happens. It has to be mentioned that, in order to continue simulation in case of an infeasible solutions, a slack variable has been added to (14).

As it can be seen in Table II, the P-ACC controller achieves in the considered scenario a higher fuel efficiency than the PD-ACC algorithm. Moreover, maximum acceleration and jerk are reduced significantly which indicates that the presented controller improves comfort. It has to be mentioned, that the cost function does not include any terms representing driving comfort and therefore the applied braking is still intense and lacks smoothness. This could be improved by adding a weight on the jerk.

TABLE II

COMPARISON TABLE

Contr.	α	$\hat{t}^*_{\mathrm{c\,min}}$[s]	$\Delta a_{\mathrm{tot}}[\mathrm{m/s^2}]$	$a_{\mathrm{tot}}[\mathrm{m/s^2}]$	$Q_{\mathrm{f}}[\mathrm{L/100km}]$
PD	—	2.15	0.57	1.69	8.99
MPC	0.001	13.78	0.19	0.79	6.22
	0.01	7.92	0.16	0.74	5.97
	0.5	5.53	0.14	0.69	5.82

D. Computational Effort

The necessary computation time of the approach consists of the prediction time and the time for solving the optimization problem. The computational effort for prediction is $t_{c,\mathrm{pred}} = 0.18\,\mathrm{s}$ and solving the optimization problem is $t_{c,\mathrm{solve}} = 0.3\,\mathrm{s}$ which indicates that the approach is currently not real-time capable.

VI. CONCLUSION

This paper presents an approach for fuel efficient longitudinal ACC on multi-lane highways. A chance constrained MPC has been developed and deterministic equivalent constraints for the risk function TTC have been formulated. Obviously, constraints on TTC allow closer inter-vehicle distances if the relative velocity is low which indicates that time to collision constraints are less conservative than constraining the time headway. The evaluation indicates that the approach

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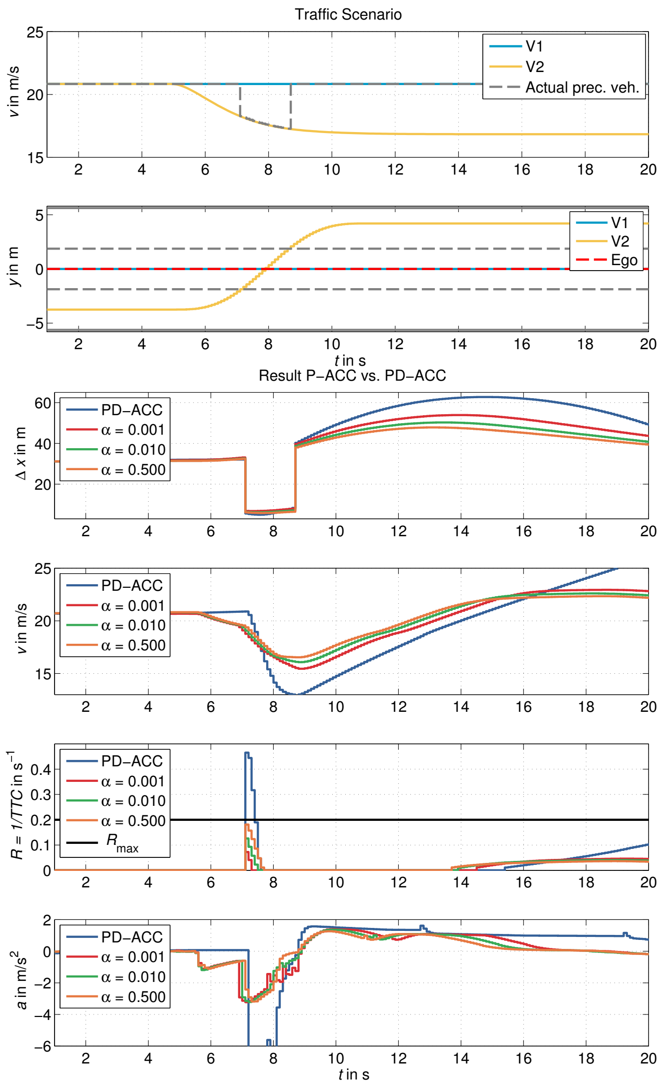

Fig. 3. Comparison between standard ACC (PD-ACC) and risk constrained approach with different values for α. Notice that Δx and TTC refer to the prevailing preceding vehicle.

improves comfort and fuel-efficiency significantly when it comes to sharp cut-off maneuvers of other vehicles.

Future work will investigate the effects of different risk functions in connection with stochastic Model Predictive Control and motion prediction models. Moreover, a decent evaluation using real-world driving cycles and a stochastic traffic environment will be examined.

VII. Acknowledgments

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