What is a Category?

sarah, leonhard, Andreas Meyer

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Category

TODO

Epis and Monis

TODO

Isomorphisms

Examples

- TODO Set
- TODO Rel
- TODO Grp
- TODO Vec
- TODO Pos
- TODO Discrete, 1, 2

Dual categories

Terminal and Initial Objects

> **Definition (Initial and terminal Objects):** In any category C, an object 0 is called initial iff for any object $A \in C$, there is an unique morphism $0 \mapsto A$. an object 1 is called terminal iff for any object $A \in C$, there is an unique morphism $A \mapsto 1$. A terminal object in C is initial in C^{op}

Proposition:

Initial and terminal objects are unique up to isomorphism.

Proof:

Assume 0,0' are both inital objects in some category C and show that $f : 0 \mapsto 0', g : 0' \mapsto 0$ form an unique isomorphism $f \circ g$ between 0,0'. One can draw the following diagram:

$$\begin{tikzcd}
& 0' \arrow[rr, "id_{0'}"] \arrow[dr, "g"] & & 0' \\
0 \arrow[ur, "f"] \arrow[rr, "id_0"] & & 0 \arrow[ur, "f"] &
\end{tikzcd}$$

Since 0 is initial, we know that f is unique, from the same argument follows uniqueness of $g = f^{-1}$. Therefore $f \circ g$ and $g \circ f$ is unique.
The same holds for terminal objects by duality. $\square$

Categories, in which the terminal is identical to the initial object, are called pointed category. Such objects zero objects.

Example:

How to show that $\emptyset$ is initial in Set and the one- element set $\{x\}$ terminal?

- There is only the binary union function from $\emptyset$ to any other set, since there are no arguments in the domain to use.
- Assume some f with
  $\forall A$: set with $A \neq \emptyset,\ f :\mapsto \{x\}$ f is obviously a constant function, since $\forall y \in A,\ f\ y = x$ holds, and therefore unique.