Adaptive Cruise Control Based on Multistage Predictive Control Approach

Aries Subiantoro
Department of Electrical Engineering
Universitas Indonesia
Depok, Indonesia
aries.subiantoro@ ui.ac.id

Fauzan Muzakir
Department of Electrical Engineering
Universitas Indonesia
Depok, Indonesia
fauzan.muzakir@ui.ac.id

Feri Yusivar
Department of Electrical Engineering
Universitas Indonesia
Depok, Indonesia
feri.yusivar@ ui.ac.id

***Abstract*—The paper deals with the design of adaptive cruise control by integrating longitudinal motion control and a safety logic control. The proposed control algorithm consists of an inner loop to manipulate the brake pedal and throttle opening position, and an outer loop to maintain the safe distance from leading vehicle. A multistage local linear model is identified using least squares method to describe nonlinear dynamic of longitudinal motion model. The objective of multistage model based predictive control is to minimize square of longitudinal vehicle speed error and square of brake pedal and throttle torque through receding horizon. By using the proposed control algorithm, the predictive control problem is still maintained as a quadratic programming problem. The safety logic control is designed base on the reference value of vehicle speed and distance from heading vehicle. Simulation results are given to demonstrate the effectiveness of the proposed control strategies.**

Keywords—adaptive cruise control, model predictive, vehicle modeling, multistage identification

I. INTRODUCTION

Cruise control and driver assistance are already known as some routes to enhance driving safety. Nowadays, there are even more countries which have active safety legislation. Every new car released by industry must have active safety element inside to prevent road accidents and car fatalities. Cruise control (CC) in vehicle has been developed to maintain a constant speed at a specific level. Adaptive cruise control (ACC) is an extension of the CC systems and manipulates braking and engine throttle position to maintain a predefined ratio of the distance between the ACC vehicle and the front vehicle in front to the speed of the ACC vehicle. ACC offers benefit in reducing stress of driving in highly traffic by serving as a longitudinal control pilot, because the vehicle will automatically adjust the vehicle speed to keep a safe distance from the vehicle in front and avoid collisions between the cars.

In recent years, a number of ACC methods has been carried out and published. A nonlinear model predictive control (NMPC) is developed by incorporating the objective of the distance tracking into extended original LTI models [1]. This approach eliminates the need to design the outer loop controller. In [2] a vehicle-to-vehicle communication based hybrid ACC is designed by considering two different strategies for generating the safe interdistance between vehicles. In the work reported by [3] a supervised adaptive dynamic programming algorithm is used for a full-range ACC systems. Moon proposed an integrating ACC and collision avoidance to improve driver's comfort during three different situations i.e. safe, warning, and dangerous modes [4]. Another work of Zhenhai et.al. [5] has shown continuous and smooth vehicle acceleration during control mode switching. However, most ACC methods have similiar problems encountered i.e. difficult to design and computationally expensive. This is because the vehicles have highly nonlinear behavior and their mathematical models are very difficult to obtain.

This paper presents a nonlinear model predictive control (NMPC)-based, which is called multistage predictive control, ACC system integrated with a safety logic control. The main contribution of this work lies on utilizing a multistage linear model to describe nonlinear behavior of ACC system, but the relationship between parameter model matrices is still kept linear. Therefore, the predictive control problem is maintained as a quadratic programming rather than dynamic programming problem. In order to verify the performance of proposed controller, some simulation results are provided.

II. SYSTEM DYNAMICS AND MULTISTAGE LINEAR MODEL IDENTIFICATION

A. Nonlinear Dynamics

In this paper, the following equations are used to describe the nonlinear dynamic of the vehicle which will be identified by using the multistage least-squares algorithm for predictive control design purpose, as follows [6][7]

$$
\begin{aligned}
\dot{v}_x &= \frac{1}{r}\frac{(T_w - T_b)}{m} \\
\dot{d} &= v_r \\
\dot{v}_r &= \dot{v}_{xh} - a_l \\
\Delta\dot{\psi} &= \left(\frac{d_y^2}{d^2}\right)\left(\frac{v_{ry}d_x - v_{rx}d_y}{d_x^2}\right) - \dot{\psi}_h
\end{aligned}
\tag{1}
$$

where

$$\dot{\psi}_h = \frac{v_{xh}\cos(\beta)}{l_f + l_r}\left(\tan(\delta_f) - \tan(\delta_r)\right) \tag{2}$$

The throttle controlling signal on the engine u_t and the brake controlling signal u_b are chosen as the input variables. The torque generated in the wheel through the engine is formulated by following equation

$$T_w = R_f R_{tr} C_r \left(T_e(u_t, N_e) I_{ei} \alpha_e\right) \tag{3}$$

And the braking torque will be calculated as follows

$$T_b = k_b . u_b \tag{4}$$

B. Multistage Linear Model Identification

Referred to [8], a multistage linear model is a nonlinear model that consists of *N* linear models as shown in Figure (1). In order to compensate error between the nonlinear output and the output of *i*th-stage linear model, the higher *i+1*th-stage of linear model has a correcting matrix $\mathbf{k}_{xi+1}$. For

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example, the linear multi model structure with two stages can be represented by two local linear models:

$$\begin{aligned} \mathbf{x}_1(k+1) &= \mathbf{A}_1\mathbf{x}_1(k)+\mathbf{B}_1\mathbf{u}(k) \\ \mathbf{y}_1(k) &= \mathbf{C}_1\mathbf{x}_1(k)+\mathbf{D}_1\mathbf{u}(k) \end{aligned} \tag{5}$$

$$\begin{aligned} \mathbf{x}_2(k+1) &= \mathbf{A}_2\mathbf{x}_2(k)+\mathbf{B}_2\mathbf{u}(k)+\mathbf{k}_{x2}\mathbf{e}_1(k) \\ \mathbf{y}_2(k) &= \mathbf{C}_2\mathbf{x}_2(k)+\mathbf{D}_2\mathbf{u}(k)+\mathbf{K}_{y2}\mathbf{e}_1(k) \end{aligned} \tag{6}$$

This model has input $\mathbf{u}(k)$ and output $\mathbf{y}_2(k)$ with model matrices ($\mathbf{A}_1$,$\mathbf{B}_1$,$\mathbf{C}_1$,$\mathbf{D}_1$,$\mathbf{A}_2$,$\mathbf{B}_2$,$\mathbf{C}_2$,$\mathbf{D}_2$,$\mathbf{k}_{x2}$,$\mathbf{K}_{y2}$) that must be estimated based on the experimental data $\{\mathbf{u}(j),\mathbf{y}(j)\}$ j=0, 1, ..., N-1. The prediction error signal $\mathbf{e}_1(k)$ is generated by the difference between the measured output system from the sensor $\mathbf{y}(k)$ and the first stage linear model output $\hat{\mathbf{y}}_1(k)$.

$$\mathbf{e}_1(k) = \mathbf{y}(k)-\hat{\mathbf{y}}_1(k) \tag{7}$$

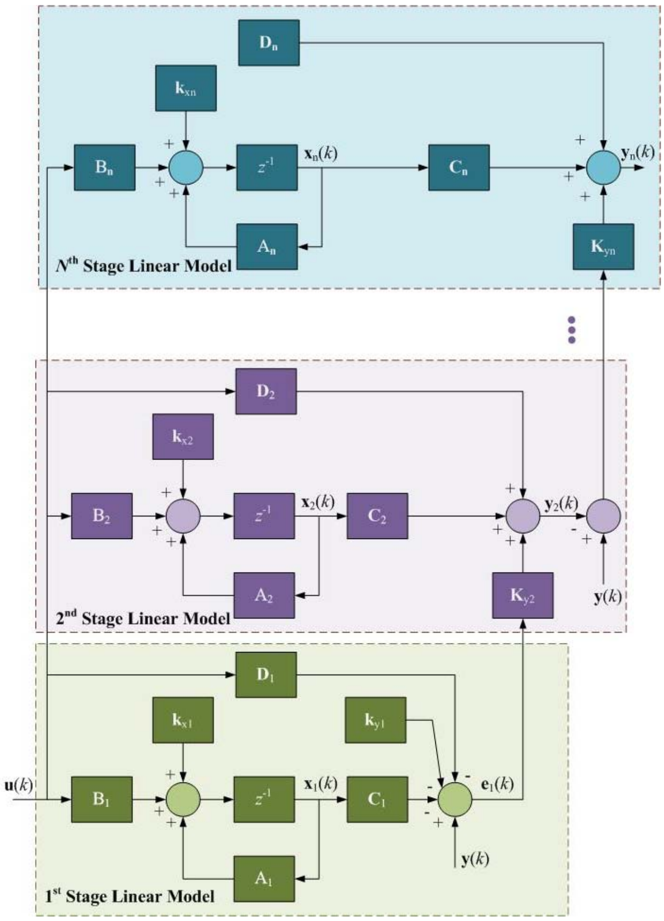

Figure 1. Structure of multistage linear model

According to the prediction error equation in the second stage linear model $\mathbf{e}_2(k)$, the identification problem for this multistage linear model has a nonlinear relation between the model parameters as follows

$$\mathbf{e}_2(k)=\left(\mathbf{I}-\hat{\mathbf{K}}_{y2}\right)\mathbf{y}(k)-\hat{\mathbf{C}}_2\mathbf{x}_2(k)-\hat{\mathbf{D}}_2\mathbf{u}_2(k)+\hat{\mathbf{K}}_{y2}\hat{\mathbf{C}}_1\mathbf{x}_1(k)+\cdots$$
$$+\hat{\mathbf{K}}_{y2}\hat{\mathbf{D}}_1\mathbf{u}_1(k)$$

The main idea of the identification solution of the multistage linear model structure comes from the identification technique using autoregressive moving average exogeneous (ARMAX) model [9]. The existence of the polynomial noise makes the relation between each of the model parameters becoming nonlinear. But with the prediction error method, the multistage least square estimation method is also practical to be used to estimate the model parameters.

By assuming state variables $\mathbf{x}(k)$ are measureable, the multistage linear model can be estimated using the four stage least square method. The first two stages of the identification are used to estimate the state equation and the output equation of the first linear model. The other two are used to identify the second linear model. For a system with p number of input and q number of output and n state variables, the first stage of state space model

$$\begin{aligned} \mathbf{x}_1^T(k+1) &= \begin{bmatrix}\mathbf{x}_1^T(k) & \mathbf{u}^T(k)\end{bmatrix}\begin{bmatrix}\mathbf{A}^T \\ \mathbf{B}^T\end{bmatrix} \\ \mathbf{y}_1^T(k) &= \begin{bmatrix}\mathbf{x}_1^T(k) & \mathbf{u}^T(k)\end{bmatrix}\begin{bmatrix}\mathbf{C}^T \\ \mathbf{D}^T\end{bmatrix} \end{aligned} \tag{8}$$

can be written as

$$\begin{aligned} \boldsymbol{\chi}_1^T &= \varphi_1^T\boldsymbol{\theta}_1 \\ \boldsymbol{\chi}_2^T &= \varphi_2^T\boldsymbol{\theta}_2 \end{aligned} \tag{9}$$

It is seen that the state equations are a dynamic system with data vector $\boldsymbol{\varphi}_1^{\mathrm{T}}$ that consists of the current time data compared to $\boldsymbol{\chi}_1^{\mathrm{T}}$ with future time data, and the output equations are static system models. For a set of system data $\{\mathbf{u}(j),\mathbf{x}(j),\mathbf{y}(j)\}$ j = 0, 1, ..., N the state space model is written as,

$$\begin{aligned} \mathbf{X}_1 &= \boldsymbol{\Phi}_1\boldsymbol{\theta}_1 \\ \mathbf{X}_2 &= \boldsymbol{\Phi}_2\boldsymbol{\theta}_2 \end{aligned} \tag{10}$$

where

$$\mathbf{X}_1=\begin{bmatrix}\mathbf{x}_1^T(1) \\ \mathbf{x}_1^T(2) \\ \vdots \\ \mathbf{x}_1^T(N)\end{bmatrix} \qquad \boldsymbol{\Phi}_1=\begin{bmatrix}\mathbf{x}_1^T(0) & \mathbf{u}^T(0) \\ \mathbf{x}_1^T(1) & \mathbf{u}^T(1) \\ \vdots & \vdots \\ \mathbf{x}_1^T(N-1) & \mathbf{u}^T(N-1)\end{bmatrix} \tag{11}$$

and

$$\mathbf{X}_2=\begin{bmatrix}y_1^T(0) \\ y_1^T(1) \\ \vdots \\ y_1^T(N)\end{bmatrix} \qquad \boldsymbol{\Phi}_2=\begin{bmatrix}\mathbf{x}_1^T(0) & \mathbf{u}^T(0) \\ \mathbf{x}_1^T(1) & \mathbf{u}^T(1) \\ \vdots & \vdots \\ \mathbf{x}_1^T(N) & \mathbf{u}^T(N)\end{bmatrix} \tag{12}$$

Thus, the first stage of linear model matrix can be calculated with the following least squares formula

$$\hat{\boldsymbol{\theta}}_1=\begin{bmatrix}\hat{\mathbf{A}}_1^T \\ \hat{\mathbf{B}}_1^T\end{bmatrix}=\left(\boldsymbol{\Phi}_1^T\boldsymbol{\Phi}_1\right)^{-1}\boldsymbol{\Phi}_1^T\mathbf{X}_1 \tag{13}$$

$$\hat{\boldsymbol{\theta}}_2=\begin{bmatrix}\hat{\mathbf{C}}_1^T \\ \hat{\mathbf{D}}_1^T\end{bmatrix}=\left(\boldsymbol{\Phi}_2^T\boldsymbol{\Phi}_2\right)^{-1}\boldsymbol{\Phi}_2^T\mathbf{X}_2 \tag{14}$$

The estimation matrix is then used to generate the output of the first linear model and produces the prediction error signal $\mathbf{e}_1(k)$. The second stage linear model is then

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representated again by combining the input signal $\mathbf{u}(k)$ with error prediction signal $\mathbf{e}_1(k)$ in the following equation form:

$$\mathbf{x}_2^T(k+1) = \begin{bmatrix}\mathbf{x}_2^T(k) & \mathbf{u}^T(k) & 1\end{bmatrix}\begin{bmatrix}\mathbf{A}_2^T \\ \mathbf{B}_2^T \\ \mathbf{k}_{x2}^T\end{bmatrix} \tag{15}$$

$$\mathbf{y}_2^T(k) = \begin{bmatrix}\mathbf{x}_2^T(k) & \mathbf{u}^T(k) & \mathbf{e}_1^T(k)\end{bmatrix}\begin{bmatrix}\mathbf{C}_2^T \\ \mathbf{D}_2^T \\ \mathbf{k}_{y2}^T\end{bmatrix} \tag{16}$$

The prediction error signal $\mathbf{e}_1(k)$ makes the second stage of linear model have an additional input signal, so the size of the regression matrix will increase. With the same technique like estimating the first stage of linear model, the second stage of linear model estimation also consists of two stages to formulate the matrices ($\mathbf{A}_2$,$\mathbf{B}_2$,$\mathbf{k}_{x2}$) in the first level and ($\mathbf{C}_2$,$\mathbf{D}_2$,$\mathbf{K}_{y2}$) in the second stage. By defining the new regression matrices as $\mathbf{\Phi}_3$ and $\mathbf{\Phi}_4$ and the output matrices $\mathbf{X}_3$ and $\mathbf{X}_4$ for the state equation and output equation respectively, the second stage of linear model estimation by least squares formula is written as

$$\hat{\boldsymbol{\theta}}_3 = \begin{bmatrix}\hat{\mathbf{A}}_2^T \\ \hat{\mathbf{B}}_2^T \\ \hat{\mathbf{k}}_{x2}^T\end{bmatrix} = \left(\mathbf{\Phi}_3^T\mathbf{\Phi}_3\right)^{-1}\mathbf{\Phi}_3^T\mathbf{X}_3 \tag{17}$$

$$\hat{\boldsymbol{\theta}}_4 = \begin{bmatrix}\hat{\mathbf{C}}_2^T \\ \hat{\mathbf{D}}_2^T \\ \hat{\mathbf{K}}_{y2}^T\end{bmatrix} = \left(\mathbf{\Phi}_4^T\mathbf{\Phi}_4\right)^{-1}\mathbf{\Phi}_4^T\mathbf{X}_4 \tag{18}$$

III. Multistage Predictive Control Based Adaptive Cruise Systems

The proposed ACC structure consists of the inner loop designed by using multistage predictive control approach, and the outer loop which is developed to calculate safety distance and relative speed between lead-car and host-car, as shown in Figure (2).

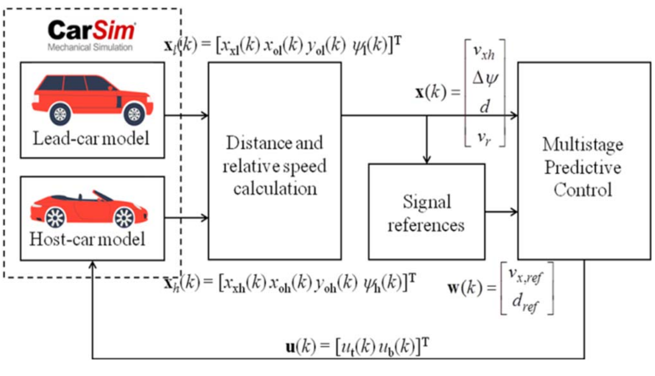

Fig. 2. Structure of the proposed Adaptive Cruise Control

A. Multistage Predictive Control

The control objective is to minimize the criterion function

$$V(k) = \sum_{j=1}^{q}\sum_{i=1}^{Hp}\left\|\hat{y}_j(k+i) - w_j(k+i)\right\|_{\mathbf{Q}}^2 + \sum_{j=1}^{p}\sum_{i=0}^{Hu-1}\left\|\Delta\hat{u}_j(k+i)\right\|_{\mathbf{R}}^2$$

where $\mathbf{Q} > 0$ is the symmetrical positive definite matrix, $\mathbf{R} \geq 0$ is a positive semidefinite matrix, H_p is prediction horizon, H_u is control horizon; subjected to the constraints of the input and the change of the input system

$$\mathbf{u}_{\min} \leq \mathbf{u}(k) \leq \mathbf{u}_{\max}$$

$$\Delta\mathbf{u}_{\min} \leq \Delta\mathbf{u}(k) \leq \Delta\mathbf{u}_{\max}$$

The solution of optimizing the predictive controller specification by involving the constraints of the inequalities becomes the problem of quadratic programming which can be solved by an active-set method.

In this paper, the authors only consider the controlled system has deterministic characteristic, two inputs $\mathbf{u}(k)=[u_t(k)\ u_b(k)]^T$, two output $\mathbf{y}(k) = [v_{xh}(k)\ d(k)]^T$, and defining some assumptions:

- The system is stable and has its eigenvalues inside the *unit circle* $|\lambda_i| \leq 1$
- The system has perfect controllable ($\mathbf{A}$, $\mathbf{B}$) and observable ($\mathbf{A}$, $\mathbf{C}$) characteristics
- The number of the state variables n is known and measureable $\mathbf{x}(k) = [v_{xh}(k)\ \Delta\psi(k)\ d(k)\ v_r(k)]^T$

First step of predictive control solution is to develop prediction model base on multistage linear model. The estimated first stage of linear model is used to generate an error signal $\mathbf{e}_1(k)$ between the nonlinear output and the local linear model. The second state space model is used to form a prediction model and iterated for H_p next step time as follows:

$$\begin{aligned}
\hat{\mathbf{x}}_2(k+1) &= \mathbf{A}_2\mathbf{x}_2(k) + \mathbf{B}_2\hat{\mathbf{u}}(k\,|\,k) + \mathbf{k}_{x2} \\
\hat{\mathbf{x}}_2(k+2) &= \mathbf{A}_2\hat{\mathbf{x}}_2(k+1) + \mathbf{B}_2\hat{\mathbf{u}}(k+1) + \mathbf{k}_{x2} \\
&= \mathbf{A}_2^2\mathbf{x}_2(k) + \mathbf{A}_2\mathbf{B}_2\hat{\mathbf{u}}(k) + \mathbf{B}_2\hat{\mathbf{u}}(k+1) + (\mathbf{A}_2 + \mathbf{I})\mathbf{k}_{x2} \\
&\vdots \\
\hat{\mathbf{x}}_2(k+H_p) &= \mathbf{A}_2\hat{\mathbf{x}}_2(k+H_p-1) + \mathbf{B}_2\hat{\mathbf{u}}(k+H_p-1) + \mathbf{k}_{x2} \\
&= \mathbf{A}_2^{H_p}\mathbf{x}_2(k) + \mathbf{A}_2^{H_p-1}\mathbf{B}_2\hat{\mathbf{u}}(k) + \cdots + \mathbf{B}_2\hat{\mathbf{u}}(k+H_p-1) + \cdots \\
&\quad + \left(\mathbf{A}_2^{H_p-1} + \cdots + \mathbf{A}_2 + \mathbf{I}\right)\mathbf{k}_{x2}
\end{aligned}$$

The state variable prediction model is later converted to a function of a change of control signal $\Delta\mathbf{u}(k+i|k)$. By defining $\Delta\mathbf{u}(k+i|k) = \mathbf{u}(k+i|k) - \mathbf{u}(k+i\text{-}1|k)$, the control signal prediction value along the control horizon can be written as

$$\begin{aligned}
\hat{\mathbf{u}}(k) &= \Delta\hat{\mathbf{u}}(k) + \mathbf{u}(k-1) \\
\hat{\mathbf{u}}(k+1) &= \Delta\hat{\mathbf{u}}(k+1) + \Delta\hat{\mathbf{u}}(k) + \mathbf{u}(k-1) \\
&\vdots \\
\hat{\mathbf{u}}(k+H_u-1) &= \Delta\hat{\mathbf{u}}(k+H_u-1) + \cdots + \Delta\hat{\mathbf{u}}(k) + \mathbf{u}(k-1)
\end{aligned}$$

Substituting both of the equations above, we have state variable prediction equations that are function of the measured state variables $\mathbf{x}(k)$, the past control signal data $\mathbf{u}(k\text{-}1)$, and the change of the control signal along the control horizon H_u

$$\begin{aligned}
\hat{\mathbf{x}}_2(k+1) &= \mathbf{A}_2\mathbf{x}_2(k) + \mathbf{B}_2\Delta\hat{\mathbf{u}}(k) + \mathbf{B}_2\mathbf{u}(k-1) + \mathbf{k}_{x2} \\
\hat{\mathbf{x}}_2(k+2) &= \mathbf{A}_2^2\mathbf{x}_2(k) + (\mathbf{A}_2 + \mathbf{I})\mathbf{B}_2\Delta\hat{\mathbf{u}}(k) + \mathbf{B}_2\Delta\hat{\mathbf{u}}(k+1) + \cdots \\
&\quad + (\mathbf{A}_2 + \mathbf{I})\mathbf{B}_2\mathbf{u}(k-1) + (\mathbf{A}_2 + \mathbf{I})\mathbf{k}_{x2} \\
&\vdots \\
\hat{\mathbf{x}}_2(k+H_p) &= \mathbf{A}_2^{H_p}\mathbf{x}_2(k) + \left(\mathbf{A}_2^{H_p-1} + \cdots + \mathbf{A}_2 + \mathbf{I}\right)\mathbf{B}_2\Delta\hat{\mathbf{u}}(k) + \cdots \\
&\quad + \left(\mathbf{A}_2^{H_p-H_u} + \cdots + \mathbf{A}_2 + \mathbf{I}\right)\mathbf{B}_2\Delta\hat{\mathbf{u}}(k+H_u-1) + \cdots \\
&\quad + \left(\mathbf{A}_2^{H_p-1} + \cdots + \mathbf{A}_2 + \mathbf{I}\right)\mathbf{B}_2\mathbf{u}(k-1) + \cdots \\
&\quad + \left(\mathbf{A}_2^{H_p-1} + \cdots + \mathbf{A}_2 + \mathbf{I}\right)\mathbf{k}_{x2}
\end{aligned}$$

The state variable prediction equation can be written in the form of matrix and vector notation

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$$\begin{bmatrix}\hat{\mathbf{x}}_2(k+1)\\ \hat{\mathbf{x}}_2(k+2)\\ \vdots\\ \hat{\mathbf{x}}_2(k+H_p)\end{bmatrix} = \begin{bmatrix}\mathbf{A}_2\\ \mathbf{A}_2^2\\ \vdots\\ \mathbf{A}_2^{H_p}\end{bmatrix}\mathbf{x}_2(k) + \begin{bmatrix}\mathbf{B}_2\\ \mathbf{A}_2\mathbf{B}_2\\ \vdots\\ \sum_{i=0}^{H_p-1}\mathbf{A}_2^i\mathbf{B}_2\end{bmatrix}\mathbf{u}(k-1)+\cdots$$

$$+\begin{bmatrix}\mathbf{B}_2 & \cdots & \mathbf{0}\\ \mathbf{A}_2\mathbf{B}_2+\mathbf{B}_2 & \cdots & \mathbf{0}\\ & \ddots & \\ \sum_{i=0}^{H_p-1}\mathbf{A}_2^i\mathbf{B}_2 & \cdots & \sum_{i=0}^{H_p-H_u}\mathbf{A}_2^i\mathbf{B}_2\end{bmatrix}\begin{bmatrix}\Delta\hat{\mathbf{u}}(k)\\ \vdots\\ \Delta\hat{\mathbf{u}}(k+H_u-1)\end{bmatrix}+\cdots$$

$$+\begin{bmatrix}\mathbf{I}\\ \mathbf{A}_2+\mathbf{I}\\ \vdots\\ \sum_{i=0}^{H_p-1}\mathbf{A}_2^i\end{bmatrix}\begin{bmatrix}\mathbf{k}_{x2}\\ \mathbf{k}_{x2}\\ \vdots\\ \mathbf{k}_{x2}\end{bmatrix}$$

The iteration of the output prediction equation along the prediction horizon uses the second stage model

$$\begin{aligned}\hat{\mathbf{y}}_2(k+1) &= \mathbf{C}_2\hat{\mathbf{x}}_2(k+1)+\mathbf{D}_2\hat{\mathbf{u}}(k+1)+\mathbf{K}_{y2}\mathbf{e}_1(k+1)\\ &= \mathbf{C}_2\hat{\mathbf{x}}_2(k+1)+\mathbf{D}_2\big(\Delta\hat{\mathbf{u}}(k+1)+\Delta\hat{\mathbf{u}}(k)+\cdots\\ &\quad +\mathbf{u}(k-1)\big)+\mathbf{K}_{y2}\mathbf{e}_1(k+1)\\ \hat{\mathbf{y}}_2(k+2) &= \mathbf{C}_2\hat{\mathbf{x}}_2(k+2)+\mathbf{D}_2\hat{\mathbf{u}}(k+2)+\mathbf{K}_{y2}\mathbf{e}_1(k+2)\\ &= \mathbf{C}_2\hat{\mathbf{x}}_2(k+2)+\mathbf{D}_2\big(\Delta\hat{\mathbf{u}}(k+2)+\Delta\hat{\mathbf{u}}(k+1)+\cdots\\ &\quad +\Delta\hat{\mathbf{u}}(k)+\mathbf{u}(k-1)\big)+\mathbf{K}_{y2}\mathbf{e}_1(k+2)\\ &\ \vdots\\ \hat{\mathbf{y}}_2(k+H_p) &= \mathbf{C}_2\hat{\mathbf{x}}_2(k+H_p)+\mathbf{D}_2\hat{\mathbf{u}}(k+H_u-1)+\cdots\\ &\quad +\mathbf{K}_{y2}\mathbf{e}_1(k+H_p)\\ &= \mathbf{C}_2\hat{\mathbf{x}}_2(k+H_p)+\mathbf{D}_2\big(\Delta\hat{\mathbf{u}}(k+H_u-1)+\cdots\\ &\quad +\Delta\hat{\mathbf{u}}(k)+\mathbf{u}(k-1)\big)+\mathbf{K}_{y2}\mathbf{e}_1(k+1)\end{aligned}$$

It is assumed that the steady state error of the linear model is constant along the prediction horizon, thus $\hat{\mathbf{e}}_1(k+1|k) = \hat{\mathbf{e}}_1(k+2|k) = ... = \hat{\mathbf{e}}_1(k+H_p|k) = \hat{\mathbf{e}}_1(k|k)$.

$$\underbrace{\begin{bmatrix}\hat{\mathbf{y}}_2(k+1)\\ \hat{\mathbf{y}}_2(k+2)\\ \vdots\\ \hat{\mathbf{y}}_2(k+H_p)\end{bmatrix}}_{\mathbf{Y}} = \mathbf{C}_y\underbrace{\begin{bmatrix}\mathbf{A}_2\\ \mathbf{A}_2^2\\ \vdots\\ \mathbf{A}_2^{H_p}\end{bmatrix}}_{\mathbf{\Psi}}\mathbf{x}(k)+\mathbf{C}_y\underbrace{\begin{bmatrix}\mathbf{B}_2\\ \mathbf{A}_2\mathbf{B}_2+\mathbf{B}_2\\ \vdots\\ \sum_{i=0}^{H_p-1}\mathbf{A}_2^i\mathbf{B}_2\end{bmatrix}}_{\mathbf{\Gamma}}\mathbf{u}(k-1)+\mathbf{C}_y\underbrace{\begin{bmatrix}\mathbf{A}_2\\ \mathbf{A}_2+\mathbf{I}\\ \vdots\\ \sum_{i=0}^{H_p-1}\mathbf{A}_2^i\end{bmatrix}}_{\mathbf{\Phi}}\mathbf{k}_{x2}$$

$$+\mathbf{C}_y\underbrace{\begin{bmatrix}\mathbf{B}_2 & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{A}_2\mathbf{B}_2+\mathbf{B}_2 & \mathbf{B}_2 & \cdots & \mathbf{0}\\ \vdots & \vdots & & \vdots\\ \sum_{i=1}^{H_p-1}\mathbf{A}_2^i\mathbf{B}_2 & \sum_{i=1}^{H_p-2}\mathbf{A}_2^i\mathbf{B}_2 & \cdots & \sum_{i=1}^{H_p-H_u}\mathbf{A}_2^i\mathbf{B}_2\end{bmatrix}}_{\mathbf{\Theta}}\underbrace{\begin{bmatrix}\Delta\hat{\mathbf{u}}(k)\\ \Delta\hat{\mathbf{u}}(k+1)\\ \vdots\\ \Delta\hat{\mathbf{u}}(k+H_u-1)\end{bmatrix}}_{\Delta\mathbf{\mu}(k)}+\underbrace{\begin{bmatrix}\mathbf{D}_2\\ \mathbf{D}_2\\ \vdots\\ \mathbf{D}_2\end{bmatrix}}_{\mathbf{\Lambda}}\mathbf{u}(k-1)$$

$$+\underbrace{\begin{bmatrix}\mathbf{D}_2 & \mathbf{D}_2 & \mathbf{0} & \mathbf{0}\\ \mathbf{D}_2 & \mathbf{D}_2 & \mathbf{D}_2 & \mathbf{0}\\ \vdots & \vdots & & \vdots\\ \mathbf{D}_2 & \mathbf{D}_2 & \mathbf{D}_2 & \mathbf{D}_2\end{bmatrix}}_{\mathbf{\Omega}}\begin{bmatrix}\Delta\hat{\mathbf{u}}(k)\\ \Delta\hat{\mathbf{u}}(k+1)\\ \vdots\\ \Delta\hat{\mathbf{u}}(k+H_u-1)\end{bmatrix}+\underbrace{\begin{bmatrix}\mathbf{K}_{y2}\\ \mathbf{K}_{y2}\\ \vdots\\ \mathbf{K}_{y2}\end{bmatrix}}_{\mathbf{\Xi}}\hat{\mathbf{e}}_1(k) \qquad (19)$$

The prediction error $\mathbf{e}_1(k)$ is formulated by comparing the output system $\mathbf{y}$(k) with the one step ahead prediction output that is already calculated at the previous time sampling $\mathbf{y}_1(k|k\text{-}1)$. The predicted output system is

$$\begin{aligned}\hat{\mathbf{y}}_1(k+1|k) &= \mathbf{C}_1\hat{\mathbf{x}}_1(k+1)+\mathbf{D}_1\hat{\mathbf{u}}(k+1)+\mathbf{K}_{y1}\\ &= \mathbf{C}_1\mathbf{A}_1\mathbf{x}(k)+\mathbf{C}_1\mathbf{B}_1\hat{\mathbf{u}}(k)+\mathbf{C}_1\mathbf{k}_{x1}+\mathbf{D}_1\hat{\mathbf{u}}(k+1)+\mathbf{K}_{y1}\end{aligned}$$

All of the elements of the prediction input is changed into changes of the input system prediction and hence the above equation can be rewritten as

$$\begin{aligned}\hat{\mathbf{y}}_1(k+1|k) &= \mathbf{C}_1\mathbf{A}_1\mathbf{x}(k)+(\mathbf{C}_1\mathbf{B}_1+\mathbf{D}_1)\mathbf{u}(k-1)+\cdots\\ &\quad +(\mathbf{C}_1\mathbf{B}_1+\mathbf{D}_1)\Delta\hat{\mathbf{u}}(k)+\mathbf{D}_1\Delta\hat{\mathbf{u}}(k+1)+\mathbf{C}_1\mathbf{k}_{x1}+\mathbf{K}_{y1}\end{aligned}$$

The output prediction result is used in the next sampling to be compared to to the measured output system from the sensor, therefore we have a prediction error equation form:

$$\hat{\mathbf{e}}_1(k|k) = \mathbf{y}(k)-\hat{\mathbf{y}}(k|k-1) \qquad (20)$$

So the prediction model equation can be redefined as a matrix and vector notation as follows:

$$\mathbf{Y} = \mathbf{\Psi}\mathbf{x}(k)+(\mathbf{\Gamma}+\mathbf{\Lambda})\mathbf{u}(k-1)+\mathbf{\Xi}\mathbf{e}_1(k|k)+(\mathbf{\Theta}+\mathbf{\Omega})\Delta\mathbf{\mu}(k)+\mathbf{\Phi}$$

The second step is to formulate the criterion function $V(k)$ as a function of the prediction model equation. By defining $\mathbf{Y}(k)$, $\mathbf{W}(k)$, and $\Delta\mathbf{\mu}(k)$ as the output prediction, the set-point, and the change of control signal, respectively

$$\mathbf{Y}(k) = \begin{bmatrix}\hat{\mathbf{y}}(k+1|k)\\ \hat{\mathbf{y}}(k+2|k)\\ \vdots\\ \hat{\mathbf{y}}(k+H_p|k)\end{bmatrix},\ \mathbf{W}(k) = \begin{bmatrix}\mathbf{w}(k+1)\\ \mathbf{w}(k+2)\\ \vdots\\ \mathbf{w}(k+H_p)\end{bmatrix}$$

$$\Delta\mathbf{\mu}(k) = \begin{bmatrix}\Delta\hat{\mathbf{u}}(k|k)\\ \Delta\hat{\mathbf{u}}(k+1|k)\\ \vdots\\ \Delta\hat{\mathbf{u}}(k+H_u-1|k)\end{bmatrix} \qquad (21)$$

Then, the cost function $V(k)$ can be represented in matrix notation as follows

$$\begin{aligned}V(k) &= \|\mathbf{\zeta}(k)-(\mathbf{\Theta}+\mathbf{\Omega})\Delta\mathbf{\mu}(k)\|_{\mathbf{Q}}^2+\|\Delta\mathbf{\mu}(k)\|_{\mathbf{R}}^2\\ &= \mathbf{\zeta}^T(k)\mathbf{Q}\mathbf{\zeta}(k)-2\Delta\mathbf{\mu}^T(k)(\mathbf{\Theta}^T+\mathbf{\Omega}^T)\mathbf{Q}\mathbf{\zeta}(k)+\cdots\\ &\quad +\Delta\mathbf{\mu}^T((\mathbf{\Theta}^T+\mathbf{\Omega}^T)\mathbf{Q}(\mathbf{\Theta}+\mathbf{\Omega})+\mathbf{R})\mathbf{\mu}(k)\end{aligned}$$

where $\mathbf{\zeta}(k)$ is free response as a function of available data i.e. $\mathbf{x}(k)$, $\mathbf{u}(k\text{-}1)$, $\mathbf{e}_1(k|k)$, and constant matrix $\mathbf{\Phi}$ defined as

$$\mathbf{\zeta}(k) = \mathbf{W}(k)-\mathbf{\Psi}\mathbf{x}(k)-(\mathbf{\Gamma}+\mathbf{\Lambda})\mathbf{u}(k-1)-\mathbf{\Xi}\mathbf{e}_1(k|k)-\mathbf{\Phi}$$

It is shown that the optimization problem is still kept as a quadratic programming (QP) problem, so it can be solved by using active set method.

The next step is to develop constraints in terms of the control signal $\mathbf{u}(k|k)$ and the slew rate $\Delta\mathbf{u}(k|k)$, and formulated as two inequalities

$$\mathbf{E}\Delta\mathbf{\mu}(k) \le \mathbf{e} \qquad (22)$$

$$\mathbf{F}\mathbf{\mu}(k) \le \mathbf{f} \qquad (23)$$

It is assumed that the constraint matrix has a form $\mathbf{F} = [\mathbf{F}_1\ \mathbf{F}_2\ ...\ \mathbf{F}_{Hu}]$, so the equation (23) can be written as

$$\sum_{i=1}^{Hu}\mathbf{F}_i\hat{\mathbf{u}}(k+i-1|k) \le \mathbf{f} \qquad (24)$$

It is necessary to formulate the prediction of control signal as a function of summation of the slew rate and the past control signal as follows

$$\hat{\mathbf{u}}(k+i-1|k) = \mathbf{u}(k-1)+\sum_{j=0}^{i-1}\Delta\hat{\mathbf{u}}(k+j|k) \qquad (25)$$

Therefore, equation (24) can be written in the following form

$$\begin{bmatrix}\sum_{j=1}^{Hu}\mathbf{F}_j & \sum_{j=2}^{Hu}\mathbf{F}_j & \cdots & \mathbf{F}_{Hu}\end{bmatrix}\begin{bmatrix}\Delta\hat{\mathbf{u}}(k|k)\\ \Delta\hat{\mathbf{u}}(k+1|k)\\ \vdots\\ \Delta\hat{\mathbf{u}}(k+H_u-1|k)\end{bmatrix} \le \mathbf{f}-\sum_{j=1}^{Hu}\mathbf{F}_j\mathbf{u}(k-1)$$

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By defining $\mathbf{H}_i = \sum_{j=i}^{Hu} \mathbf{F}_j$ and $\mathbf{H} = [\mathbf{H}_1, \ \cdots, \ \mathbf{H}_{Hu}]$, the equation above is given by

$$\mathbf{H}\Delta\boldsymbol{\mu}(k) \quad \leq \quad \mathbf{f} - \mathbf{H}_1 \mathbf{u}(k-1) \tag{26}$$

Now, the two of inequalities are already formulated as a function of slew rate.

In order to solve the QP problem, the cost function should be composed as constant part and dynamic part which contain slew rate matrix as follows

$$V(k) \quad = \quad \text{const} - \Delta\boldsymbol{\mu}^T(k)\boldsymbol{\eta} + \Delta\boldsymbol{\mu}^T(k)\boldsymbol{\kappa}\Delta\boldsymbol{\mu}(k) \tag{27}$$

where

$$\begin{aligned} \text{const} \quad &= \quad \boldsymbol{\zeta}^T(k)\mathbf{Q}\boldsymbol{\zeta}(k) \\ \boldsymbol{\eta} \quad &= \quad 2(\boldsymbol{\Theta}^T + \boldsymbol{\Omega}^T)\mathbf{Q}\boldsymbol{\zeta}(k) \\ \boldsymbol{\kappa} \quad &= \quad ((\boldsymbol{\Theta}^T + \boldsymbol{\Omega}^T)\mathbf{Q}(\boldsymbol{\Theta} + \boldsymbol{\Omega}) + \mathbf{R}) \end{aligned} \tag{28}$$

So, the control signal can be caluculated by optimizing the quadratic function

$$\min_{\Delta\boldsymbol{\mu}(k)} \Delta\boldsymbol{\mu}(k)^T \boldsymbol{\kappa}\Delta\boldsymbol{\mu}(k) - \Delta\boldsymbol{\mu}(k)^T \boldsymbol{\eta} + \text{const} \tag{29}$$

subjected to

$$\boldsymbol{\Omega}_c \Delta\boldsymbol{\mu}(k) \quad < \quad \boldsymbol{\sigma} \tag{30}$$

where

$$\begin{aligned} \boldsymbol{\Omega}_c \quad &= \quad \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} \\ \boldsymbol{\sigma} \quad &= \quad \begin{bmatrix} \mathbf{e} \\ \mathbf{f} - \mathbf{H}_1\mathbf{u}(k-1) \end{bmatrix} \end{aligned} \tag{31}$$

The optimization algorithm is calculated every sampling period after acquiring new measurement data $\mathbf{x}(k)$. From the prediction control vector $\Delta\boldsymbol{\mu}(k)$ that is calculated by QP algorithm, only $\Delta\hat{\mathbf{u}}(k|k)$ and $\Delta\hat{\mathbf{u}}(k+1|k)$ that are used and the rests are negligible. The signal $\hat{\mathbf{u}}(k|k) = [\mathbf{I}_p \ \mathbf{0}_p \ \ldots \ \mathbf{0}_p]^T\Delta\boldsymbol{\mu}(k) + \mathbf{u}(k\text{-}1)$ is the control signal and $\hat{\mathbf{u}}(k+1|k) = [\mathbf{I}_p \ \mathbf{I}_p \ \ldots \ \mathbf{0}_p]^T\Delta\boldsymbol{\mu}(k) + \mathbf{u}(k\text{-}1)$ is used to predict the output of the linear model.

B. Switching Logic and Set-Point Calculation

The safe distance between the host- and leading-vehicle can be calculated by using the following equations [8]

$$d_{\text{ref}} = 1 + d_s + \tau_h . v_{xh} \tag{32}$$

where l is the vehicle length, ds is the correcting factor of distance between the host- and leading-vehicle in order to avoid collision, and τ_h is a constant-time headway. The desired speed of host vehicle is determined by using following logic rule

$$v_{xh,ref} \quad = \quad \begin{cases} v_{xl}, & v_r \geq 0, d \leq d_{ref}, \varphi \leq \Delta\psi \\ v_{xh}, & \text{otherwise} \end{cases} \tag{33}$$

IV. RESULTS AND DISCUSSION

A training data set of ACC system is divided into estimation data set (60% of total data) and validation data set (40%). A multistage linear model is identified by using the least squares algorithm. It is because the control performance depends strongly on the quality of model, The identified model should be analyzed in terms of stability, controllability, observability. The purpose of model validation is to verify the similarity between the identified model with the nonlinear behavior of ACC system.

The identified model is analyzed quantitatively by considering the eigen value,

$$\lambda_i \quad = \quad |\lambda\mathbf{I} - \mathbf{A}_i| \tag{34}$$

the controllability property

$$\text{rank}(\mathbf{Q}_c) = \text{rank}([\mathbf{B} \quad \mathbf{AB} \quad \cdots \quad \mathbf{A}^{n-1}\mathbf{B}]) \rightarrow \max \tag{35}$$

and the observability property

$$\text{rank}(\mathbf{Q}_o) = \text{rank}\left(\begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix}\right) \rightarrow \max \tag{36}$$

The quality of the identified model is also measured using two performance indicators, the loss function J_{ee}

$$J_{ee} \quad = \quad \frac{1}{N}\sum_{i=1}^{N}(y(i) - \hat{y}(i))^2 \tag{37}$$

and the final prediction error (FPE) based on Akaike criterion

$$FPE \quad = \quad \frac{N+m}{N-m}\sum_{i=1}^{N}(y(i) - \hat{y}(i))^2 \tag{38}$$

Table (1) shows a good model of ACC system in terms of performance indicators in equations (34)-(38).

TABLE 1. THE QUALITY OF IDENTIFIED MODEL AT 2ND STAGE

Eigen values λ_i		0.99998, 0.99998, 0.9999, 0.9999
Rank $\{\mathbf{Q}_c\}$		4 (max), full controllable
Rank $\{\mathbf{Q}_o\}$		4 (max), full observable
J_{ee}	Estimation	1.6252×10^{-4}
	Validation	0.0029
FPE	Estimation	1.4284
	Validation	25.6918

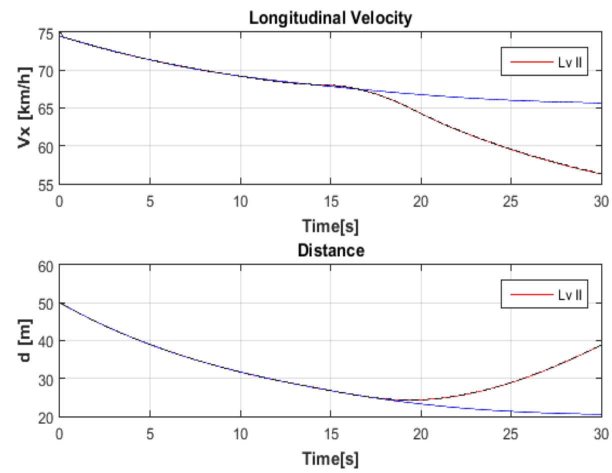

Fig. 3. Comparation between identification model and vehicle model

Performance indicators J_{ee} and FPE show the errors that occur between the results of an estiamsi system with the actual system value. In both estimation and validation, the second-level identification model has a smaller J_{ee} and FPE indicator value than the J_{ee} and FPE indicator values in the first-level identification model. Figure (3) shows the ability of multistage linear model in mimicking nonlinear properties of dynamic vehicle qualitatively.

After identification process, the model is used in designing a multistage predictive controller. The quality of control results depends highly on the quality of the model. If the model shows bad performance in mimicing the ACC system dynamic, it will lead to a bad control performance. As tuning parameter of multistage predictive controller, the

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weighting matrices $\mathbf{Q} = 100\mathbf{I}$, $\mathbf{R} = 0.01\mathbf{I}$ and receding horizon parameter $H_p = 5$ $H_u = 2$ are determined.

After controller design task is accomplished, the system was tested in tracking target vehicle within a safe distance during traveling at specific speed 60 km/h. It is assumed that the initial distance between vehicles is 50 m and initial traveling speed of following vehicle set at 75 km/h. As the results shown in Figure (4), after 18 seconds the following vehicle traces the heading vehicle by maintaining the desired distance. With the ACC system, the host vehicle can avoid collision toward the leading vehicle as shown in Figure (5). Compared to the host vehicle without ACC system, it cannot maintain a safe distance which will lead to crash the leading vehicle.

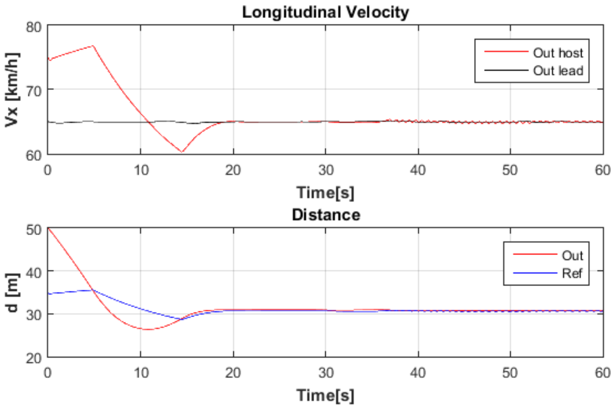

Fig. 4. Response of longitudinal velocity and distance following references

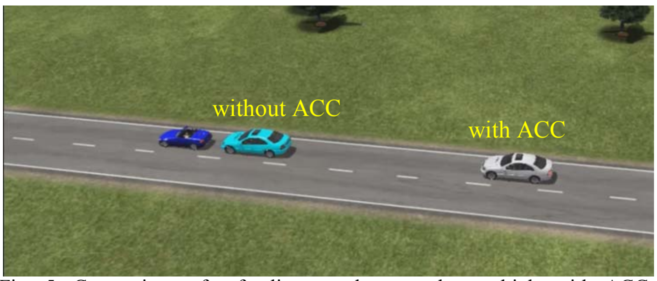

Fig. 5. Comparison of safe distances between host-vehicle with ACC (white) and without ACC (cyan) to the leading-vehicle (blue) using Carsim

V. Conclusions

This paper demonstrates that the proposed multistage predictive control combined with the time gap logic control provides increasing vehicle safety and it has been aproved through simulation that both the actual vehicle speed and actual distance converge to the desired values.

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