

A Sliding-Mode-Control-Based Adaptive Cruise Controller

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Abstract— This paper presents an alternative solution to the conventional cruise controller of a hybrid electric vehicle based on the sliding mode control approach. The mathematical model of a hybrid electric vehicle cruise control system is developed. Then, the sliding mode control approach is applied as the controller. The sliding mode control stability is investigated and demonstrated. Thereafter, the system is simulated and the results are presented.

I. INTRODUCTION

A robust and reliable approach to controlling a nonlinear and uncertain system is the sliding mode control methodology. It was first proposed in 1950s [1], but later on used in different applications of control engineering. In the sliding control methodology, a notational simplification is introduced which allows nth-order problems to be substituted by corresponding 1st-order problems. Then, for the altered problem, perfect performances can be considered. The basic scheme of sliding mode control can be outlined as follows: (i) the system dynamics is primary delineated on a sliding mode surface in the state space, (ii) a controller is then created to constrain the closed-loop system to reach the sliding mode surface, and (iii) the defined dynamics of the closed-loop system is then attained on the sliding mode surface.

The dynamic of a controllable nth-order single input system can be described as follows:

$$\dot{x} = \varphi(x) + \beta(x) u \quad (1)$$

where x is scalar and the variable of interest, for instance the position of a mechanical system, or in our case, the speed of a vehicle, and $x = [x \ \dot{x} \ \ddot{x} \ x^{(n-1)}]^T$ is the state vector. In Equation (1), the function $\varphi(x) \in R^{n \times 1}$ is usually nonlinear and not exactly known. However, it is upper bounded by a known continuous function of x . $\beta(x) \in R^{n \times 1}$ is the control gain, and similar to $\varphi(x)$, is not exactly known, but is bounded by known continuous functions of x . For example, the inertia of a mechanical system is only known to a certain precision in a model describing only part of the actual friction forces. The finite control variable u gets the state x to track a specific time varying state $x = [x \ \dot{x} \ \ddot{x} \ x^{(n-1)}]$ in the presence of model imprecision on $\varphi(x)$ and $\beta(x)$.

In order to achieve the tracking task by the finite control u , the initial desired state $x_d(0)$ must satisfy the following:

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$$x_d(0) = x(0) \quad (2)$$

For design of a sliding mode controller, it is required to identify a sliding variable s :

$$s = c^T x = c_0 x + c_1 \dot{x} + c_2 \ddot{x} + \dots + c_{n-2} x^{(n-2)} + x^{(n-1)} \quad (3)$$

where $c = [c_0 \ c_1 \ c_2 \ \dots \ c_{n-2} \ 1]$ is the vector of sliding mode parameter, and should be selected such that the solution of the following differential equation is asymptotically stable.

$$c^T \dot{x} = c_0 \dot{x} + c_1 \ddot{x} + c_2 \dddot{x} + \dots + c_{n-2} x^{(n-2)} + x^{(n-1)} = 0 \quad (4)$$

According to the linear control theory, Equation (4) is asymptotically stable when its eigenvalues have the negative real parts. With the fulfilment of the stated principle, the desired dynamic of the closed-loop system will be satisfied with the chosen sliding variable s . Consequently, the task of the sliding mode control method is to design a controller, in order for the sliding variable s to converge to zero. In the sliding mode control, the desired system dynamics stated by Equation (4) is called "sliding mode or sliding mode surface", and the controller designed to conduct the sliding variable s to converge to zero is called "sliding mode controller" [2]. The next step is to define the concept of stability based on the second method of Lyapunov which is implemented here to carry out the stability of the cruise control system. Hence, the first candidate of Lyapunov function can be defined as:

$$H = \frac{1}{2} s^2 \quad (5)$$

By differentiating H with respect to time, then we will obtain:

$$\dot{H} = s \dot{s} = s(c^T \dot{x}) = s[c^T \varphi(x) + c^T \beta(x) u] \quad (6)$$

As it expressed before, the functions $\varphi(x)$ and $\beta(x)$ are not exactly known. However, they are upper and lower bounded by a known continuous function of x as they are showed below.

$$\|\varphi(x)\| < \varphi_0(x) \quad (7)$$

and

$$\alpha < c^T \beta(x) \quad (8)$$

where α is a positive constant. The above bounded information of $\varphi(x)$ and $\beta(x)$ are used to design the controller. Therefore, one possible way of selecting the control signal u can be as the following form:

$$u = -\alpha^{-1}(\|c\|\varphi_0(x) + \delta) \text{sign}(s) \quad (9)$$

where δ is a constant and $\delta > 0$. The sign function $\text{sign}(s)$ is defined as follows:

$$\text{sign}(s) = \begin{cases} 1 & \text{for } s > 0 \\ 0 & \text{for } s = 0 \\ -1 & \text{for } s < 0 \end{cases} \quad (10)$$

Substituting Equation (9) into Equation (6) will give:

$$\begin{aligned}
\dot{H} &= s[c^T \varphi(x) + c^T \beta(x) u] \\
&= s[c^T \varphi(x) - c^T \beta(x)(\alpha^{-1}(\|c\|\varphi_0(x) + \delta)\text{sign}(s))] \\
&= s[c^T \varphi(x) - |s|(c^T \beta(x)\alpha^{-1})\|c\|\varphi_0(x) \\
&\quad - \delta|s|(c^T \beta(x)\alpha^{-1})] \\
&\leq |s|\|c\|\|\varphi(x)\| - |s|(c^T \beta(x)\alpha^{-1})\|c\|\varphi_0(x) \\
&\quad - \delta|s|(c^T \beta(x)\alpha^{-1}) \\
&\leq |s|\|c\|\|\varphi(x)\| - |s|\|c\|\varphi_0(x) - \delta|s| \\
&\leq -|s|\|c\|(\varphi_0(x) - \|\varphi(x)\|) - \delta|s| \\
&\leq -\delta|s| \leq 0
\end{aligned} \tag{11}$$

In the obtained result that is given in Equation (11), the following criteria are considered:

$$\text{sign}(s)s = |s|, \quad (c^T \beta(x)\alpha^{-1}) > 1, \quad \text{and} \quad \varphi_0(x) - \|\varphi(x)\| > 0 \tag{12}$$

Hence, based on the second method of Lyapunov stability, the "sliding variable s " will asymptotically converge to zero. By obtaining $s=0$, then the desired closed-loop system dynamics shown by Equation (4) is achieved. Accordingly, the state variable vector x will asymptotically converge to zero on the sliding mode surface, $s=0$.

The sliding mode control has been used to control various systems including robot manipulator [3], automotive transmission [4, 5], under water vehicle [6, 7], electric motor and uninterrupted-power-supply (UPS) [8, 9]. However, in an adaptive cruise controller especially in a HEV, it is a novel approach which is considered and investigated in this paper. Before, modelling and discussion of the implemented sliding mode control in cruise control, at first the mathematical form of the simplified model of cruise control is devised.

II. CRUISE CONTROL MODEL

The cruise control in a vehicle can be considered as a common feedback system in the process control class. The controller which traditionally is a PID controller attempts to maintain a constant speed in the presence of disturbances such as wind and slope of the road. Different from a conventional cruise controller, in which the speed set-point is a constant parameter, in an adaptive cruise controller the speed is variable factor. This variation of the speed is an additional burden to the cruise controller system from the standpoint of reliability. In practice, a cruise control system with a PID controller is not robust against the swift speed changes, and usually falls to instability. This is the main reason for switching to the manual mode in the ACC vehicle due to the rapid change in the speed.

A conceptual presentation of a forward model of the cruise control in a hybrid electric vehicle is shown in Figure 1. In the forward model, the desired speed starts from accelerator or brake, and then the control systems of different components act such that the desired speed is transferred to the wheels. It can be seen in the figure that the command for acceleration and deceleration is initiated from the accelerator, brake or the ACC. This command is interpreted as the control signal, U in the block diagram. The control signal determines the required traction force F_d provided by the electric motor and/or the internal combustion engine in the vehicle. Subtracting the resistive forces such as gravitational, aerodynamic and friction from the traction force, the momentum force can be obtained. Accordingly, the speed can

be calculated and sent back to the ACC as shown in the figure.

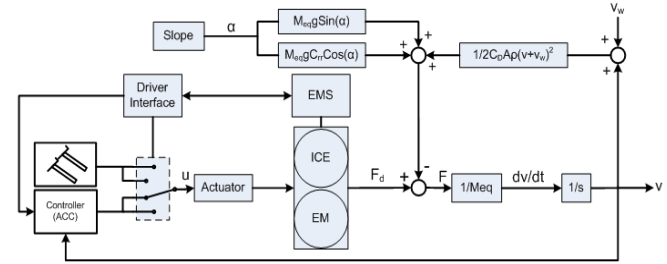


Figure 1. Conceptual model of the vehicle and the cruise control.

III. SLIDING MODE CONTROLLER IN PLANT

In order to design the sliding mode controller, the engine and/or the electric motor are considered as a first order system with the constant time of T . Therefore, the power demand F_d based on the control signal u can be presented as follows:

$$F_d = \frac{U(t)C_1}{TS+1} \tag{13}$$

If we rewrite the state equation of the system, the following relations are obtained:

$$\begin{cases} \dot{v} = \frac{1}{M_{eq}} F_d - g \sin \alpha - g c_{rr} \cos \alpha - \frac{1}{2} \frac{c_p A \rho}{M_{eq}} v^2 \\ \dot{F}_d = \frac{C_1}{T} U(t) - \frac{1}{T} F_d \\ y = v \end{cases} \tag{14}$$

In practice, in order to be able to design a suitable sliding surface, the demand power F_d must be a measurable and observable variable. In reality, from the engine torque meter, F_d can be figured out. Accordingly the sliding mode is defined as follows:

$$\begin{cases} s_1 = v - v_d \\ s_2 = F_d - x_{2d} \end{cases} \tag{15}$$

where v_d and x_{2d} are the variation of the measurable parameters. By substituting Equation (15) in Equation (14), we get:

$$\dot{s}_1 = \frac{1}{M_{eq}} F_d - g \sin \alpha - g c_{rr} \cos \alpha - \frac{1}{2} \frac{c_p A \rho}{M_{eq}} v^2 - \dot{v}_d \tag{16}$$

In Equation (16), if F_d is substituted by its equivalent $s_2 + x_{2d}$ from Equations (15), Equation (16) can be rewritten as follows:

$$\dot{s}_1 = \frac{1}{M_{eq}} (s_2 + x_{2d}) - g \sin \alpha - g c_{rr} \cos \alpha - \frac{1}{2} \frac{c_p A \rho}{M_{eq}} v^2 - \dot{v}_d \tag{17}$$

In order to find an effectual solution to the system, x_{2d} in the following format is a satisfactory selection:

$$x_{2d} = M_{eq} (g \sin \alpha + g c_{rr} \cos \alpha + \frac{1}{2} \frac{c_p A \rho}{M_{eq}} v^2 + \dot{v}_d - k_1 s_1) \tag{18}$$

where k_1 is a selectable constant. In order to satisfy the stability of the system, the constant k_1 must be positive, i.e. $k_1 > 0$. If the relation in Equation (18) is substituted in Equation (17) then:

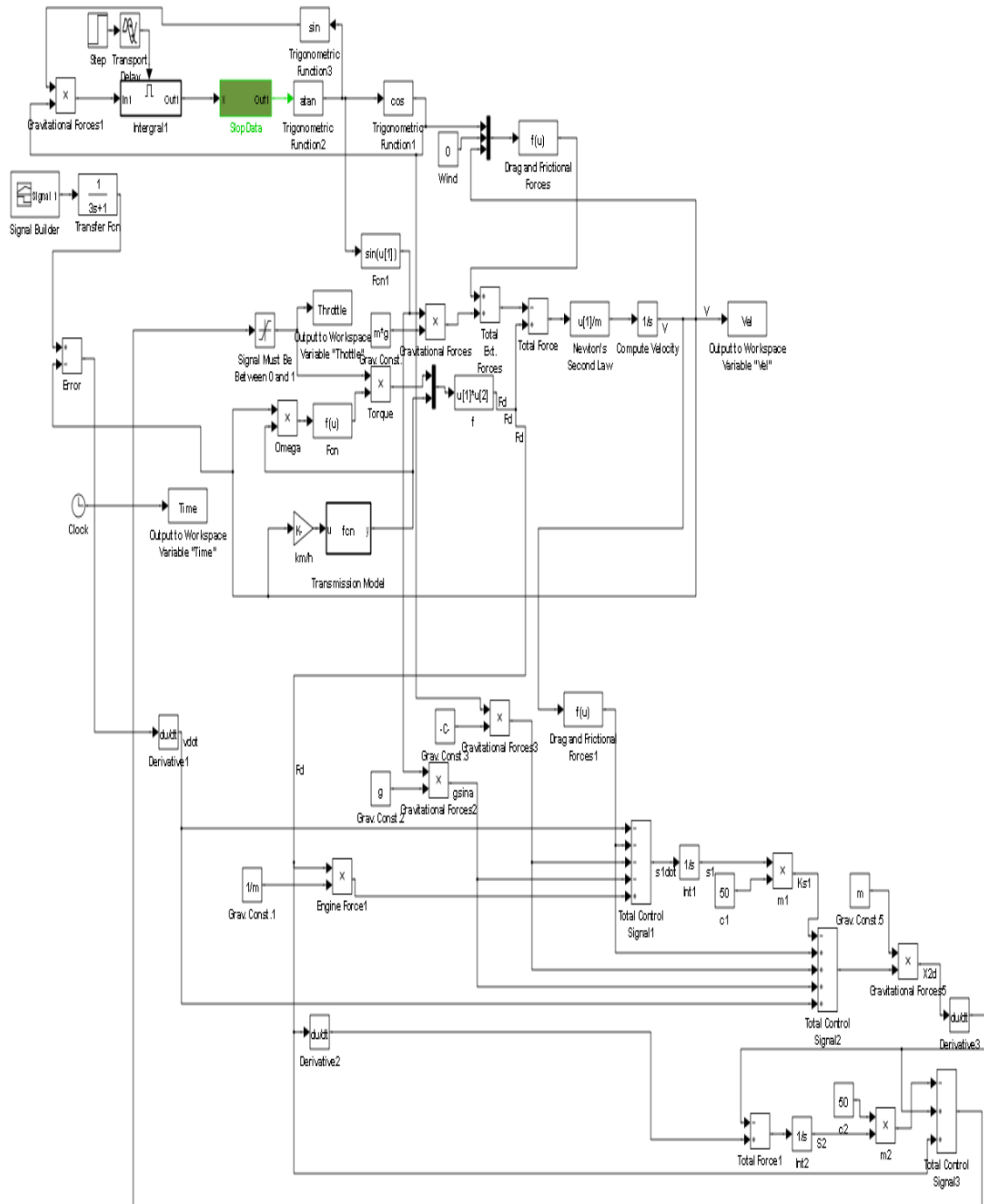


Figure 2. Conceptual model of the vehicle and the cruise control.

$$\dot{s}_1 = \frac{1}{M_{eq}} s_2 - k_1 s_1 \quad (19)$$

If we assume that s_2 is a differentiable parameter then from Equation (15):

$$\dot{s}_2 = \dot{F}_d - \dot{x}_{2d} \quad (20)$$

and by considering the system presented by Equation (14), therefore:

$$\dot{s}_2 = \frac{c_1}{T} u(t) - \frac{1}{T} F_d - \dot{x}_{2d} \quad (21)$$

If $u(t)$ is selected as the following equation:

$$u(t) = \frac{1}{c_1} (F_d + T \dot{x}_{2d} - k_2 s_2) \quad (22)$$

where k_2 is a positive selectable constant. By substitution Equation (22) in Equation (21), we conclude:

$$\dot{s}_2 = \frac{-k_2}{c_1} s_{2d} \quad (23)$$

It is noticeable that the solution of the differential Equation (23) converges to zero exponentially. Taking into account the revealed result in Equation (23) and from Equation (19), it can be concluded that:

$$\dot{s}_1 \approx -k_1 s_1 \quad (24)$$

Also, the solution of the differential Equation (24) converges to zero exponentially, which proves the stability of the sliding motion and the convergence of the error to zero.

The nonlinear model of the cruise control system with using the real data of the road is modelled in MATLAB/SIMULINK (see Figure 2). The described method of control is applied to the cruise control model. In the simulation, the desired speed input which is a variable speed controller is used. The output speed and reference speed are presented in Figure 3.

As can be seen from the figure, the output response reveals 0% overshoot that it shows the sliding mode control provides a robust control without overshoots and undershoots.

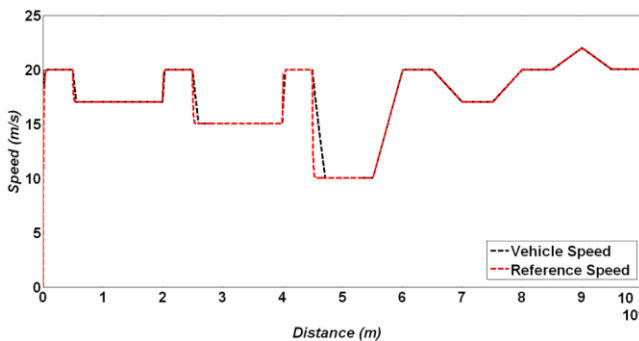


Figure 3. The input and the response signal of the non-linear model of the cruise controller based on the sliding mode control method.

IV. CONCLUSION

This paper presented a mathematical model for a cruise controller which is applicable to both conventional and hybrid electric vehicles. The sliding mode control which is a powerful methodology to provide a reliable solution was applied as the controller in plant. The stability of the system

based on the second method of Lyapunov was demonstrated. The desired speed was applied as the set-point to the controller. The output speed followed the desired speed with 0% overshoots and undershoot and negligible delay.

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REFERENCES

- [1] J. J. E. Slotin and W. Li, *Applied nonlinear control*. Englewood Cliffs: Prentice-Hall, 1991.
- [2] M. Zhihong, *Robotics for Computer Engineering Students*. Jurong, Singapore: Printice Hall, Pearson Education South Asia, 2004.
- [3] L. M. Capisani and A. Ferrara, "Trajectory planning and second-order sliding mode motion/interaction control for robot manipulators in unknown environments," *IEEE Transactions on Industrial Electronics*, vol. 59, pp. 3189-3198, 2012.
- [4] G. W. Kim and K. W. Wang, "Switching sliding mode force tracking control of piezoelectric-hydraulic pump-based friction element actuation systems for automotive transmissions," *Smart Materials and Structures*, vol. 18, 2009.
- [5] X. Song and Z. Sun, "Pressure-based clutch control for automotive transmissions using a sliding-mode controller," *IEEE/ASME Transactions on Mechatronics*, vol. 17, pp. 534-546, 2012.
- [6] X. Q. Bian, X. Q. Cheng, H. M. Jia, Z. P. Yan, and L. J. Zhang, "A bottom-following controller for underactuated AUV based on iterative sliding and increment feedback," *Kongzhi yu Juece/Control and Decision*, vol. 26, pp. 289-292+296, 2011.
- [7] J. Javadi-Moghaddam and A. Bagheri, "An adaptive neuro-fuzzy sliding mode based genetic algorithm control system for under water remotely operated vehicle," *Expert Systems with Applications*, vol. 37, pp. 647-660, 2010.
- [8] M. L. Corradini, G. Ippoliti, S. Longhi, and G. Orlando, "A quasi-sliding mode approach for robust control and speed estimation of PM synchronous motors," *IEEE Transactions on Industrial Electronics*, vol. 59, pp. 1096-1104, 2012.
- [9] H. Komurcugil, "Rotating-Sliding-Line-Based Sliding-Mode Control for Single-Phase UPS Inverters," *IEEE Transaction on Industrial Electronics*, vol. 59, pp. 3719-3726, OCTOBER 2012 2012.
- [10] S. Sheikholeslam and C. Desoer, "Longitudinal Control of a Platoon of Vehicles," Institute of Transportation Studies University of California at Berkeley August 18, 1989.
- [11] H. Hjalmarsson and T. Birkeland, "Iterative feedback tuning of linear time-invariant mimo systems," in *37th IEEE Conference on Decision and Control*, 1998, pp. 3893-3898.
- [12] M. A. Johnson and M. H. Moradi, "PID Control," in *Science+Business Media*, Springer, Ed., 2005.
- [13] M.-H. Yoon and C.-H. Shin, "Design of online auto-tuning PID controller for power plant " in *Proceedings of the 36th SICE Annual Conference*, Jul 1997, pp. 1221-1224.
- [14] V. Aggarwal, M. Mao, and U.-M. O'Reilly, "A Self-Tuning Analog Proportional-Integral-Derivative (PID) Controller," *AHS*, pp. 12-19, 2006.
- [15] R. C. Eberhart and J. Kennedy, "A new optimizer using particle swarm theory," in *Proceedings of the Sixth International Symposium on Micro Machine and Human Science*, Nagoya, Japan, Piscataway, 1995, pp. 39-43.
- [16] Y. Shi and R. Eberhart, "A modified particle swarm optimizer " in *IEEE International Conference on Evolutionary Computation*, Piscataway, NJ, USA, 1998, pp. 69-73.
- [17] Y. Shi and R. Eberhart, "Empirical study of particle swarm optimization," in *IEEE Congress on Evolutionary Computation*, Piscataway, NJ, USA, 1999, pp. 1945-1950.
- [18] B. Ganji, A. Kouzani, and M. A. Hesami, "Backward modeling and look-ahead fuzzy energy management controller for a parallel hybrid electric vehicle," *Control and Intelligent Systems*, vol. 39, 2011.