Prepared for submission to JHEP

YITP-24-71

AdS/CFT correspondence for the $O(N)$ invariant critical φ^4 model in 3-dimensions by the conformal smearing

Sinya Aoki[a] **Kiyoharu Kawana**[b] **Kengo Shimada**[a]

[a] *Center for Gravitational Physics and Quantum Information, Yukawa Institute for Theoretical Physics, Kyoto University, Kitashirakawa Oiwakecho, Sakyo-Ku, Kyoto 606-8502, Japan*

[b] *School of Physics, Korea Institute for Advanced Study, Seoul 02455, Korea*

E-mail: saoki@yukawa.kyoto-u.ac.jp

Abstract: We investigate a structure of a 4-dimensional bulk space constructed from the $O(N)$ invariant critical φ^4 model in 3-dimension using the conformal smearing. We calculate a bulk metric corresponding to the information metric and the bulk-to-boundary propagator for a composite scalar field φ^2 in the large N expansion. We show that the bulk metric describes an asymptotic AdS space at both UV (near boundary) and IR (deep in the bulk) limits, which correspond to the asymptotic free UV fixed point and the Wilson-Fischer IR fixed point of the 3-dimensional φ^4 model, respectively. The bulk-to-boundary scalar propagator, on the other hand, encodes Δ_{φ^2} (the conformal dimension of φ^2) into its z (a coordinate in the extra direction of the AdS space) dependence. Namely it correctly reproduces not only $\Delta_{\varphi^2} = 1$ at UV fixed point but also $\Delta_{\varphi^2} = 2$ at the IR fixed point for the boundary theory. Moreover, we confirm consistency with the GKP-Witten relation in the interacting theory that the coefficient of the $z^{\Delta_{\varphi^2}}$ term in $z \to 0$ limit agrees exactly with the two-point function of φ^2 including an effect of the φ^4 interaction.

Contents

1 Introduction

The AdS/CFT correspondence [1–3] plays a crucial role in understanding the holographic nature of gravity and may give a hint on its quantization, but it is still mysterious even though plenty of evidences and applications exist after the first proposal. While a large part of the AdS/CFT correspondence can be understood in the context of the closed string/open string duality with D branes, a complete understanding of this duality has not been attained yet. It is widely accepted, however, that a particular type of CFT can be holographic dual to bulk gravity theories, and such theories are called holographic CFT.

On the other hand, it is generally believed that the AdS radial direction emerges as the energy scale of a renormalization group transformation applied to the dual CFT at the boundary. (For example, see Ref. [4]). Among studies in such a direction, the continuum version of multi-scale entanglement renormalization ansatz (cMERA) has been employed as a real space quantum renormalization group, in order to generate the extra dimension as a level of the coarse-graining [5–7]. In their approach, the geometric structure of the bulk spacetime at a given time slice is determined from a quantum information metric for a CFT state on the equal-time (boundary) surface. Indeed the bulk space-time becomes AdS for the vacuum state, and thus the AdS spacetime naturally emerges from the boundary CFT. More interestingly, their bulk construction can be applied not only to holographic CFT but also to generic CFT or even to non-conformal field theories.

Following the philosophy of Refs. [5–7], one of the present authors has proposed a similar but different method to construct the Euclidean AdS space from Euclidean CFT by employing a different coarse-graining technique called flow equations and applied it to various cases [8–16]. Recently, an improved version of the flow equations has been found [17], and we call the corresponding method a conformal smearing since the conformal transformations applied to CFT fields on the boundary are literally mapped to a part of the general coordinate transformations of the smeared fields in the bulk, which is the full isometry of the AdS space.

In this paper, in order to obtain deeper understanding of a mechanism for an emergent extra dimension in AdS/CFT correspondences, we apply the conformal smearing to the $O(N)$ invariant critical $\lambda\varphi^4$ model in 3-dimensions, which has the asymptotic free UV fixed point and the Wilson-Fischer IR fixed point, where λ has mass dimension one and breaks conformal symmetry. This model is thought to be dual to higher spin theories [18], and this duality has been investigated in terms of a bi-local field, where a magnitude of its relative coordinate is interpreted as an extra bulk dimension [19, 20]. The conformal smearing approach is different from theirs. In particular, while the bulk geometry is assumed to be AdS in their approach, it is determined by the information metric in our approach. Indeed, the previous study employing the Gaussian smearing [9] has shown that the bulk space becomes the asymptotic AdS space at both UV limit(near boundary) and IR limit (deep in the bulk), whose AdS radii are different in two limits. Since the Gaussian smearing keeps only a part of the relation between the conformal transformation and the AdS isometry [11], however, results in Ref. [9] are insufficient to understand the duality between the bulk theory and the boundary CFT. For example, inequality for the AdS radii between two limits can not be determined, and a change of the conformal dimension of the composite scalar operator between two limits has not been investigated in terms of the bulk language. We, therefore, investigate this duality using the conformal smearing in this paper.

We here summarize the main results from our study.

(1) The bulk space constructed from the interacting $O(N)$ invariant critical φ^4 model at $d = 3$ by the conformal smearing is the 4-dimensional AdS space at the leading order (LO) in the large N expansion. While, at the next-to-leading order (NLO), it becomes the asymptotic AdS space both in the UV and the IR limit, which correspond to the asymptotic free UV fixed point and the Wilson-Fischer IR fixed point, respectively. We also observe that $R_{\rm AdS}^{\rm UV} < R_{\rm AdS}^{\rm IR}$ at the NLO, which reflects the fact that the number of degrees of freedom decreases from UV to IR by the renormalization group.

(2) The bulk-to-boundary propagator of the $O(N)$ invariant scalar field has been calculated. In the $z \to 0$ limit where z is the radial coordinate of the bulk space, this propagator behaves as z^1, showing that the conformal dimension of the corresponding $O(N)$ invariant scalar operator at the boundary is one [21]. This values agrees correctly with $\Delta_{\varphi^2} = 1$, the conformal dimension of the composite scalar operator φ^2 (the spin zero "current" J in Ref. [18]) at the UV fixed point. On the other hand, in the IR limit ($z \to \infty$), the bulk-to-boundary propagator behaves as

$$z^{-2} \simeq \left(\frac{z}{z^2 + x^2}\right)^2, \tag{1.1}$$

where x is the boundary coordinates. This behavior corresponds to $\Delta_{\varphi^2} = 2$, which is the conformal dimension of φ^2 at the IR fixed point.

These two findings show that the non-trivial dynamics of the boundary theory generated by the non-conformal interaction term is correctly encoded in the bulk geometry and dynamics.

2 Model and Conformal smearing

Throughout the paper, we are working on $d = 3$, where d is the dimension of the boundary.

2.1 $O(N)$ model in 3 dimensions

We consider an $O(N)$ invariant model for scalar fields, whose action is given by

$$S(\varphi) = N \int d^3x \left[\frac{Z_\varphi}{2}\partial^\mu\varphi\cdot\partial_\mu\varphi + \frac{m^2 Z_m}{2} Z_\varphi \varphi\cdot\varphi + \frac{\lambda Z_\lambda}{4} Z_\varphi^2 (\varphi\cdot\varphi)^2\right], \tag{2.1}$$

where $\varphi\cdot\varphi := \sum_{a=1}^N \varphi^a\varphi^a$ with a being an $O(N)$ index, and Z_φ, Z_m and Z_λ are renormalization constants which relate bare to renormalized quantities as $\varphi_0^a = \sqrt{Z_\varphi}\varphi^a$, $m_0^2 = m^2 Z_m$, and $\lambda_0 = \lambda Z_\lambda$, respectively. Note that we have extracted a factor N to consider the large N expansion, and $m^2 Z_m$ in our notation includes the additive mass counter terms.

We calculate correlation functions necessary in this paper in the large N expansion, by employing the Schwinger-Dyson equation in Appendix A, and results are summarized below.

In this paper, we consider the critical case, where the renormalized mass is tuned to be zero. The 2-pt function $N\langle\varphi^a(x)\varphi^b(y)\rangle := \delta^{ab}\Gamma(x-y)$ in this case ($m^2=0$) is given at the NLO in the large N expansion as

$$\Gamma(x) = \int_p e^{ipx}\left[\tilde\Gamma_0(p) + \frac{1}{N}\tilde\Gamma_1(p)\right], \quad \int_p := \int \frac{d^3p}{(2\pi)^3}, \tag{2.2}$$

where

$$\tilde\Gamma_0(p) = \frac{1}{p^2}, \quad \tilde\Gamma_1(p) = \frac{X(p^2)}{(p^2)^2}, \quad X(p^2) := \int_Q \left[\frac{1}{(Q-p)^2} - \frac{1}{Q^2}\right]\frac{-2\lambda_0}{1+\lambda_0 B(Q^2)} \tag{2.3}$$

with $B(p^2) = 1/(8|p|)$.

The 4-pt function is decomposed as

$$\begin{aligned} K^{a_1a_2a_3a_4}(x_1,x_2,x_3,x_4) &:= N^3\langle\varphi^{a_1}(x_1)\varphi^{a_2}(x_2)\varphi^{a_3}(x_3)\varphi^{a_4}(x_4)\rangle \\ &= \delta^{a_1a_2}\delta^{a_3a_4}K(x_1,x_2;x_3,x_4) + (2\leftrightarrow 3) + (2\leftrightarrow 4), \end{aligned} \tag{2.4}$$

where K at the LO is given by

$$K_0(x_1,x_2;x_3,x_4) = \prod_{i=1}^4 \int_{p_i} \frac{e^{ip_ix_i}}{p_i^2} \times (2\pi)^3\delta^{(3)}\left(\sum_{i=1}^4 p_i\right)\frac{-2\lambda_0}{1+\lambda_0 B((p_1+p_2)^2)}. \tag{2.5}$$

2.2 Conformal smearing

In Ref. [17], the conformal smearing has been introduced to construct bulk field ϕ^a from φ^a as

$$\phi^a(X) := \int d^3y\, S(x-y,z)\varphi^a(y) = \int_p S(p,z)\tilde\varphi^a(p)e^{ipx}, \tag{2.6}$$

where $X := (z,x)$, $\tilde\varphi^a(p)$ and $S(p,z)$ are the Fourier transforms of $\varphi^a(x)$ and $S(x,z)$, respectively, and the smearing kernel in the momentum space is given with the modified Bessel function K_1 as

$$S(p,z) := pzK_1(pz), \quad p := |\vec p|\ . \tag{2.7}$$

It is easy to see that $\phi^a(X)$ is the solution to the (conformal) flow equation [17] as

$$-\eta\partial_\eta^2\phi^a(X) = \Box_x\phi^a(X), \quad \phi^a(0,x) = \varphi^a(x), \quad \eta := \frac{z^2}{4}. \tag{2.8}$$

Furthermore, we define the normalized smeared field as

$$\sigma^a(X) := \frac{\phi^a(X)}{\sqrt{\gamma(z)}}, \quad \gamma(z) := \sum_a\langle\phi^a(X)\phi^a(X)\rangle, \tag{2.9}$$

where $\langle\cdots\rangle$ is a vacuum expectation value in the $O(N)$ model. It was shown that the conformal transformations to $\varphi^a(x)$ generate a part of general coordinate transformations applied to the scalar $\sigma^a(X)$ [17]. The translational invariance tells us that γ only depends on z.

At the NLO in the large N expansion, we explicitly obtain

$$\gamma(z) = \gamma_0(z) + \frac{1}{N}\gamma_1(z), \tag{2.10}$$

where

$$\gamma_0(z) = \int_p \frac{S^2(p.z)}{p^2}, \quad \gamma_1(z) = \int_p S^2(p,z)(p^2)^2 X(p^2). \tag{2.11}$$

3 Bulk metric via the conformal smearing

In the smearing approach, the bulk metric corresponding to the vacuum state of the boundary theory[1] can be defined in terms of the normalized smeared field [8] as

$$g_{AB}(X) := \ell^2 \langle \partial_A \sigma^a(X) \partial_B \sigma^a(X) \rangle, \tag{3.1}$$

which can be interpreted as the Bures (quantum) information metric [11, 17], where ℓ is some constant of the length scale. Note that a similar definition using Bures metric was employed in Ref. [5].

Non-zero components of the metric are given at the NLO as

$$\begin{aligned} g_{\mu\nu}(z) &= \frac{\delta_{\mu\nu}}{3\gamma(z)} \left[F_0(z) + \frac{1}{N} F_1(z) \right], \\ g_{zz}(z) &= \left(\frac{\gamma_z(z)}{2\gamma(z)} \right)^2 - \frac{\gamma_z(z)}{\gamma^2(z)} \left[G_0(z) + \frac{1}{N} G_1(z) \right] + \frac{1}{\gamma(z)} \left[H_0(z) + \frac{1}{N} H_1(z) \right], \end{aligned} \tag{3.2}$$

where $\gamma_z(z) := \dfrac{d\gamma(z)}{dz}$ and the LO contributions are expressed as

$$F_0(z) = \int_p S^2(p,z), \quad G_0(z) = \int_p \frac{S_z(p,z)S(p.z)}{p^2}, \quad H_0(z) = \int_p \frac{S_z(p,z)S_z(p.z)}{p^2}, \tag{3.3}$$

with $S_z(p,z) := \partial_z S(p,z)$. On the other hand, NLO corrections are

$$\begin{aligned} F_1(z) &= \int_p \frac{S^2(p,z)}{p^2} X(p^2), \\ G_1(z) &= \int_p \frac{S_z(p,z)S(p,z)}{(p^2)^2} X(p^2), \quad H_1(z) = \int_p \frac{S_z(p,z)S_z(p,z)}{(p^2)^2} X(p^2). \end{aligned} \tag{3.4}$$

3.1 Results at the LO

Using the integration formula (C.1), we obtain

$$\gamma_0(z) = \frac{3}{64z}, \quad F_0(z) = \frac{45}{1024z^3}, \quad G_0(z) = -\frac{3}{128z^2}, \quad H_0(z) = \frac{27}{1024z^3}. \tag{3.5}$$

Therefore, the metric at the LO is given by

$$g_{\mu\nu}^{\rm LO}(z) = \frac{5\ell^2}{16} \frac{\delta_{\mu\nu}}{z^2}, \qquad g_{zz}^{\rm LO} = \frac{5\ell^2}{16} \frac{1}{z^2}, \tag{3.6}$$

which is the Euclidean AdS metric with the AdS radius [22]

$$R_{\rm AdS}^{\rm LO} = \ell \frac{\sqrt{5}}{4} = \ell \sqrt{\frac{\Delta_\varphi (d - \Delta_\varphi)}{d+1}} \tag{3.7}$$

at $d = 3$, where $\Delta_\varphi = (d-2)/2$ is the conformal dimension of a free massless scalar.

[1]The bulk metric depends on the boundary state. See [22] in the case of the metric for the thermal state.

3.2 Results at the NLO

The metric at the NLO is given by

$$
\begin{aligned}
g^{\mathrm{NLO}}_{\mu\nu}(z) &= g^{\mathrm{LO}}_{\mu\nu}(z)\left[1+\frac{1}{N}G_s(z)\right], \\
g^{\mathrm{NLO}}_{zz}(z) &= g^{\mathrm{LO}}_{zz}(z)\left[1+\frac{1}{N}G_\sigma(z)\right],
\end{aligned} \tag{3.8}
$$

where $G_s(z)$ and $G_\sigma(z)$ are defined in Eqs. (B.2) and (B.3).

In the UV limit that $z \to 0$, from explicit expression of $F^n_{ij}(g)$ in Eq. (C.14), we see that the D^1_{11} term in $F^1_{11}(g)$ dominates in the limit. Therefore the metric becomes

$$
\begin{aligned}
g^{\mathrm{NLO}}_{\mu\nu}(z) &\simeq g^{\mathrm{LO}}_{\mu\nu}(z)\left[1-\frac{g}{N}\frac{128 D^1_{11}}{3\pi^4}\right], \\
g^{\mathrm{NLO}}_{zz}(z) &\simeq g^{\mathrm{LO}}_{zz}(z)\left[1-\frac{g}{N}\frac{128 D^1_{11}}{15\pi^4}\right],
\end{aligned} \tag{3.9}
$$

as $g := \lambda_0 z/8 \to 0$, where $D^1_{11} = -\pi^2/4$ is given in Eq. (C.17) with $c_1 = 1$. This means that the NLO correction is sub-leading of the order z in the UV limit so that the AdS radius is unchanged: $R^{UV}_{\mathrm{AdS}} = R^{\mathrm{LO}}_{\mathrm{AdS}}$ and the metric describes the asymptotic AdS space.

In the IR limit that $z \to \infty$, on the other hand, Eq. (C.20) leads to

$$
\begin{aligned}
g^{\mathrm{NLO}}_{\mu\nu}(z) &\simeq g^{\mathrm{LO}}_{\mu\nu}(z)\left[1-\frac{4}{3\pi^2 N}C_{\mathrm{IR}}\right], \\
g^{\mathrm{NLO}}_{zz}(z) &\simeq g^{\mathrm{LO}}_{zz}(z)\left[1+\frac{4}{3\pi^2 N}\left(\frac{48}{5}+3C_{\mathrm{IR}}\right)\right]
\end{aligned} \tag{3.10}
$$

as $g \to \infty$, where

$$
C_{\mathrm{IR}} = \frac{512}{45\pi^2}\int_0^\infty dp\, p^2\left(\frac{15}{16}-p^2\right)K_1^2(p)\ln(p^2) = -\frac{77}{30}. \tag{3.11}
$$

By the change of the z coordinate as

$$
\tilde{z} = z\left[1+\frac{4}{3\pi^2 N}\left(\frac{24}{5}+2C_{\mathrm{IR}}\right)\right], \tag{3.12}
$$

we finally obtain

$$
\tilde{g}^{\mathrm{NLO}}_{AB}(\tilde{z}) \simeq (R^{\mathrm{LO}}_{\mathrm{AdS}})^2\frac{\delta_{AB}}{\tilde{z}^2}\left[1+\frac{4}{3\pi^2 N}\left(\frac{48}{5}+3C_{\mathrm{IR}}\right)\right], \tag{3.13}
$$

which describes the AdS space with the radius given by

$$
R^{\mathrm{IR}}_{\mathrm{AdS}} := R^{\mathrm{LO}}_{\mathrm{AdS}}\left[1+\frac{2}{3\pi^2 N}\left(\frac{48}{5}+3C_{\mathrm{IR}}\right)\right]. \tag{3.14}
$$

Since

$$
\frac{48}{5}+3C_{\mathrm{IR}} = \frac{19}{10}, \tag{3.15}
$$

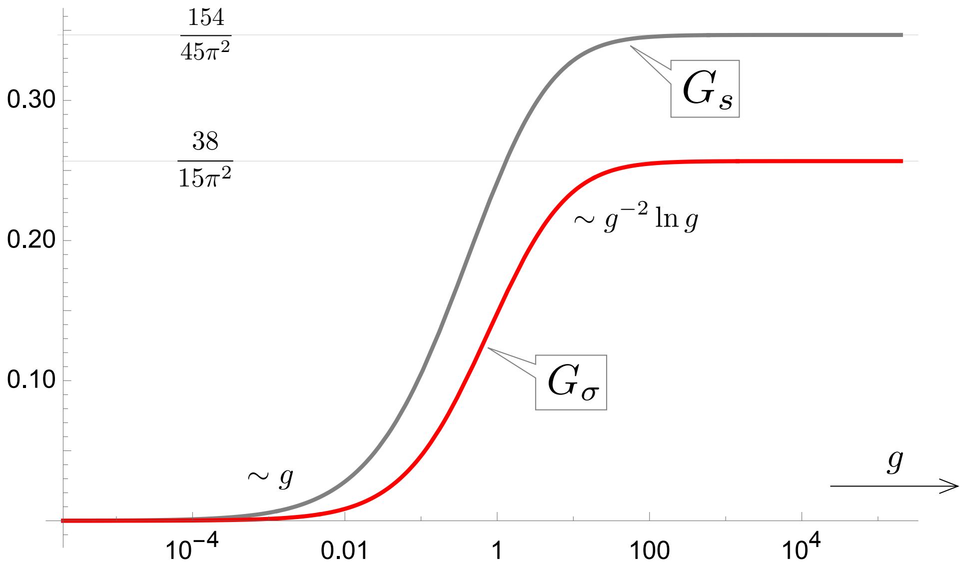

Figure 1. The $1/N$ corrections to the metric, $G_s(z)$ (gray) and $G_\sigma(z)$ (red), as a function of $g = \lambda_0 z/8$. The horizontal axis is on a logarithmic scale for g.

the radius increases from UV to IR by the $1/N$ correction as $R_{\rm AdS}^{\rm IR} - R_{\rm AdS}^{\rm UV} = O(1/N) > 0$. This can be understood as follows. Since a number of effective degrees of freedom decreases from UV to IR by the renormalization group (smearing in our case), corresponding contributions to the negative cosmological constant in the bulk also decrease, so that the AdS radius increases.[2]

Fig. 1 shows behaviors of $1/N$ correction terms G_s and G_σ in the metric as a function of $g = \lambda_0 z/8$.

4 Bulk-to-boundary scalar propagator

In this section, we consider the $O(N)$ invariant scalar field

$$O(y) := \ :\varphi^b(y)\varphi^b(y): \ , \tag{4.1}$$

where : : denotes the normal ordering. The bulk-to-boundary propagator for this operator can be defined as the correlation function with the corresponding bulk (smeared) operator:

$$\Pi(X,y) := \langle \sigma^a(X)\sigma^a(X)O(y)\rangle\,. \tag{4.2}$$

At the LO in the large N expansion, we obtain

$$\Pi(X,y) = \frac{1}{N}\left[2\Pi_0(x-y,z) + \Pi_\lambda(x-y,z)\right] + O\left(\frac{1}{N^2}\right) \tag{4.3}$$

[2]This is related to the F theorem in quantum field theories at $d=3$. See Ref. [23].

where

$$\Pi_0(x-y,z) = \frac{1}{\gamma_0(z)}\left[\int_p e^{ip(x-y)}S(p,z)\Gamma_0(p)\right]^2 \tag{4.4}$$

$$= \frac{1}{\gamma_0(z)}\left[\int d^3y_1\, S(x-y_1,z)\Gamma_0(y_1-y)\right]^2 = \frac{4}{3\pi^2}\left[\frac{z}{z^2+(x-y)^2}\right], \tag{4.5}$$

which is nothing but the bulk-to-boundary propagator (except a factor two) in the free theory [17]. This behavior tells us that the conformal dimension of the $O(N)$ invariant composite scalar operator φ^2 is given by $\Delta_{\varphi^2}=1$ in the free theory.

On the other hand, the contribution coming from the interaction is given by

$$\Pi_\lambda(x-y,z) := \frac{1}{\gamma_0(z)}\int d^3y_1\, d^3y_2\, S(x-y_1,z)S(x-y_2,z)K_0(y_1,y_2;y,y)\ , \tag{4.6}$$

which is evaluated as

$$\Pi_\lambda(x-y,z) = \frac{1}{\gamma_0(z)}\int_{p_1,p_2} e^{i(p_1+p_2)(x-y)}\frac{S(p_1,z)S(p_2,z)}{p_1^2p_2^2}\frac{-2\lambda_0 B(p_{12}^2)}{1+\lambda_0 B(p_{12}^2)} \tag{4.7}$$

$$= \frac{1}{\gamma_0(z)z^2}\int_{p_1,p_2}\frac{K_1(p_1)K_1(p_2)}{|p_1||p_2|}e^{i(p_1+p_2)(x-y)/z}\frac{-2g}{p_{12}+g}, \tag{4.8}$$

where $p_{12} := |p_1+p_2|$.

Combining (4.4) and (4.7), the LO contribution in Eq. (4.3) turns out to be

$$\Pi(x-y,z) = \frac{2}{N\gamma_0(z)}\int_{p_1,p_2} e^{i(p_1+p_2)(x-y)}\frac{S(p_1,z)S(p_2,z)}{p_1^2p_2^2}\frac{1}{1+\lambda_0 B(p_{12}^2)}$$

$$= \frac{2}{N\gamma_0(z)z^2}\int_{p_1,p_2}\frac{K_1(p_1)K_1(p_2)}{|p_1||p_2|}e^{i(p_1+p_2)(x-y)/z}\frac{p_{12}}{p_{12}+g}\ . \tag{4.9}$$

Note that the factor $1+\lambda_0 B(p_{12}^2)$ on the first line is nothing but the wavefunction renormalization Z_O of the composite operator $O=\varphi^2$ in Eq. (A.52) with the renormalization scale μ replaced by p_{12}, as expected from the construction (4.2). We use this expression to discuss the IR limit later. Now we start with the UV limit.

4.1 UV limit

Since $S(p,z)=1+O(z^2)$ in the UV limit ($z\to 0$), Eq. (4.7) leads to

$$\Pi_\lambda(X,z) = -\frac{128\lambda_0 z}{3}\left[\Omega(x-y)+O(z^2)\right], \tag{4.10}$$

where

$$\Omega(x) = \int_{p_1,p_2}\frac{e^{i(p_1+p_2)x}}{p_1^2p_2^2}\frac{1}{8p_{12}+\lambda_0}\ . \tag{4.11}$$

The explicit form of Ω is given in Eq. (A.58). In total, we obtain

$$\Pi(X,y) \simeq \frac{64z}{3}\frac{1}{N}\left[\frac{1}{8\pi^2}\frac{1}{z^2+(x-y)^2}-2\lambda_0\Omega(x-y)\right] \tag{4.12}$$

as $z \to 0$, where the second term represents a correction due to the non-zero coupling λ_0.

Therefore, even in the presence of the interaction, we see

$$\Pi(X, y) \simeq z^{\Delta^{\rm UV}_{\varphi^2}} \times \frac{64}{3} \left\langle \varphi^a(x)\varphi^a(x)\varphi^b(y)\varphi^b(y) \right\rangle_c + O(z^3), \quad z \to 0 \tag{4.13}$$

where the connected correlation function of φ^2 is obtained in Eq. (A.57), and $\Delta^{\rm UV}_{\varphi^2} = 1$ corresponds to its conformal dimension at the asymptotic free UV fixed point of the boundary theory. Moreover, as shown in appendix A.3.2, we see

$$\left\langle \varphi^a(x)\varphi^a(x)\varphi^b(y)\varphi^b(y) \right\rangle_c \propto \begin{cases} |x-y|^{-2}, \ \ |x-y| \ll \dfrac{1}{\lambda_0} \quad ({\rm UV}), \\[2ex] |x-y|^{-4}, \ \ |x-y| \gg \dfrac{1}{\lambda_0} \quad ({\rm IR}). \end{cases} \tag{4.14}$$

4.2 IR limit

Let us expand the factor $p_{12}/(p_{12}+g)$ in Eq. (4.9) for $g \propto z \to \infty$. Then we get

$$\Pi(x-y, z) = \sum_{n=1}^{\infty} \Pi_n(x-y, z), \tag{4.15}$$

where

$$\Pi_n(x, z) := \frac{2}{N\gamma_0(z) z^2} \frac{(-1)^{n+1}}{g^n} \int_{p_1, p_2} \frac{K_1(p_1)K_2(p_2)}{p_1 p_2} e^{i(p_1+p_2)\cdot x/z} p_{12}^n. \tag{4.16}$$

The leading contribution is the $n = 1$ term, which can be further expanded as

$$\Pi_1(x, z) = \frac{1}{Nz^2} \frac{512}{3\pi^4 \lambda_0} \sum_{k=0}^{\infty} \frac{(k+1)(-1)^k A_k}{(2k+3)!} \left(\frac{x^2}{z^2} \right)^k, \tag{4.17}$$

where

$$A_k := \int_0^\infty dp_1 \int_0^{p_1} dp_2 \, K_1(p_1)K_1(p_2) \left[(p_1+p_2)^{2k+3} - (p_1-p_2)^{2k+3} \right] . \tag{4.18}$$

This integral converges, and especially, we find $A_0 = 3\pi^2/2$.

Thus, at large z, the bulk-to-boundary propagator behaves as

$$\Pi(X, y) = \frac{1}{z^2} \frac{256 A_0}{9\pi^4 \lambda_0 N} \left[1 + \mathcal{O}\left(z^{-1}\right) \right] = \frac{1}{z^2} \frac{128}{3\pi^2 \lambda_0 N} \left[1 + \mathcal{O}\left(z^{-1}\right) \right], \tag{4.19}$$

where the NLO contribution $\propto z^{-3}$ in the large z comes from $\Pi_2(x-y, z)$. This behavior is consistent with the LO behavior of the bulk-to-boundary propagator for the scalar field with the conformal dimension $\Delta^{\rm IR}_{\varphi^2}$ in the presence of conformal symmetry, which is

$$\left[\frac{z}{z^2 + (x-y)^2} \right]^{\Delta^{\rm IR}_{\varphi^2}} \sim z^{-\Delta^{\rm IR}_{\varphi^2}}, \quad z \to \infty. \tag{4.20}$$

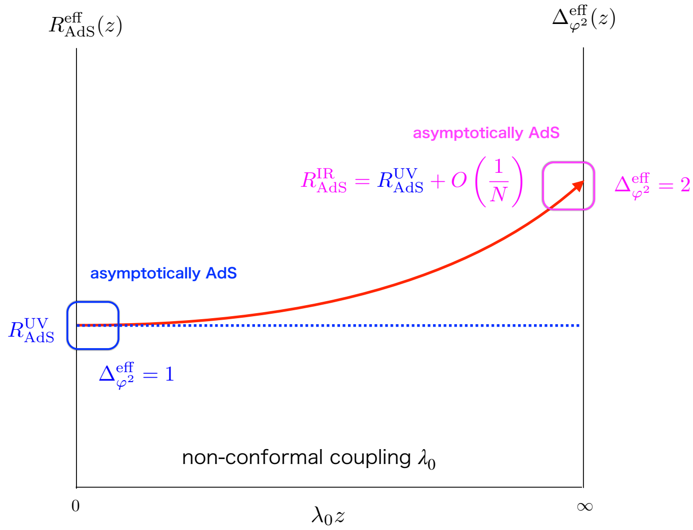

Figure 2. A schematic figure for the (effective) AdS radius ($R^{\rm eff}_{\rm AdS}(z)$) and the (effective) conformal dimension of φ^2 ($\Delta^{\rm eff}_{\varphi^2}(z)$) as a function of $\lambda_0 z$.

The $z^{-\Delta^{\rm IR}_{\varphi^2}}$ behavior also satisfies the EOM of the free scalar field at $z \to \infty$, whose mass m is given by $m^2 \propto \Delta^{\rm IR}_{\varphi^2}(\Delta^{\rm IR}_{\varphi^2} - d)$. Therefore the z dependence of the bulk-to-boundary scalar propagator at $z \to \infty$ in Eq. (4.19) correctly reproduces $\Delta^{\rm IR}_{\varphi^2} = 2$ (the conformal dimension of φ^2) at the Wilson-Fischer IR fixed point.

In this section, we have shown that $z \to 0$ and $z \to \infty$ behaviors of the bulk-to-boundary propagator constructed by the conformal smearing correspond to conformal dimensions of φ^2 at the asymptotic free UV and Wilson-Fischer IR fixed points, respectively, in the interacting theory. Note that this property is robust in the sense that it holds even for a general smearing function $S(x, z)$ rather than the conformal smearing, as discussed in the appendix D.

5 Conclusion

In this paper, we have investigated the bulk space dual to $O(N)$ invariant critical φ^4 model in 3-dimensions combining the conformal smearing with the large N expansion, and obtained the following results, which are also schematically summarized in Fig. 2.

The metric in the bulk space at the NLO is given by Eq. (3.8), which describes the asymptotic AdS space both UV ($z \to 0$) limit in Eq. (3.9) and IR ($\to \infty$) limit in Eq. (3.10). Moreover the AdS radii satisfy $R^{\rm IR}_{\rm AdS} - R^{\rm UV}_{\rm AdS} = O(1/N) > 0$, which reflects the fact that the number of the degrees of freedom decreases from UV to IR by the renormalization group, as suggested by the F-theorem [23].

The bulk-to-boundary propagator in Eq. (4.2) at the LO encodes Δ_{φ^2} (the conformal dimension of the composite scalar operator φ^2) in its z dependence at UV as Eq. (4.13) and at IR as Eq. (4.19), corresponding to $\Delta_{\varphi^2} = 1$ at the asymptotic UV fixed point and

$\Delta_{\varphi^2} = 2$ at the Wilson-Fischer IR fixed point, respectively. Interestingly, the UV limit in Eq. (4.13) reproduces an expected GKP-Witten relation for the interacting theory with non-zero λ_0, whose $z^{\Delta_{\varphi^2}}$ behavior is controlled by $\Delta_{\varphi^2} = 1$, the value at the UV fixed point, while the $|x-y|$ behavior of the two-point function for φ^2 shows complicated behavior that $|x-y|^{-2}$ at $\lambda_0|x-y| \ll 1$ (UV in the $O(N)$ model)) or $|x-y|^{-4}$ at $\lambda_0|x-y| \gg 1$ (IR in the $O(N)$ model).

As the conformal smearing approach works well for the $O(N)$ invariant critical φ^4 model in 3-dimensions, one may use it to derive some properties of the higher spin theories in 4-dimensions [24], which is expected to be dual to the $O(N)$ model in 3-dimensions.

Acknowledgments

This work has been supported in part by KIAS Individual Grants, Grant No. 090901, and by the JSPS Grant-in-Aid for Scientific Research (No. JP22H00129).

References

[1] J. M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231–252, [hep-th/9711200].

[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett. B* **428** (1998) 105–114, [hep-th/9802109].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253–291, [hep-th/9802150].

[4] I. Heemskerk and J. Polchinski, *Holographic and Wilsonian Renormalization Groups*, *JHEP* **06** (2011) 031, [arXiv:1010.1264].

[5] M. Nozaki, S. Ryu, and T. Takayanagi, *Holographic Geometry of Entanglement Renormalization in Quantum Field Theories*, *JHEP* **10** (2012) 193, [arXiv:1208.3469].

[6] M. Miyaji and T. Takayanagi, *Surface/State Correspondence as a Generalized Holography*, *PTEP* **2015** (2015), no. 7 073B03, [arXiv:1503.03542].

[7] M. Miyaji, T. Numasawa, N. Shiba, T. Takayanagi, and K. Watanabe, *Continuous Multiscale Entanglement Renormalization Ansatz as Holographic Surface-State Correspondence*, *Phys. Rev. Lett.* **115** (2015), no. 17 171602, [arXiv:1506.01353].

[8] S. Aoki, K. Kikuchi, and T. Onogi, *Geometries from field theories*, *PTEP* **2015** (2015), no. 10 101B01, [arXiv:1505.00131].

[9] S. Aoki, J. Balog, T. Onogi, and P. Weisz, *Flow equation for the scalar model in the large N expansion and its applications*, *PTEP* **2017** (2017), no. 4 043B01, [arXiv:1701.00046].

[10] S. Aoki, J. Balog, T. Onogi, and P. Weisz, *Flow equation for the large N scalar model and induced geometries*, *PTEP* **2016** (2016), no. 8 083B04, [arXiv:1605.02413].

[11] S. Aoki and S. Yokoyama, *Flow equation, conformal symmetry, and anti-de Sitter geometry*, *PTEP* **2018** (2018), no. 3 031B01, [arXiv:1707.03982].

[12] S. Aoki and S. Yokoyama, *AdS geometry from CFT on a general conformally flat manifold*, *Nucl. Phys. B* **933** (2018) 262–274, [arXiv:1709.07281].

[13] S. Aoki, J. Balog, and S. Yokoyama, *Holographic computation of quantum corrections to the bulk cosmological constant*, *PTEP* **2019** (2019), no. 4 043B06, [arXiv:1804.04636].

[14] S. Aoki, S. Yokoyama, and K. Yoshida, *Holographic geometry for nonrelativistic systems emerging from generalized flow equations*, *Phys. Rev. D* **99** (2019), no. 12 126002, [arXiv:1902.02578].

[15] S. Aoki, J. Balog, S. Yokoyama, and K. Yoshida, *Nonrelativistic hybrid geometries with gravitational gauge-fixing terms*, *Phys. Rev. Res.* **2** (2020), no. 1 013169, [arXiv:1910.11032].

[16] S. Aoki, T. Onogi, and S. Yokoyama, *What does a quantum black hole look like?*, *Phys. Lett. B* **814** (2021) 136104, [arXiv:2004.03779].

[17] S. Aoki, J. Balog, T. Onogi, and S. Yokoyama, *Special flow equation and the GKP–Witten relation*, *PTEP* **2023** (2023), no. 1 013B03, [arXiv:2204.06855].

[18] I. R. Klebanov and A. M. Polyakov, *AdS dual of the critical O(N) vector model*, *Phys. Lett. B* **550** (2002) 213–219, [hep-th/0210114].

[19] S. R. Das and A. Jevicki, *Large N collective fields and holography*, *Phys. Rev. D* **68** (2003) 044011, [hep-th/0304093].

[20] O. Aharony, S. M. Chester, and E. Y. Urbach, *A Derivation of AdS/CFT for Vector Models*, *JHEP* **03** (2021) 208, [arXiv:2011.06328].

[21] T. Banks, M. R. Douglas, G. T. Horowitz, and E. J. Martinec, *AdS dynamics from conformal field theory*, hep-th/9808016.

[22] S. Aoki, K. Shimada, J. Balog, and K. Kawana, *Bulk modified gravity from a thermal CFT by the conformal flow*, *Phys. Rev. D* **109** (2024), no. 4 046006, [arXiv:2308.01076].

[23] I. R. Klebanov, S. S. Pufu, and B. R. Safdi, *F-Theorem without Supersymmetry*, *JHEP* **10** (2011) 038, [arXiv:1105.4598].

[24] M. A. Vasiliev, *Higher spin gauge theories in four-dimensions, three-dimensions, and two-dimensions*, *Int. J. Mod. Phys. D* **5** (1996) 763–797, [hep-th/9611024].

[25] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products. – Eighth Edition.* Academic Press, 1943.

A $O(N)$ model in 3 dimensions

A.1 Schwinger-Dyson equation

The Schwinger-Dyson equation (SDE) for the action $S(\varphi)$ in eq. (2.1) is compactly written as

$$\left\langle \frac{\delta O(\varphi)}{\delta\varphi(x)} \right\rangle = \left\langle O(\varphi)\frac{\delta S(\varphi)}{\delta\varphi(x)} \right\rangle \tag{A.1}$$

where

$$\langle O(\varphi)\rangle := \frac{1}{Z}\int \mathcal{D}\varphi\, O(\varphi)e^{-S(\varphi)}, \quad Z := \int \mathcal{D}\varphi\, e^{-S(\varphi)}. \tag{A.2}$$

We define the connected part of n-point functions with appropriate powers of N as

$$\Gamma^{ab}(x,y) := N\langle \varphi^a(x)\varphi^b(y)\rangle, \tag{A.3}$$

$$K^{a_1a_2\cdots a_n} = N^{n-1}\langle \varphi^{a_1}(x_1)\varphi^{a_2}(x_2)\cdots\varphi^{a_n}(x_n)\rangle_c \tag{A.4}$$

for $n = 4, 6, , \cdots$. The $O(N)$ symmetry tells us

$$\Gamma^{ab}(x,y) := \delta^{ab}\Gamma(x-y) \tag{A.5}$$

$$K^{a_1a_2a_3a_4} = \delta^{a_1a_2}\delta^{a_3a_4}K(x_1,x_2;x_3,x_3) + (2\leftrightarrow 3) + (2\leftrightarrow 4), \tag{A.6}$$

$$K^{a_1a_2\cdots a_6}(x_1,x_2,\cdots,x_6) = \delta^{a_1a_2}\delta^{a_3a_4}\delta^{a_5a_6}H(x_1,x_2;x_3,x_4;x_5.x_6) + (14\text{ perm.}). \tag{A.7}$$

Taking $O = \phi^a(x)$, the Schwinger-Dyson equation leads to

$$\begin{aligned}\delta^{(3)}(x-y) &= \left[-\Box + m^2 Z_m + \lambda Z_\lambda Z_\varphi \Gamma(0)\right] Z_\varphi \Gamma(x-y)\\ &+ \frac{\lambda Z_\lambda Z_\varphi^2}{N}\left[\left(1+\frac{2}{N}\right)K(x,y;x,x) + 2\Gamma(0)\Gamma(x-y)\right],\end{aligned} \tag{A.8}$$

while the one for $O = \varphi^{a_2}(x_2)\varphi^{a_3}(x_3)\varphi^{a_4}(x_4)$ gives

$$\begin{aligned}0 &= \left[-\Box + m^2 Z_m + \lambda Z_\lambda Z_\varphi\left(1+\frac{2}{N}\right)\Gamma(0)\right] Z_\varphi^2 K(x_1,x_2;x_3,x_4)\\ &+ \lambda Z_\lambda Z_\varphi^3 \Gamma(x_1-x_2)\left[2\Gamma(x_1-x_3)\Gamma(x_1-x_4) + \left(1+\frac{2}{N}\right)K(x_1,x_1;x_3,x_4) + \frac{2}{N}K(x_1,x_3;x_1,x_4)\right]\\ &+ \frac{\lambda Z_\lambda Z_\varphi^3}{N}\left[\left(1+\frac{2}{N}\right)H(x_1,x_1;x_1,x_2;x_3,x_4) + \frac{2}{N}H(x_1,x_2;x_1,x_3;x_1,x_4)\right]\\ &+ \frac{2\lambda Z_\lambda Z_\varphi^3}{N}\left[\Gamma(x_1-x_3)K(x_1,x_2;x_1,x_4) + \Gamma(x_1-x_4)K(x_1,x_2;x_1,x_3)\right],\end{aligned} \tag{A.9}$$

where eq. (A.8) has already been used.

A.2 Large N expansion

A.2.1 2-pt function at the LO

Using the Fourier transformation of the 2-pt function that

$$\Gamma(x) = \int \frac{d^3p}{(2\pi)^3} e^{ipx}\tilde{\Gamma}(p), \quad \tilde{\Gamma}(p) = \tilde{\Gamma}_0(p) + \frac{1}{N}\Gamma_1(p) + \cdots, \tag{A.10}$$

the 2-pt function at the LO satisfies

$$1 = \left[p^2 + m^2 Z_m + \lambda Z_\lambda Z_\varphi \Gamma_0(0)\right] Z_\varphi \tilde{\Gamma}_0(p), \quad \Gamma_0(0) = \int \frac{d^3p}{(2\pi)^3}\tilde{\Gamma}_0(p). \tag{A.11}$$

We adopt the renormalization conditions for the 2-pt function that

$$\left.\tilde{\Gamma}_0^{-1}(p)\right|_{p^2=\mu^2} = m^2 + \mu^2, \quad \left.\frac{d}{dp^2}\tilde{\Gamma}_0^{-1}(p)\right|_{p^2} = 1, \tag{A.12}$$

which lead to

$$Z_\varphi = 1, \quad m^2 Z_m = m^2 + \lambda Z_\lambda\left(\frac{m}{4\pi} - \frac{\Lambda}{2\pi^2}\right), \tag{A.13}$$

where we use

$$\Gamma_0(0) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{p^2+m^2} = \frac{\Lambda}{2\pi^2} - \frac{m}{4\pi} \tag{A.14}$$

with Λ being the momentum cut-off and $m \geq 0$ being assumed.

A.2.2 4-pt function at the LO

The SDE relevant for the 4-pt function at the LO is written as

$$(-\Box_1 + m^2)K_0(12;34) + \lambda Z_\lambda \Gamma_0(12)K_0(11;34) = -2\lambda Z_\lambda \Gamma_0(12)\Gamma_0(13)\Gamma_0(14), \tag{A.15}$$

where we expand $K = K_0 + K_1/N + \cdots$, and employ short-handed notations such as $\Gamma_0(ij) = \Gamma(x_i - x_j)$ and $K_0(ij;kl) = K(x_i, x_j; x_k, x_l)$.

Introducing the amputated 4-pt function in the momentum space as

$$K_0(12;34) := \prod_{i=1}^{4}\left(\int \frac{d^3p_i}{(2\pi)^3}\frac{e^{ip_ix_i}}{p_i^2+m^2}\right)(2\pi)^3\delta^{(3)}\left(\sum_{i=1}^{4} p_i\right) G_0(p_1,p_2;p_3,p_4), \tag{A.16}$$

the SDE becomes

$$\begin{aligned} G_0(p_1,p_2;p_3,p_4) \;+\; \lambda Z_\lambda \prod_{i=1}^{2}\left(\int \frac{d^3q_i}{(2\pi)^3}\frac{1}{q_i^2+n^2}\right) G_0(q_1,q_2;p_3,p_4)\delta^{(3)}\left(q_1+q_2+p_3+p_4\right) \\ = -2\lambda Z_\lambda, \end{aligned} \tag{A.17}$$

where $p_1 + p_2 + p_3 + p_4 = 0$.

The solution to the above equation is given by $G_0(p_1,p_2;p_3+p_4) = G_0(p_{12}^2)$ with $p_{12} := p_1 + p_2$ and

$$G_0(p^2) = \frac{-2\lambda Z_\lambda}{1+\lambda Z_\lambda B(p^2)}, \tag{A.18}$$

where

$$B(p^2) = \int \frac{d^3q}{(2\pi)^3}\frac{1}{q^2+m^2}\frac{1}{(p-q)^2+m^2} = \frac{\pi - 2\arctan(2m/|p|)}{8\pi|p|}, \tag{A.19}$$

which becomes simple in the massless limit as

$$\lim_{m^2\to 0} B(p^2) = \frac{1}{8|p|}, \tag{A.20}$$

and thus

$$\lim_{m^2\to 0} G_0(p_1,p_2;p_3,p_4) = \frac{-2\lambda Z_\lambda}{1+\lambda Z_\lambda/(8|p_{12}|)}, \tag{A.21}$$

We take the renormalization condition for the amputated 4-pt function as

$$G(p_1,p_2;p_3,p_4)|_{p_{12}^2=\mu^2} = -2\lambda, \tag{A.22}$$

which gives $G_0(\mu^2) = -2\lambda$ at the LO. We thus obtain

$$Z_\lambda = \frac{1}{1-\lambda B(\mu^2)} \longrightarrow \frac{1}{1-\lambda/(8\mu)}, \quad m^2 \to 0. \tag{A.23}$$

Now all renormalization constants are fixed at the LO.

A.2.3 2-pt function at the NLO

The 2-pt function at the NLO, necessary in the main text, is also calculated here.

At the NLO, we write

$$Z_\varphi = 1 + \frac{1}{N} Z_\varphi^{(1)}, \quad Z_m = Z_m^{(0)} + \frac{1}{N} Z_m^{(1)}, Z_\lambda = Z_\lambda^{(0)} + \frac{1}{N} Z_\lambda^{(1)}, \tag{A.24}$$

where the LO parts have already been determined.

The SDE relevant for the 2-pt function at the NLO reads

$$\begin{aligned}0 = &(-\Box + m^2)\Gamma_1(12) + \left[m^2 (Z_m^{(1)} + Z_m^{(0)} Z_\varphi^{(1)}) + \lambda_0 (\Gamma_1(0) + \frac{Z_\lambda^{(1)}}{Z_\lambda^{(0)}} + 2 Z_\varphi^{(1)} \right] \\ &+ \lambda_0 \left[K_0(12;11) + 2\Gamma_0(0)\Gamma_0(12) \right].\end{aligned} \tag{A.25}$$

where $\lambda_0 := \lambda Z_\lambda^{(0)}$ at this order. Using the relation obtained from eq. (A.15) as

$$\lambda_0 \left[K_0(x,x;0x,0) + 2\Gamma_0(0)\Gamma_0(x) \right] = -\frac{1}{\Gamma_0(0)} (-\Box_1 + m^2) K_0(x_1,x;x,0) \Big|_{x_1=x}, \tag{A.26}$$

the SDE becomes

$$\begin{aligned}(-\Box + m^2)\Gamma_1(12) = &-\left[m^2 \left(Z_m^{(1)} + Z_m^{(0)} Z_\varphi^{(1)} \right) + \lambda_0 \left(\Gamma_1(0) + \frac{Z_\lambda^{(1)}}{Z_\lambda^{(0)}} + 2 Z_\varphi^{(1)} \right) \right] \\ &+ \frac{-\Box_1 + m^2}{\Gamma_0(0)} K_0(x_1,x;x,0) \mid_{x_1=x},\end{aligned} \tag{A.27}$$

whose last term is further evaluated as

$$\begin{aligned}\frac{-\Box_1 + m^2}{\Gamma(0)} K_0(x_1,x;x,0) \Big|_{x_1=x} &= \int \frac{d^3p}{(2\pi)^3} \frac{e^{ipx}}{p^2+m^2} \int \frac{d^3Q}{(2\pi)^3} \frac{G_0(Q)}{(Q-p)^2+m^2} \\ = \int \frac{d^3p}{(2\pi)^3} \frac{e^{ipx}}{p^2+m^2} &\int \frac{d^3Q}{(2\pi)^3} \frac{1}{(Q-p)^2+m^2} \frac{-2\lambda_0}{1+\lambda_0 B(Q^2)}.\end{aligned} \tag{A.28}$$

Using the expression in the momentum space as

$$\Gamma_1(x) = \int \frac{d^3p}{(2\pi)^3} \frac{X(p^2)}{(p^2+m^2)^2} e^{ipx}, \tag{A.29}$$

the SDE leads to

$$X(p^2) = -\left[m^2 \left(Z_m^{(1)} + Z_m^{(0)} Z_\varphi^{(1)} \right) + \lambda_0 \left(\Gamma_1(0) + \frac{Z_\lambda^{(1)}}{Z_\lambda^{(0)}} + 2 Z_\varphi^{(1)} + 2Y(p^2) \right) \right], \tag{A.30}$$

where by definition

$$\Gamma_1(0) = \int \frac{d^3p}{(2\pi)^3} \frac{X(p^2)}{(p^2+m^2)^2}, \tag{A.31}$$

while

$$Y(p^2) = \int \frac{d^3Q}{(2\pi)^3} \frac{1}{(Q-p)^2+m^2} \frac{1}{1+\lambda_0 B(Q^2)}. \tag{A.32}$$

Inserting eq. (A.30) into eq. (A.31), we obtain

$$\Gamma_1(0) = -\frac{m^2\left(Z_m^{(1)} + Z_m^{(0)}Z_\varphi^{(1)}\right) + \lambda_0\left(\Gamma_1(0) + \frac{Z_\lambda^{(1)}}{Z_\lambda^{(0)}} + 2Z_\varphi^{(1)} + 2Y(p^2)\right)}{1+\lambda_0 I(m^2)}, \tag{A.33}$$

where

$$I(m^2) := \int \frac{d^3p}{(2\pi)^3}\frac{1}{(p^2+m^2)^2} = -\frac{d}{dm^2}\Gamma_0(0) = \frac{1}{8\pi}\frac{1}{\sqrt{m^2}}. \tag{A.34}$$

The 2-pt function at the NLO in the momentum space is expressed as

$$Z_\varphi\tilde{\Gamma}(p) = \left(1+\frac{1}{N}Z_\varphi^{(1)}\right)\tilde{\Gamma}_0(p) + \frac{1}{N}\tilde{\Gamma}_1(p) = \frac{1+\frac{1}{N}Z_\varphi^{(1)}}{p^2+m^2-\frac{1}{N}X(p^2)}, \tag{A.35}$$

and thus the renormalization condition at $p^2=\mu^2$ reads

$$\left(1-\frac{1}{N}Z_\varphi^{(1)}\right)(\mu^2+m^2) - \frac{1}{N}X(p^2) = \mu^2+m^2, \tag{A.36}$$

$$1-\frac{1}{N}Z_\varphi^{(1)} - \frac{1}{N}\frac{d}{dp^2}X(p^2)\bigg|_{p^2=\mu^2} = 1, \tag{A.37}$$

which leads to relations among renormalization constants at the NLO as

$$m^2\left(Z_m^{(1)} + Z_m^{(0)}Z_\varphi^{(1)}\right) = Z_\varphi^{(1)}(\mu^2+m^2) - \lambda_0\left(\Gamma_1(0) + \frac{Z_\lambda^{(1)}}{Z_\lambda^{(0)}} + 2Z_\varphi^{(1)} + 2Y(\mu^2)\right), \tag{A.38}$$

$$Z_\varphi^{(1)} = 2\lambda_0\frac{d}{dp^2}Y(p^2)\bigg|_{p^2=\mu^2}. \tag{A.39}$$

We then finally obtain

$$X(p^2) = -2\lambda_0 Y_r(p^2) - Z_\varphi^{(1)}(\mu^2+m^2), \quad Y_r(p^2) := Y(p^2) - Y(\mu^2), \tag{A.40}$$

which is UV-finite, thanks to the subtraction.

In the main text, we consider the case with $m^2=\mu^2=0$, which leads to

$$X(p^2) = \int\frac{d^3Q}{(2\pi)^3}\left[\frac{1}{(Q-P)^2} - \frac{1}{Q^2}\right]\frac{-2\lambda_0}{1+\lambda_0 B(Q^2)} = \frac{2\lambda_0 p}{(2\pi)^2}L\left(\frac{\lambda_0}{8p}\right), \tag{A.41}$$

where

$$\begin{aligned} L(x) = &\frac{x^2}{2}\left[\ln^2\frac{x+1}{x} - \left(\ln\frac{x-1}{x} - 2i\pi\right)\ln\frac{x-1}{x} + 4\frac{\ln x+1}{x}\right] \\ &+ x^2\left[\mathrm{Li}_2\left(\frac{x}{x+1}\right) - \mathrm{Li}_2\left(\frac{x}{x-1}\right)\right] \end{aligned} \tag{A.42}$$

with

$$\mathrm{Li}_2(x) := -\int_0^x dt\,\frac{\ln(1-t)}{t}. \tag{A.43}$$

In two limits ($x\to 0$ or $x\to\infty$), we have

$$L(x) \longrightarrow \begin{cases} 2(1+\ln x)x - \dfrac{\pi^2}{2}x^2 + \cdots, & x\to 0, \\ -\dfrac{2}{9x}(1+3\ln x) + \cdots, & x\to\infty. \end{cases} \tag{A.44}$$

A.3 Renormalization Group (RG) analysis in the massless case at the LO

A.3.1 Beta function

We define a dimension less coupling constant as $g_R := \lambda/\mu$ at the LO, which is written in terms of λ_0 at $m^2 = 0$ as

$$g_R = \frac{\lambda_0/\mu}{1 + \lambda_0/(8\mu)}. \tag{A.45}$$

Since λ_0 is μ independent, the β function for g_R can be calculated as

$$\beta(g_R) := \mu \frac{d}{d\mu} g_R(\mu) = -g_R \left(1 - \frac{g_R}{8}\right). \tag{A.46}$$

Therefore $g_R = 0$ corresponds to an asymptotic free (UV) fixed point, while $g_R = 8$ is the Wilson-Fischer (IR) fixed point.

A.3.2 Anomalous mass dimension of the composite scalar operator

We calculate the anomalous dimension of the $O(N)$ invariant scalar operator, given by

$$O_R := Z_O \varphi_0^a \varphi_0^a = Z_O Z_\varphi \varphi^a \varphi^a, \tag{A.47}$$

where Z_O is the renormalization factor, from which the anomalous mass dimension of O is defined as

$$\gamma_O := -\mu \frac{d}{d\mu} \ln Z_O. \tag{A.48}$$

At the LO where $Z_\varphi = 1$, we have

$$\langle O(x_1)\varphi^{a_3}(x_3)\varphi^{a_4}(x_4)\rangle = \frac{Z_O \delta_{a_3 a_4}}{N^2} \left[2\Gamma_0(13)\Gamma_0(14) + K(_0(11;34)\right], \tag{A.49}$$

whose Fourier transformation should be equal to the tree-level contribution that

$$\prod_{i=3}^{4} \int d^3x_i e^{ip_i x_i} \langle O(x_1 = 0)\varphi^{a_3}(x_3)\varphi^{a_4}(x_4)\rangle|_{\text{tree}} = \frac{\delta_{a_3 a_4}}{N^2} 2\tilde{\Gamma}_0(p_3)\tilde{\Gamma}_0(p_4) \tag{A.50}$$

at $p_{34}^2 = \mu^2$. Since

$$\prod_{i=3}^{4} \int d^3x_i e^{ip_i x_i} K_0(11;34) = \tilde{\Gamma}_0(p_3)\tilde{\Gamma}_0(p_4) \int \frac{d^3p}{(2\pi)^3} \frac{G_0(p_{34}^2)}{[p^2 + m^2]\left[(p_{34} - p)^2 + m^2\right]}, \tag{A.51}$$

we obtain

$$Z_O = \left[1 + \frac{1}{2} G_0(\mu^2) B(\mu^2)\right]^{-1} = 1 + \lambda_0 B(\mu^2). \tag{A.52}$$

Therefore the anomalous dimension is calculated as

$$\gamma_O = -\frac{\lambda_0}{1 + \lambda_0 B(\mu^2)} \mu \frac{d}{d\mu} B(\mu^2). \tag{A.53}$$

In the massless case ($m^2 = 0$), we obtain

$$\gamma_O = \frac{g_R}{8} = \frac{\lambda_0}{8\mu + \lambda_0} = \begin{cases} 0 & (\mu \to \infty), \\ 1 & (\mu \to 0). \end{cases} \tag{A.54}$$

Therefore, the total conformal (mass) dimension of O in the IR fixed point at $\mu \to 0$ becomes

$$\Delta_O := 2\Delta_{\varphi_0} + \gamma_O = 2, \tag{A.55}$$

where $\Delta_{\varphi_0} = 1/2$ is the conformal dimension of φ_0, while $\Delta_O = 1$ in the UV fixed point ($\mu \to \infty$).

From the above result, it is straightforward to calculate the connected 2-pt function of φ^2 without Z_O at the LO in the large N expansion. In the massless limit, it becomes

$$\langle \varphi^2(x)\varphi^2(y)\rangle_c = \frac{2}{N}\int_{p_1,p_2} \frac{e^{i(p_1+p_2)\cdot(x-y)}}{p_1^2 p_2^2} \frac{1}{1+\lambda_0 B(p_{12}^2)} \tag{A.56}$$

$$= \frac{2}{N}\left[\frac{1}{16\pi^2(x-y)^2} - \lambda_0 \Omega(x-y)\right], \tag{A.57}$$

where $\Omega(x)$ is defined in eq. (4.11), whose explicit form can be written as

$$\Omega(x) = \frac{|x|^{-1}}{128\pi^2}[\mathrm{Ci}(\chi)\sin(\chi) - \mathrm{si}(\chi)\cos(\chi)] \tag{A.58}$$

with $\chi := |x|\lambda_0/8$ and the trigonometric integrals

$$\mathrm{Ci}(z) := -\int_z^\infty dt \frac{\cos t}{t}, \quad \mathrm{si}(z) := -\int_z^\infty dt \frac{\sin t}{t}. \tag{A.59}$$

In the UV limit that $|x-y| \ll 8/\lambda_0$, we get $\Omega(x-y) \approx |x-y|^{-1}/256\pi$, and thus, the tree level contribution $\propto (x-y)^{-2}$ in (A.57) dominates the 2-pt function.

In the IR limit that $|x-y| \gg 8/\lambda_0$, on the other hand, we have

$$\Omega(x-y) = \frac{(x-y)^{-2}}{16\pi^2\lambda_0}\left(1 - \frac{2(8/\lambda_0)^2}{(x-y)^2} + \mathcal{O}\big((x-y)^{-4}\big)\right). \tag{A.60}$$

Therefore, the 2-pt function (A.57) behaves as

$$\langle \varphi^2(x)\varphi^2(y)\rangle_c \approx \frac{1}{N}\frac{16}{\pi^2\lambda_0^2}\frac{1}{(x-y)^4}, \tag{A.61}$$

as expected from (A.55).

B Metric at the NLO

The metric at the NLO is given in eq. (3.8) in the main text as

$$g_{\mu\nu}^{\mathrm{NLO}}(z) = g_{\mu\nu}^{\mathrm{LO}}(z)\left[1 + \frac{1}{N}G_s s(z)\right], \quad g_{zz}^{\mathrm{NLO}}(z) = g_{zz}^{\mathrm{LO}}(z)\left[1 + \frac{1}{N}G_\sigma(z)\right], \tag{B.1}$$

where

$$G_s(z) := \frac{F_1(z)}{F_0(z)} - \frac{\gamma_1(z)}{\gamma_0(z)} = \frac{32g}{\pi^4}\left[\frac{64}{45}F^3_{11}(g) - F^1_{11}(g)\right], \tag{B.2}$$

$$G_\sigma(z) := \frac{1}{5}\left(\frac{9H_1(z)}{H_0(z)} - \frac{\gamma_1(z)}{\gamma_0(z)} - \frac{8G_1(z)}{G_0(z)}\right) = \frac{32g}{5\pi^4}\left[\frac{64}{3}\left\{F^3_{00}(g) - F^2_{01}(g)\right\} - F^1_{11}(g)\right], \tag{B.3}$$

with $g := \lambda_0 z/8$ and

$$F^n_{ij}(g) := \int_0^\infty dp\, p^n K_i(p) K_j(p) L\left(\frac{g}{p}\right). \tag{B.4}$$

Analytic expressions of $F^n_{ij}(g)$ are given in the appendix C in two limits that $g \to 0$ (UV) or $\to \infty$ (IR).

C Momentum integrals

Here we present several momentum integrals used in the main text.

C.1 Integral formula for Bessel functions

We present useful formulas for integrals of Bessel functions.

(1) 6.576-4 in [25]:

$$\int_0^\infty dx\, x^{-\lambda} K_\mu(ax) K_\nu(bx) = \frac{2^{-2-\lambda} a^{-\nu+\lambda-1} b^\nu}{\Gamma(1-\lambda)} \prod_\pm \Gamma\left(\frac{1-\lambda \pm \mu + \nu}{2}\right) \Gamma\left(\frac{1-\lambda \pm \mu - \nu}{2}\right)$$
$$\times\ {}_2F_1\left(\frac{1-\lambda+\mu+\nu}{2}, \frac{1-\lambda-\mu+\nu}{2}; 1-\lambda; 1-\frac{b^2}{a^2}\right) \tag{C.1}$$

for $\mathrm{Re}(a+b) > 0$, $\mathrm{Re}\,\lambda < |\,\mathrm{Re}\,\mu| - |\,\mathrm{Re}\,\nu|$.

(2) 6.671-5 in [25]:

$$\int_0^\infty dx\, K_\nu(ax)\sin(bx) = \frac{\pi}{4}\frac{a^{-\nu}}{\sin(\nu\pi/2)} \frac{\left[(\sqrt{b^2+a^2}+b)^\nu - (\sqrt{b^2+a^2}-b)^\nu\right]}{\sqrt{a^2+b^2}} \tag{C.2}$$

for $\mathrm{Re}\,a > 0$, $b > 0$, $|\,\mathrm{Re}\,\nu\,| < 2$, $\nu \neq 0$.

C.2 F^n_{ij} in two limit

We split $F^n_{ij}(g)$ defined in eq. (B.4) into two part as $F^n_{ij}(g) = F^{n,a}_{ij}(g) + F^{n,b}_{ij}(g)$, where

$$F^{n,a}_{ij}(g) := \int_0^g dp\, p^n K_i(p) K_j(p) L(g/p), \quad F^{n,b}_{ij}(g) := \int_g^\infty dp\, p^n K_i(p) K_j(p) L(g/p), \tag{C.3}$$

and evaluate it in UV and IR limits.

C.2.1 IR limit

We first consider the IR ($g \to \infty$) limit.

Using an expansion in terms of $1/g$ at $g = \infty$ as

$$\int_0^g dx\, f(x) \simeq \int_0^\infty dx\, f(x) + \frac{1}{g} \lim_{g\to\infty} f(g) \frac{dg}{d(1/g)} + \cdots , \tag{C.4}$$

we have

$$\begin{aligned} F_{ij}^{n,a}(g) &\simeq \int_0^\infty dp\, p^n K_i(p) K_j(p) L(g/p) - \frac{1}{g} \lim_{g\to\infty} g^{n+2} K_i(g) K_j(g) L(1) + \cdots \\ &\simeq -\frac{2}{9g} \int_0^\infty dp p^{n+1} K_i(p) K_j(p)\,(1 + 3\ln g/p) \\ &= -\frac{1}{3g}\left[\left(\frac{2}{3} + \ln g^2\right) C_{ij}^{n+1} - L_{ij}^{n+1}\right], \end{aligned} \tag{C.5}$$

where the second term in the first line vanishes exponentially, the asymptotic behavior of $L(x)$ in (A.44) is used to obtain the second line, and constants in the third line are defined as

$$C_{ij}^n := \int_0^\infty dp\, p^n K_i(p) K_j(p), \quad L_{ij}^n := \int_0^\infty dp\, \ln p^2\, p^n K_i(p) K_j(p). \tag{C.6}$$

On the other hand, $F_{ij}^{n,b}$ is evaluated by the steepest-descent method after a change of variable $p = gy$ as

$$F_{ij}^{n,b}(g) = g^{n+1} \int_1^\infty dy\, y^n K_i(gy) K_j(gy) L(1/y) \simeq g^{n+1} y_0^n K_i(gy_0) K_j(gy_0) L(1/y_0), \tag{C.7}$$

which vanishes exponentially as $g \to \infty$, where $1 \le y_0 < \infty$ is a point which gives the largest contribution to the integral. The point y_0 is either given as a solution to $S'(y) = 0$, where

$$S(y) := \ln\left[y^n K_i(gy) K_j(gy) L(1/y)\right], \tag{C.8}$$

or $y_0 = 1$ if no solution exists.

In total we obtain

$$F_{ij}^n(g) \simeq -\frac{1}{3g}\left[\left(\frac{2}{3} + \ln g^2\right) C_{ij}^{n+1} - L_{ij}^{n+1}\right] \tag{C.9}$$

in the IR ($g \to \infty$) limit.

C.2.2 UV limit

In the UV ($g \to 0$) limit, $F_{ij}^{n,a}$ is evaluated with $p = gy$ as

$$F_{ij}^{n,a}(g) = g^{n+1} \int_0^1 dy\, y^n K_i(gy) K_j(gy) L(1/y) \simeq c_i c_j g \int_0^1 dy\, (gy)^{n-i-j} L(1/y), \tag{C.10}$$

where c_i is given in the small x expansion of the Bessel function as $K_i(x) = c_i x^{-i} + \cdots$ with x^{-i} read as $\ln x$ for $i = 0$. For example, $c_0 = -1$, $c_1 = 1$, and $c_2 = 2$. The y integral is convergent for $n - i - j > -2$ since $L(1/y) \sim y \ln y$ for small y as seen in (A.44).

For $n-i-j>0$, $F^{n,b}_{ij}$ is evaluated straightforwardly as

$$F^{n,b}_{ij}(g) \simeq 2g\int_0^\infty dp\, p^{n-1}K_i(p)K_j(p)\left[1+\ln(g/p)\right] \tag{C.11}$$

For $n=i=j=1$, we calculate $F^{n,b}_{ij}$ as

$$F^{1,b}_{11} = \int_g^\infty dp\, p\left[K_1^2(p)-\frac{c_1^2}{p^2}\right]L(g/p)+c_1^2\int_1^\infty dy\,\frac{L(1/y)}{y}, \tag{C.12}$$

where the second term is a g-independent constant, while the first term can be evaluated using the UV limit of $L(x)$ in (A.44) by

$$\int_g^\infty dp\, p\left[K_1^2(p)-\frac{c_1^2}{p^2}\right]L(g/p) \simeq 2g\int_0^\infty dp\,\left[K_1^2(p)-\frac{c_1^2}{p^2}\right](1+\ln(g/p))\,. \tag{C.13}$$

In total, we have

$$F^n_{ij}(g) \simeq -g\left[(2+\ln g^2)C^{n-1}_{ij}-L^{n-1}_{ij}\right]-g^{n+1-i-j}D^n_{ij} \tag{C.14}$$

for $n-i-j>0$, where C^n_{ij} and L^n_{ij} are already given in eq. (C.6), and

$$D^n_{ij} := c_ic_j\int_0^1 dy\, y^{n-i-j}L(1/y), \tag{C.15}$$

while for $n=i=j=1$ one should replace C^0_{11} and L^0_{11} with

$$\tilde{C}^0_{11} := \int_0^\infty dp\,\left[K_1^2(p)-\frac{c_1}{p^2}\right],\quad \tilde{L}^0_{11} := \int_0^\infty dp\,\left[K_1^2(p)-\frac{c_1}{p^2}\right]\ln p^2, \tag{C.16}$$

respectively, and

$$D^1_{11} := c_1^2\int_0^\infty dy y^{-1}L(1/y). \tag{C.17}$$

C.3 Some calculations in the IR limit

In the IR limit, we have a universal formula as

$$\frac{X_1(z)}{X_0(z)} \simeq -\frac{4}{3\pi^2}\left[\frac{2}{3}+\ln g^2-C_X\right],\quad X=\gamma,F,G,H, \tag{C.18}$$

where

$$\begin{aligned} C_\gamma &= \frac{32}{3\pi^2}L^2_{11},\quad C_F = \frac{512}{45\pi^2}L^4_{11},\\ C_G &= \frac{64}{3\pi^2}L^3_{01} = C_\gamma+2,\quad C_H = \frac{512}{27\pi^2}L^4_{00} = -\frac{5}{3}C_F+\frac{8}{3}C_\gamma+\frac{64}{9}. \end{aligned} \tag{C.19}$$

By combining these, we have

$$G_s(z) \simeq -\frac{4}{3\pi^2}C_{\rm IR},\quad G_\sigma(z) \simeq \frac{4}{3\pi^2}\left(\frac{48}{5}+3C_{\rm IR}\right), \tag{C.20}$$

where

$$C_{\rm IR} := C_\gamma - C_F = \frac{512}{45\pi^2}\left(\frac{15}{16}L^2_{11}-L^4_{11}\right). \tag{C.21}$$

Importantly, $\ln g^2$ terms are cancelled in $G_s(z)$ and $G_\sigma(z)$, so that the metric describes the AdS space in the IR limit.

C.4 $\Pi_{\lambda,1}(x,z)$ in the IR limit

We here evaluate $\Pi_{\lambda,1}(x,z)$ in the IR limit. Integrating the angle between a vector p_1+p_2 and a vector x, we have

$$\Pi_{\lambda,1}(x,z) = \frac{4}{\gamma_0(z)z^2 g(2\pi)^4 r}\int_0^\infty p_1dp_1\, p_2dp_2\, K_1(p_1)K_2(p_2)\int_0^\pi \sin\theta d\theta \sin(p_{12}r), \tag{C.22}$$

where $r := |x|/z$ and θ is an angle between p_1 and p_2, so that $p_{12} = \sqrt{p_1^2+p_2^2+2p_1p_2\cos\theta}$.

By expanding $\sin(p_{12}r)$, the θ integral is performed as

$$\begin{aligned}
&\sum_{k=0}^{\infty}\frac{(-1)^k r^{2k+1}}{(2k+1)!}\int_{-1}^{1} da\,(p_1^2+p_2^2+2p_1p_2a)^{k+1/2}\\
&=\sum_{k=0}^{\infty}\frac{(-1)^k r^{2k+1}}{(2k+3)p_1p_2(2k+1)!}\left[(p_1+p_2)^{2k+3}-|p_1-p_2|^{2k+3}\right],
\end{aligned} \tag{C.23}$$

which leads to eq. (4.17) with eq. (4.18).

D Bulk-to-boundary propagator by a general smearing

In this appendix, we investigate the UV and the IR behaviors for the bulk-to-boundary propagator of the composite scalar operator using a general smearing function $S(\vec{p},z)$, which satisfies $S(\vec{p},0)=1$ since $\phi^a(x,z=0)=\varphi^a(x)$ by construction. In order to keep the rotational symmetry at the boundary and to avoid the introduction of extra dimensionful parameters, we assume $S(\vec{p},z)=S(pz)$ with $p:=|\vec{p}|$.

The bulk-to-boundary is compactly written in terms of $S(pz)$ as

$$\Pi(X,y) = \frac{2}{N\gamma_0(z)}\int_{p_1,p_2} e^{i(p_1+p_2)\cdot(x-y)}\frac{S(p_1z)S(p_2z)}{p_1^2p_2^2}\frac{1}{1+\lambda_0 B(p_{12}^2)} \tag{D.1}$$

$$= \frac{2}{N\gamma_0(z)z^2}\int_{p_1,p_2} e^{i(p_1+p_2)\cdot(x-y)/z}\frac{S(p_1)S(p_2)}{p_1^2p_2^2}\frac{p_{12}}{p_{12}+g}, \tag{D.2}$$

where

$$\gamma_0(z) := \int_p \frac{S^2(pz)}{p^2} = \frac{c}{z}, \quad c := \int_p \frac{S^2(p)}{p^2}. \tag{D.3}$$

In the $z\to 0$ limit, the first expression leads to

$$\lim_{z\to 0}\Pi(X,y) = \frac{2z}{Nc}\int_{p_1,p_2}\frac{e^{i(p_1+p_2)\cdot(x-y)}}{p_1^2p_2^2}\frac{1}{1+\lambda_0 B(p_{12}^2)} = \frac{z}{c}\langle\varphi^2(x)\varphi^2(y)\rangle_c. \tag{D.4}$$

In the $z\to\infty$ limit, on the other hand, the second expression gives

$$\lim_{z\to\infty}\Pi(X,y) = \frac{1}{z^2}\frac{16}{Nc\lambda_0}\int_{p_1,p_2} S(p_1)S(p_2)\frac{p_{12}}{p_1^2p_2^2} \sim \frac{1}{z^2}, \tag{D.5}$$

which leads to $\Delta_{\varphi^2}=2$.