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A Path Planning and Model Predictive Control for Automatic Parking System

Xiangxi Meng, Jian Wu, Rui He, Bing Zhu, and Jian Zhao Jilin University

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Abstract

With the increasing number of urban cars, parking has become the primary problem that people face in daily life. Therefore, many scholars have studied the automatic parking system. In the existing research, most of the path planning methods use the combined path of arc and straight line. In this method, the path curvature is not continuous, which indirectly leads to the low accuracy of path tracking. The parking path designed using the fifth-order polynomial is continuous, but its curvature is too large to meet the steering constraints in some cases. In this paper, a continuous-curvature parking path is proposed. The parking path tracker based on Model Predictive Control (MPC) algorithm is designed under the constraints of the control accuracy and vehicle steering. Firstly, in order to make the curvature of the parking path continuous, this paper superimposes the fifth-order polynomial with the sigmoid function, and the curve obtained has the continuous and relatively small curvature. Therefore, the superposition curve is used as a parallel parking path while the superposition curve and its inverse function curve are combined to form a perpendicular parking path. The coefficients of the superposition curve are calculated according to the constraint condition, the parking start point and end point. Thus, the parking path is determined.. Secondly, the vehicle kinematics model is established and a parking path tracker based on Model Predictive Control (MPC) algorithm is designed. Finally, the co-simulation analysis is performed using CarSim and Simulink. The simulation results show that the parking path curvature designed in this paper is continuous and the parking path tracker has a good tracking effect. The lateral error and longitudinal error can be controlled in the centimeter scale and the heading angle error is no more than 3°.

Introduction

Parking is not only an essential part of the daily use of cars, but also a part that drivers are prone to make mistakes. For novice drivers or even some experienced drivers, parking is still a major problem in driving. And parking errors account for a large part of the total number of traffic accidents. The university of Michigan's transportation institute analysed a report on traffic accidents in American cities and found that park-related accidents accounted for 44% of all accidents [1]. In recent years, the demand of auto consumers for automatic parking is increasing day by day, and automatic parking has become a research hotspot in the field of vehicle and transportation. Automatic parking usually needs to realize environment perception, path planning and path tracking, among which path tracking control, as the final realisation of automatic parking, is a key technology related to the success rate and accuracy of parking. Compared with other path tracking control, the reference path curvature in parking process is larger and the precision is higher.

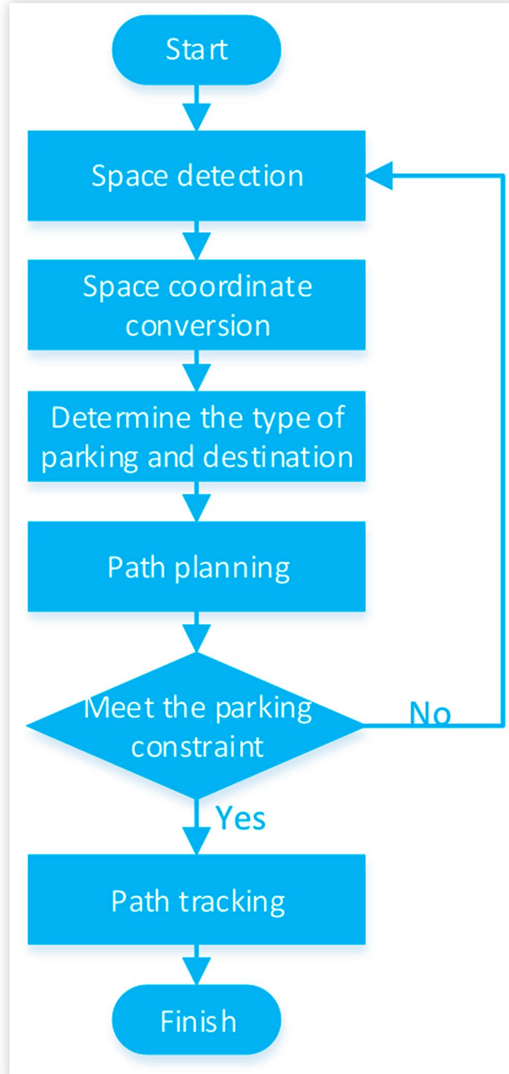
As one of the most common method, automatic parking method, which bases on path planning and tracking, is more flexible. But there often exist curvature mutation points and high requirements of starting position in the current path planning method. Thus improving the rationality of the parking path planning method and path tracking effect has a practical significance for engineering applications. For many years, scholars in various countries have done a lot of research on parking path planning and tracking. Kim [2] adoptes arc-linear curve and designs a path suitable for parking in a compact space under the vehicle obstacle avoidance constraint, but the curvature discontinuity of the parking path increases the difficulty of vehicle path tracking. Under the minimum turning radius and obstacle avoidance constraints, JM Wang [3] and Petrov [4,5] obtain the corresponding parking path curve of parallel parking and perpendicular parking through geometric derivation. However, the path curvature generated by this method is discontinuous, and it is necessary to stop and turn at the curvature discontinuity when following, so it is difficult to guarantee the path tracking accuracy. S Zhang [6] designs the parking path of the vehicle based on the initial point and the state vector of the initial point, and obtains a smooth and continuous curve that is easy for the vehicle to track, but could not guarantee the constraint of safe collision avoidance. Upadhyay [7,8] employees the logistic curve path to divide paths into Γ and S curve according to the circumstance and then generates the curvature of vehicle path according to the starting point and the target position. Although all these methods can generate smooth curves with continuous curvature, these curves cannot satisfy the safety constraint of vehicle collision avoidance. On the basis of circular parking path, Hsu [9] designs the change law of steering wheel angle under the changing vehicle travel route. The time tracking requirement of the speed varying with time is reduced. However, it doesn't consider the constraint of the steering angular speed of the vehicle steering system. Therefore, the parking and turning are needed at the path curvature discontinue points. What's more, the path following method is open loop control. There is no deviation adjustment when the vehicle deviates from the parking path tracking. Gomez-bravo [10,11] uses b-spline curve to smooth the parking path on the basis of arc parking path and establishes a path tracking controller based on fuzzy control theory. But it doesn't consider the influence of the steering angular velocity of the vehicle steering system and the vehicle travel speed on the vehicle path tracking. Shi J [12] uses the method of multi-point preview to track the parking path, which is verified in real cars. T Tashiro [13] applies MPC model prediction controller to track the parking path, but only in the case that the perpendicular parking tangent direction could not be changed. Yang W [14] designs to preview fuzzy controller to track the parking path, and the robustness and adaptability are demonstrated by the co-simulation of PreScan and Matlab.

In this paper, a parking path with continuous curvature is proposed under the minimum turning radius of vehicles and vehicle obstacle avoidance constraints. In order to improve the control accuracy, a parking path tracker based on model predictive control (MPC) algorithm is designed. First, the superposition of the fifth-order polynomial and sigmoid function is carried out. The superposition curve is obtained with continuous curvature and the curvature is relatively small. The coefficients of superposition curve are calculated according to the starting point, ending point and constraint conditions of parking. So the parking path is determined. Secondly, the vehicle kinematics model is established and the parking path tracker based on model predictive control (MPC) algorithm is designed. Finally, CarSim and Simulink are used for co-simulation analysis. Simulation results show that the curvature of the parking path is continuous. and the tracking effect of the parking path tracker is good. The lateral error and longitudinal error can be controlled in the centimeter scale and the heading angle error no more than 3°.

Path Planning

Generally speaking, the automatic parking system includes environmental perception, path planning and path tracking. In this paper, the parking space coordinates obtained by environmental perception are directly used for path planning and tracking. The process is shown in Figure 1.

FIGURE 1 The process of automatic parking



Superposition Curve

In this paper, the center point of the rear axle of the vehicle is taken as the reference point. The whole parking process is carried out in the vehicle coordinate system at the starting point of the parking, which is called the parking coordinate system (PCS). The starting point is $(0,0)$, and the ending point is (x_d, y_d). Then, the lateral displacement and longitudinal displacement of the vehicle completing the parking can be expressed as,

$$\begin{cases} d = y_d \\ l = x_d \end{cases} \tag{1}$$

Where d denotes the lateral displacement and l denotes the longitudinal displacement. If the fifth-order polynomial fitting curve is used as the parking path, the path expression can be shown in the following equation,

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 \tag{2}$$

If sigmoid function is used to fit the curve parking path, the path expression can be shown in the following equation,

$$h(x)=\frac{b_0}{1+e^{(b_1x+b_2)}}+b_3 \tag{3}$$

By substituting the position constraint of vehicle starting and ending points, heading angle constraint and steering stability constraint into equations (2) and (3), the equations (4) and (5) can be obtained. They are

$$\begin{cases} f(0)=0, f(x_d)=y_d \\ f'(0)=0, f'(x_d)=0 \\ f''(0)=0, f''(x_d)=0 \end{cases} \tag{4}$$

$$\begin{cases} h(0)=0, h(x_d)=y_d \\ h'(0)=0, h'(x_d)=0 \\ h''(0)=0, h''(x_d)=0 \end{cases} \tag{5}$$

The lateral and longitudinal displacement can be used to represent the two parking paths as follows,

$$\begin{cases} f(x)=d\left(10\left(\frac{x}{l}\right)^3-15\left(\frac{x}{l}\right)^4+6\left(\frac{x}{l}\right)^5\right) \\ h(x)=\dfrac{d}{1+e^{\left(\frac{-20x}{l}+10\right)}} \end{cases} \tag{6}$$

Both of the two paths have continuous curvature and zero curvature at both starting and ending points. However, after starting point, the curvature, fitted by the fifth-order polynomial after the starting point, is large and rapid changing. Also, the required value of the front wheel angle is likely to exceed the limit value in the process of path tracking. The curvature of the path after the path starting point of the curve fitted by the sigmoid function is so small that the desired end point can be reached only by a large curvature in the middle section of the path. However, the required value of the front wheel angle can easily exceed the limit value if the curvature of the middle section is too large. In order to keep the path curvature continuous, the zero curvatures at the starting and the end points and reduce the amplitude of curvature, this paper superimposes these two curves in a certain proportion, and the superimposed curve obtained can be expressed as shown in the following equation,

$$y(x)=kh(x)+(1-k)f(x) \tag{7}$$

Where k denotes the proportional coefficient, the value of k can be obtained according to the following constraints.

Figure 2 and 3 respectively represent the schematic diagram of the three paths and the front wheel Angle curve required for tracking the three paths when the starting point is $(0,0)$, the ending point is $(-7,-3)$, and the proportionality coefficient k is 0.17. As can be seen from figure 3, the starting and ending points of the front wheel angle required for tracking the path of superposition curve are all zero, with continuous changes and the minimum amplitude, which is easier to meet the constraint of the limit value of the front wheel angle.

FIGURE 2 The schematic diagram of the three paths

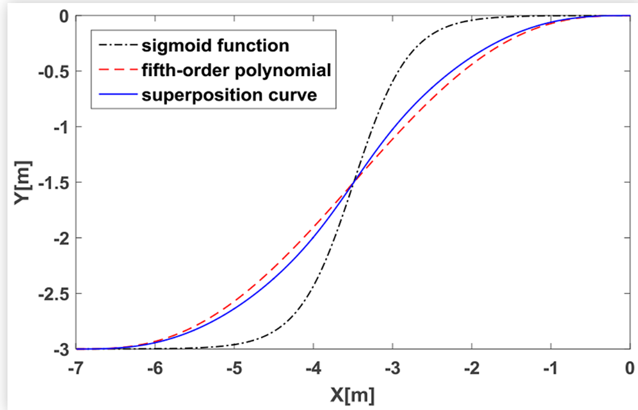
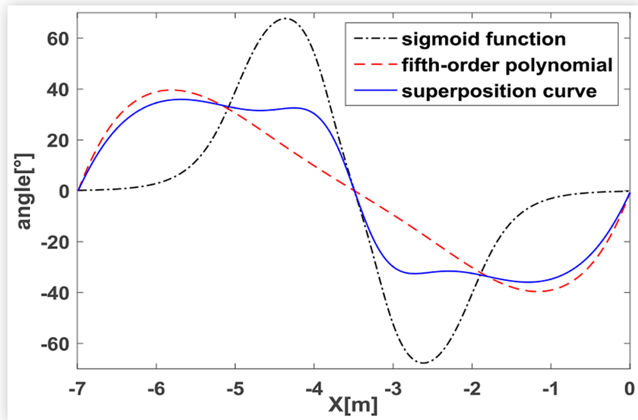


FIGURE 3 The front wheel angle required for tracking the three paths



Parking Path

In this paper, the obtained superposition curve is used as the parallel parking path, and the specific expression is shown in the following equation,

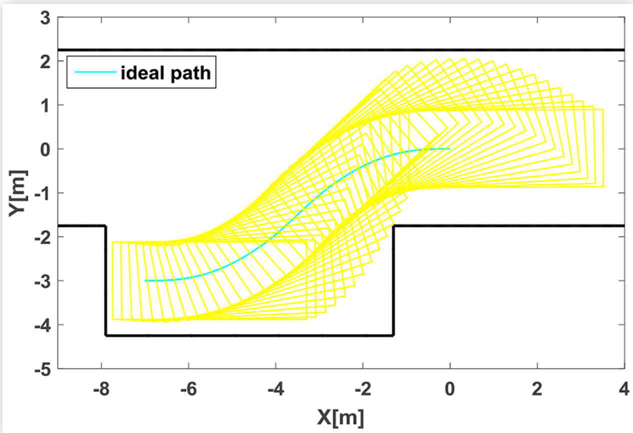
$$y(x)=\frac{d_1k}{1+e^{\left(\frac{-20x}{l}+10\right)}}+d_1(1-k)\left(\begin{array}{l}10\left(\frac{x}{l_1}\right)^3-15\left(\frac{x}{l_1}\right)^4 \\ +6\left(\frac{x}{l_1}\right)^5\end{array}\right) \tag{8}$$

Where d_1 denotes the lateral displacement and l_1 denotes the longitudinal displacement of the vehicle when parallel parking is completed.

Figure 4 is the schematic diagram of parallel parking when the starting point is $(0,0)$, the ending point is $(-7,-3)$, and the proportionality coefficient k is 0.17.

The perpendicular parking path consists of three parts: a straight line, part of the superimposed curve and the inverse function curve of the superimposed curve. Here the superposition curve is expressed by equation (9).

FIGURE 4 The schematic diagram of parallel parking



$$y(x)=\frac{d_2k}{1+e^{\left(\frac{-20x}{l_2}+10\right)}}+d_2(1-k)\begin{pmatrix}10\left(\frac{x}{l_2}\right)^3-15\left(\frac{x}{l_2}\right)^4\\+6\left(\frac{x}{l_2}\right)^5\end{pmatrix} \quad (9)$$

If the slope of $y(x)$ is 1 at the point $(l_2/2, d_2/2)$, the right half of the superimposed curve follows the line with a slope of -1 to get its inverse function curve, and the two curves are tangent at the point $(l_2/2, d_2/2)$. As is shown in Figure 5.

The proportional coefficient k is obtained according to the constraint conditions, and the initial value is 0, so l_2 and d_2 can be calculated by equation (10), so as to determine the parking path.

$$\begin{cases} y'(l_2/2)=1 \\ l_2/2-d_2/2=\Delta x \end{cases} \quad (10)$$

Where Δx denotes the difference value between the parking ending point and the starting point in the x direction. Its value can be obtained by determining the position of the starting point and the ending point. Figure 6 shows

FIGURE 5 Partial perpendicular parking path

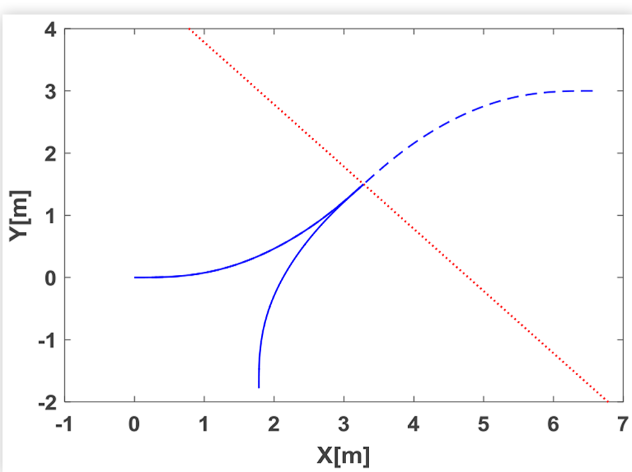
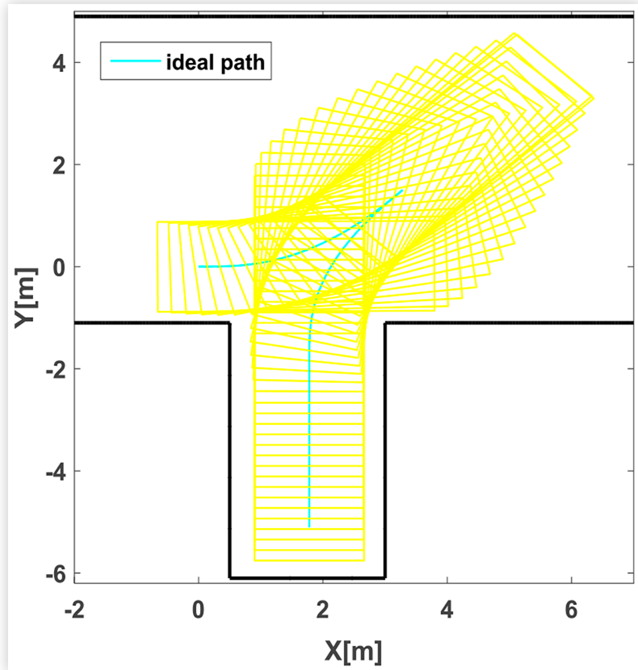


FIGURE 6 The schematic diagram of perpendicular parking



the schematic diagram of perpendicular parking when Δx = 1.78.

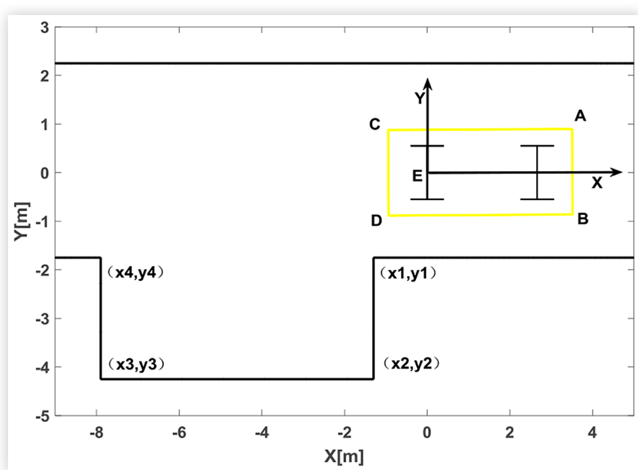
Path Constraint

After obtaining the effective parking space coordinates, firstly, converting them to the vehicle coordinate system. As is shown in the figure 7. The length-width ratio of the parking space is calculated by the formula below,

$$kw=\frac{|x1-x4|}{|y1-y2|} \quad (11)$$

Where kw denotes the length-width ratio of the parking space. If kw is greater than 1, the parking type is parallel parking, otherwise it is perpendicular parking. In the PCS,

FIGURE 7 Parking space in vehicle coordinate system



the starting point of the parking is $(0,0)$, and the ending point (x_d, y_d) is determined according to the type of parking. In case of parallel parking, the ending point calculation is shown in the following equation,

$$\begin{cases} x_d = \dfrac{x3+x4}{2} + l_g + L_r \\ y_d = \dfrac{y1+y2+y3+y4}{4} \end{cases} \tag{12}$$

Where l_g denotes the distance between the tail and the parking space line and L_r denotes tail length of vehicle.

In case of perpendicular parking, the ending point calculation is shown in the following equation,

$$\begin{cases} x_d = \dfrac{x1+x2+x3+x4}{4} \\ y_d = \dfrac{y2+y3}{2} + l_g + L_r \end{cases} \tag{13}$$

In this paper, the center rear axle of the vehicle $E(x, y)$ is the reference point. Thus, the four vertices of vehicle body ABCD can be expressed by the following equation,

$$\begin{bmatrix} x_A \\ y_A \\ x_B \\ y_B \\ x_C \\ y_C \\ x_D \\ y_D \end{bmatrix} = \begin{bmatrix} x - b\sin\varphi + (L+L_f)\cos\varphi \\ y + b\cos\varphi + (L+L_f)\sin\varphi \\ x + b\sin\varphi + (L+L_f)\cos\varphi \\ y - b\cos\varphi + (L+L_f)\sin\varphi \\ x - b\sin\varphi - L_r\cos\varphi \\ y + b\cos\varphi - L_r\sin\varphi \\ x + b\sin\varphi - L_r\cos\varphi \\ y - b\cos\varphi - L_r\sin\varphi \end{bmatrix} \tag{14}$$

Where b denotes half the vehicle width, φ denotes the heading angle of the vehicle, δ_f denotes the front wheel angle, L denotes axle distance of vehicle, L_f denotes header length of vehicle, and L_r denotes tail length of vehicle.

The figure 8 and figure 9 show the movement track of the four points of the vehicle body in the case of two kinds of parking. In this paper, the constraint conditions are angle constraint and obstacle avoidance constraint. The angle constraint means that the front wheel angle required by the track parking path is less than the limit value. Obstacle avoidance constraint means that the trajectories of A,B,C,D do not intersect with the boundary (the black bold line in figure 8 and figure 9. Taking parallel parking as an example, the constraint conditions of obstacle avoidance are shown as follows,

$$\begin{cases} y2 < y_A < y1 + W_p, x4 < x_A < x1 \\ y1 < y_A < y1 + W_p, x_A \geq x1 \\ y2 < y_B < y1 + W_p, x4 < x_B < x1 \\ y1 < y_B < y1 + W_p, x_B \geq x1 \\ y2 < y_C < y1 + W_p, x4 < x_C < x1 \\ y1 < y_C < y1 + W_p, x_C \geq x1 \\ y2 < y_D < y1 + W_p, x4 < x_D < x1 \\ y1 < y_D < y1 + W_p, x_D \geq x1 \end{cases} \tag{15}$$

FIGURE 8 The movement track of vehicle body in parallel parking

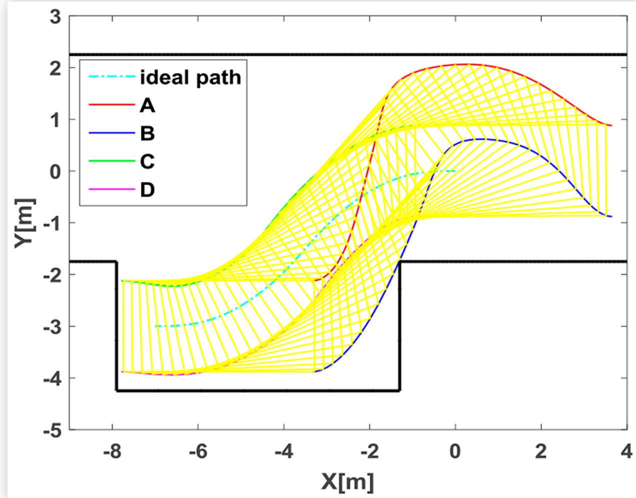
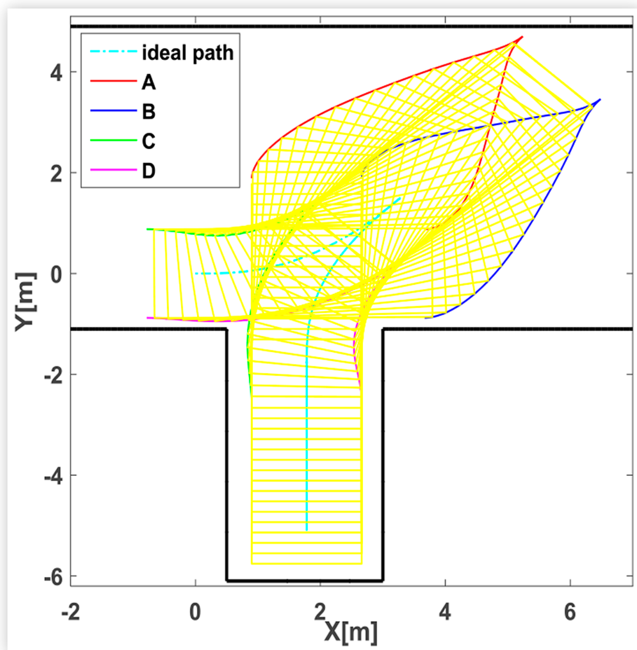


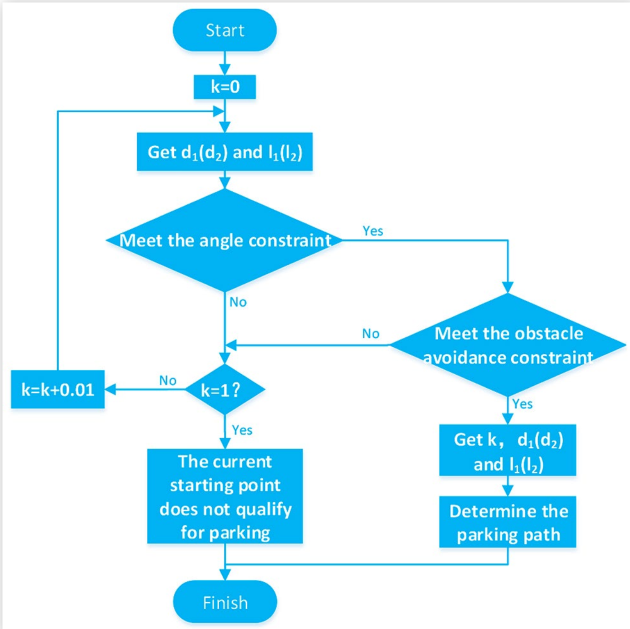
FIGURE 9 The movement track of vehicle body in perpendicular parking



Where W_p denotes the width of parallel parking passage.

After determining the parking type and the parking destination, the parameters of the parking path can be obtained according to the constraint conditions. The specific process is shown in figure 10. The initial value of the proportionality coefficient k is 0. If the constraint conditions are met, we can obtain the value of k, otherwise increase 0.01 by the current value and enter the next cycle until the value of k is 1.

FIGURE 10 The process of obtaining parking path parameters

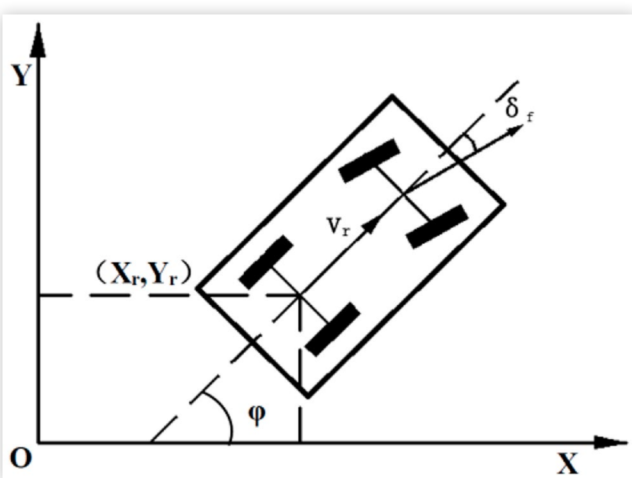


Path Tracking

Vehicle Kinematics Model

In the process of parking, the vehicle is always running at a low speed [15]. Generally, it is not necessary to consider the vehicle stability and other dynamics issues. The vehicle can be regarded as a rigid body to establish the vehicle kinematics model, as shown in figure 11. In the inertial coordinate system, (X_r, Y_r) is the axis of the rear axes of the vehicle, φ is the heading angle of the vehicle, δ_f is the angle of the front wheel, v_r is the central velocity of the rear axle of the vehicle, L is axle distance of vehicle. Assuming that the vehicle centroid side angle remains unchanged in the course of steering, that is, the instantaneous turning radius of the vehicle equals to the

FIGURE 11 Vehicle kinematics model



curvature radius of the road, the kinematic equation of the vehicle in the process of parking is obtained as follows,

$$\begin{bmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ \tan\left(\delta_f / L\right) \end{bmatrix} v_r \tag{16}$$

Path Tracker

In this paper, a path tracker is designed based on the model predictive control algorithm. Because the model predictive control algorithm has excellent characteristics such as model prediction, rolling optimization and feedback correction, the MPC controller has good adaptability and robustness. From equation (16), the system can be regarded as a control system of which the input is $u(v,\delta)$ and state vector is $\chi(x,y,\varphi)$. The general form is shown in the following equation,

$$\dot{\chi} = f\left(\chi, u\right) \tag{17}$$

The given reference path is described by the motion path of the reference vehicle. Each point on it satisfies the kinematics equation above. The general form of the system is shown in the following equation,

$$\dot{\chi}_r = f\left(\chi_r, u_r\right) \tag{18}$$

In his system, $\chi_r = [x_r \;\; y_r \;\; \varphi_r]^T, u_r = \left[v_r \quad \delta_r\right]$.

Expanding the right side of (17) in Taylor series around the point (χ_r, u_r) and ignoring the higher-order terms, the equation (19) is obtained.

$$\begin{aligned} \dot{\chi} &= f\left(\chi_r, u_r\right) \\ &+ \left.\frac{\partial f\left(\chi, u\right)}{\partial \chi}\right|_{\substack{\chi=\chi_r \\ u=u_r}} \left(\chi - \chi_r\right) + \left.\frac{\partial f\left(\chi, u\right)}{\partial u}\right|_{\substack{\chi=\chi_r \\ u=u_r}} \left(u - u_r\right) \end{aligned} \tag{19}$$

The linearised error model is obtained by subtracting equation (18) from equation (19).

$$\begin{aligned} \dot{\tilde{\chi}} = \begin{bmatrix} \dot{x} - \dot{x}_r \\ \dot{y} - \dot{y}_r \\ \dot{\varphi} - \dot{\varphi}_r \end{bmatrix} &= \begin{bmatrix} 0 & 0 & -v_r \sin\varphi_r \\ 0 & 0 & v_r \cos\varphi_r \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x - x_r \\ y - y_r \\ \varphi - \varphi_r \end{bmatrix} + \\ & \begin{bmatrix} \cos\varphi_r & 0 \\ \sin\varphi_r & 0 \\ \tan\delta_r / L & v_r / \left(L\cos^2\delta_r\right) \end{bmatrix} \begin{bmatrix} v - v_r \\ \delta - \delta_r \end{bmatrix} \end{aligned} \tag{20}$$

In order to apply the error model to the design of model prediction controller, equation (20) is discretized:

$$\tilde{\chi}\left(k+1\right) = A_{k,t}\tilde{\chi}\left(k\right) + B_{k,t}\tilde{u}\left(k\right) \tag{21}$$

Where, T denotes the sampling time,

$$A_{k,t} = \begin{bmatrix} 1 & 0 & -v_r \sin\varphi_r T \\ 0 & 1 & v_r \cos\varphi_r T \\ 0 & 0 & 1 \end{bmatrix},$$

$$B_{k,t}=\begin{bmatrix}\cos\varphi_r T & 0\\ \sin\varphi_r T & 0\\ \tan\delta_r T/L & v_r T/\left(L\cos^2\delta_r\right)\end{bmatrix}$$

In order to realise the limitation of control vector increment and control vector, the linear time-varying system shown in formula (21) is rewritten. And the state vector and control vector are combined into a new state vector.

$$\begin{aligned}&\xi\left(k+1|t\right)=\tilde{A}_{k,t}\xi\left(k|t\right)+\tilde{B}_{k,t}\Delta U\left(k|t\right)\\&\eta\left(k|t\right)=\tilde{C}_{k,t}\xi\left(k|t\right)\end{aligned}\tag{22}$$

Where,

$$\xi\left(k|t\right)=\begin{bmatrix}\tilde{\chi}\left(k|t\right)\\ \tilde{u}\left(k-1|t\right)\end{bmatrix},\tilde{A}_{k,t}=\begin{bmatrix}A_{k,t} & B_{k,t}\\ 0_{m\times n} & I_m\end{bmatrix},\tilde{B}_{k,t}=\begin{bmatrix}B_{k,t}\\ I_m\end{bmatrix},$$

where n denotes the dimension of state vector and m denotes the dimension of control vector.

The objective function is the cost function to solve the optimal control sequence in the model predictive control. This objective function is also employed to denote the accuracy and stability of the tracker. In this paper, the basic form of the objective function adopted is shown in equation (21).

$$\begin{aligned}J=&\sum_{i=1}^{N_p}\left\|\left(\eta\left(k+i|k\right)-\eta_{ref}\left(k+i|k\right)\right)\right\|_Q^2\\&+\sum_{i=0}^{N_c-1}\left\|\Delta\boldsymbol{u}\left(k+i|k\right)\right\|_R^2+\rho\varepsilon^2\end{aligned}\tag{23}$$

Where Q denotes the weight matrix of state quantity error, R denotes the weight matrix of state quantity error, ε denotes the relaxation factor, and ρ denotes the weight of relaxation factor.

One of the main advantages of model predictive control is that it can explicitly deal with various constraints of the system. So the stability of the system is improved. In MPC basic control structure, the most basic constraint is the control quantity amplitude constraint as shown in formula (24) and the control quantity increment constraint as shown in formula (25). The MPC parameters in this paper are shown in table 1.

TABLE 1 MPC parameters

Parameter	Value
Sampling period	t_{mpc} = 0.02s
Predict horizon	Np = 30
Control horizon	Nc = 10
Weighting mavrameter	$Q=\left[100\ \ 0\ \ 0;0\ \ 100\ \ 0;0\ \ 0\ \ 50\right]$
	$R=\left[5\ \ 0\ \ 0;0\ \ 5\ \ 0;0\ \ 0\ \ 5\right]$
Constraint extremum	$-0.2m/s\le v-vr\le 0.2m/s$
	$-40°\ \le\delta_f\le 40°$
Constraint incremental extremum	$-0.05m/s\le\Delta v\le 0.05m/s$
	$-0.5°\ \le\Delta\delta_f\le 0.5°$

$$\underline{u}\le u\left(k\right)\le\overline{u}\tag{24}$$

$$\underline{\Delta u}\le\Delta u\left(k\right)\le\overline{\Delta u}\tag{25}$$

Simulation Results

In this paper, CarSim and Simulink are used for co-simulation analysis. The parallel parking and perpendicular parking are simulated respectively. The vehicle parameters are shown in table 2.

Simulation Results of Parallel Parking

In the PCS, the coordinates of the four simulated parking Spaces are (−3.6, −2.75), (−3.6, −5.25), (−10.2, −5.25), (−10.2, −2.75). In parallel parking simulation, vehicle speed is set as 5 km/h.

In figure 12, the reference path is designed in the above method, and the path tracker can well track the designed reference path. In figure 13, it can be seen that both the refrence angle and tracking angle are continuous without sudden change and there is no in-situ turning. It also illustrates the curvature continuity of parallel parking path designed in this paper. In figure 14, the reference heading angle changes continuously and both the starting point and ending point are zero. The heading angle of the simulation vehicle can well follow the reference heading angle and straighten the vehicle at the ending point.

TABLE 2 Vehicle parameters

Parameter	Value(m)
Total length	l = 4.45
Total width	w = 1.76
Axle distance	L = 2.66
Header length	L_f = 0.95
Tail length	L_r = 0.84

FIGURE 12 Parallel parking path tracking results

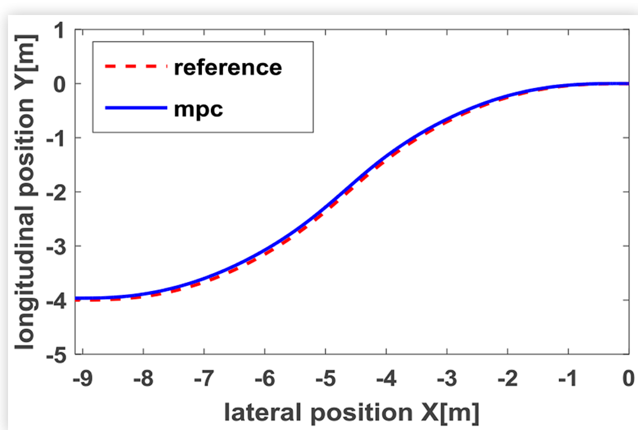


FIGURE 13 The front wheel angle of parallel parking

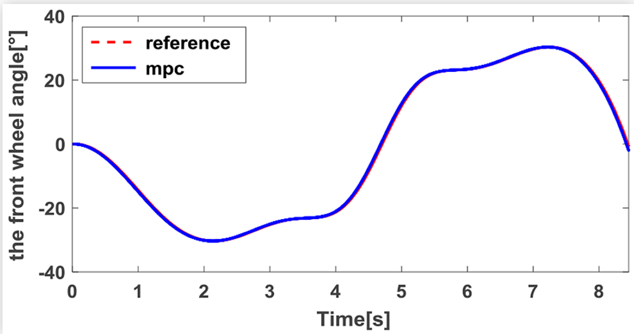


FIGURE 14 Heading angle variation of parallel parking

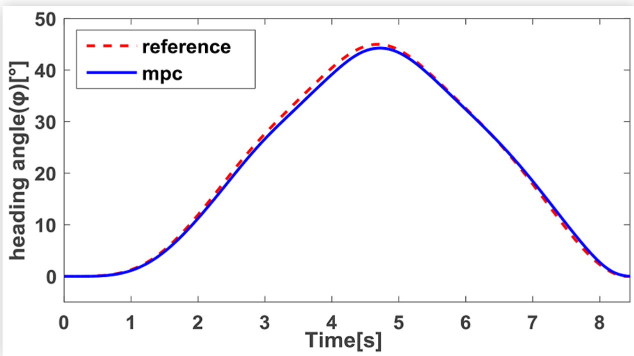


FIGURE 15 Parallel parking errors

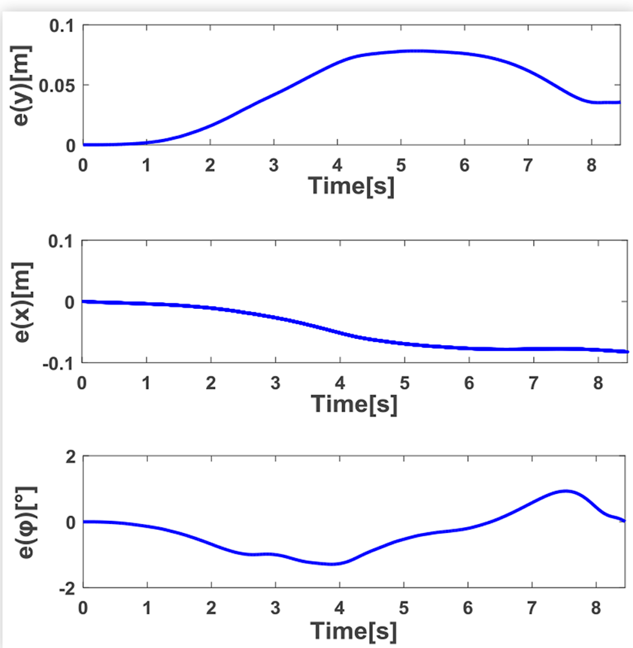


Figure 15 shows the lateral error, longitudinal error and heading angle error in parallel parking process. It can be seen that the lateral error and longitudinal error are no more than 10cm. The heading angle error is no more than 2°. The heading angle error at the end of parking is 0.4°. When the parking is completed, the vehicle can be straightened. It shows that the control precision of the controller is high and it meets the requirements of path tracking.

Simulation Results of Perpendicular Parking

In the PCS, the coordinates of the four simulated parking spaces are $(3.8, -1.1)$, $(3.8, -6.1)$, $(1.3, -6.1)$, $(1.3, -1.1)$. Vehicle speed is set as 2 km/h in perpendicular parking simulation.

In figure 16, the reference path is designed in the above method. The path tracker can well track the designed reference path. In figure 17, it can be seen that both the reference angle

FIGURE 16 Perpendicular parking path tracking results

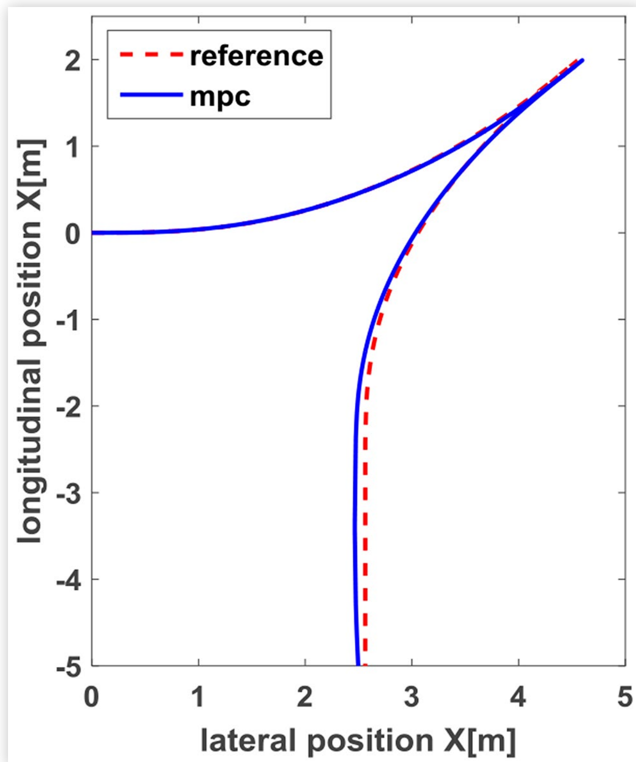


FIGURE 17 The front wheel angle of perpendicular parking

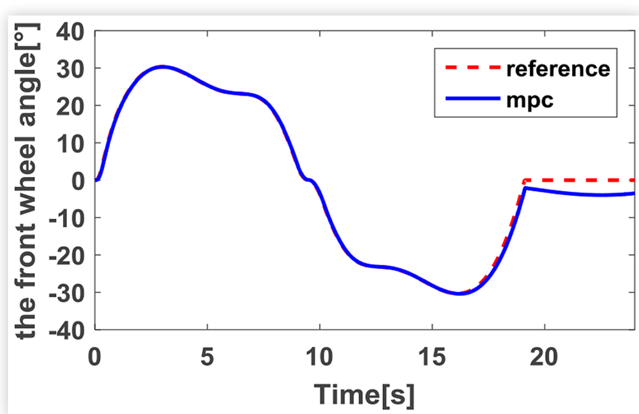
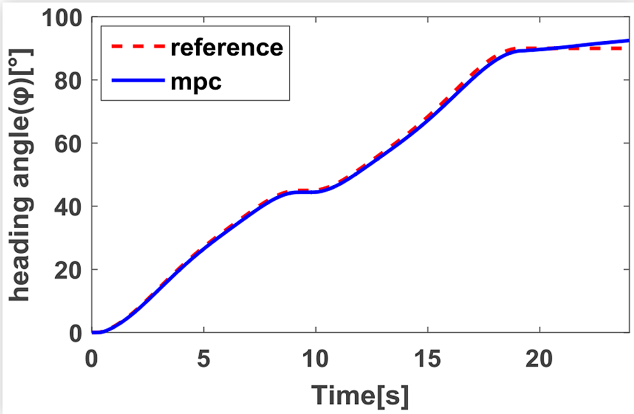


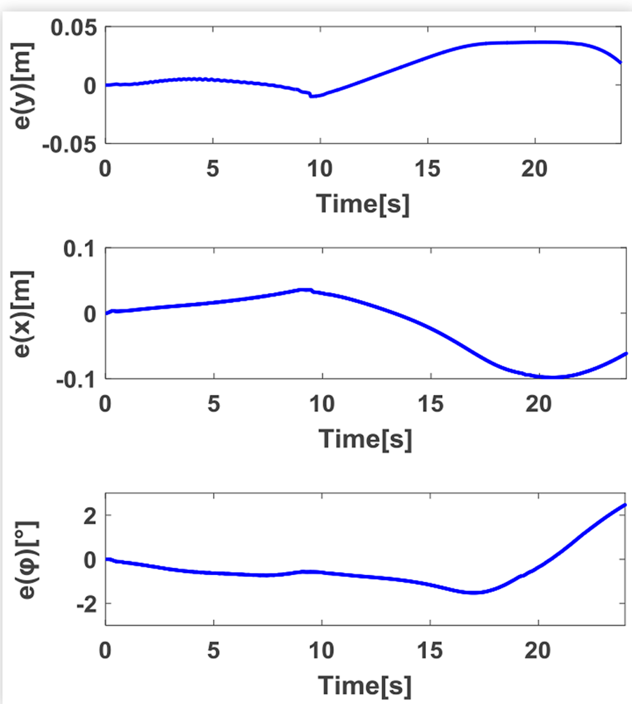
FIGURE 18 Heading angle variation of perpendicular parking



and tracking angle are continuous without sudden change and there is no in-situ turning. It also illustrates the curvature continuity of parallel parking path designed in this paper. Due to the large error in the tracking of the curve path, the front wheel angle should be controlled to reduce the lateral error and longitudinal error in the whole tracking process. In figure 18, the reference heading angle changes continuously, starting at zero and ending at 90°. The heading angle of the simulation vehicle can well follow the reference heading angle. In the process of linear path tracking, in order to reduce the lateral error and longitudinal error, the heading angle error is increased. But the error is not very large so that the vehicle can still be straightened at the parking end point.

Figure 19 shows the lateral error, longitudinal error and heading angle error of perpendicular parking. It can be seen

FIGURE 19 Perpendicular parking errors



that the lateral error and longitudinal error are no more than 10cm. Heading angle error is no more than 3°. The heading angle error is 2.3° at the end point of parking. When the parking is completed, the vehicle can be put enough to meet the requirement of path tracking.

Summary/Conclusions

This paper takes the minimum turning radius and obstacle avoidance constraints into consideration. A continuous curvature parking path is proposed, which can meet the requirements of parallel parking and perpendicular parking. A path tracker based on model predictive control (MPC) algorithm is designed to track the parking path. Simulation results of the path tracker show that the tracking effect of the parking path tracker is good enough. The lateral error and longitudinal error can be controlled in the centimeter scale. Finanlly, the vehicle can be straightened when the parking is completed.

However, there are also some problems remained to be solved. First of all, the parking path designed in this paper has requirements on the vehicle posture at the starting point. In addition, the impact of speed and gear shift on the accuracy of parking control is not considered. Further works will focus on the impact of these factors.

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Contact Information

For questions or to contact the authors, please email to **Rui He** at: herui@jlu.edu.cn.

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Definitions/Abbreviations

MPC - model predictive control

PCS - Parking Coordinate System

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