The wave kinetic theory of three wave and four wave models

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Abstract

In this thesis, we introduce new techniques for studying the random series expansion of dispersive PDEs. We take a quadratic KdV type and a cubic Klein-Gordon type equation as examples to demonstrate the different techniques in three wave and four wave models. For three wave models, we introduce a counting argument to handle the degeneracy problems of the resonance surface and the loss of derivative problem. For four wave models, we introduce a novel renormalization argument and prove a renormalized Wick theorem. We provide a heuristic argument that this renormalization is able to remove all bad terms from the L^2 mass term, combining with an almost cancellation identity of the regular pairing and the Deng-Hani's Feynman diagram analysis [1], [2].

Acknowledgements

thank you very much

To my family,

Declaration

I declare that I have not violated the Honor Code during the composition of this work. This paper represents my own work in accordance with University regulations.

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Chapter

1.1 Section

abc

1.1.1 Subsection

Text.

1.2 New Section

1.3 Outline

The following is a possible outline for your paper.

1.3.1 Introduction

- Motivation and Goal (The goal of this project is...)
- Overview of challenge and previous work
- Approach
- Summary of implementation
- Summary of results

1.3.2 Problem Background and Related Work

- Survey of prior work with similar goals
- For each previous approach, explain what has been done and why it does not meet your goal

1.3.3 Approach

- Key novel idea
- Why it is a good idea

1.4 section

1.4.1 Implementation

- System overview (flow chart of key steps?)
- Subsection for each step or issue you addressed
  - Problem statement
  - Possible approaches
  - Chosen approach and why
  - Implementaton details

1.4.2 Evaluation

- Experiment design...
- Data...
- Metrics...
- Comparisons...
- Qualitative results...
- Quantitative results...

1.4.3 Summary

- Conclusions...
- Limitations...
- Future work...

1.4.4 Subsection

More text.

Bibliography

[1] Y. Deng and Z. Hani. Full derivation of the wave kinetic equation. *arXiv preprint arXiv:2104.11204*, 2021.

[2] Y. Deng and Z. Hani. Derivation of the wave kinetic equation: full range of scaling laws. *arXiv preprint arXiv:2301.07063*, 2023.