Lecture 4:

From Linear to Logistic Regression

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Lecture Contents

Part 1/3:

Advanced Linear Regression

1. Review and Connection to Previous Lecture
2. Revisiting Linear Regression Assumptions
3. Polynomial Regression and Basis Expansion
4. Ridge Regression (L2 Regularization)
5. Lasso Regression (L1 Regularization)
6. Elastic Net
7. Feature Selection and Importance
8. Limitations of Linear Regression

Linear Regression: Foundation & Challenges

Linear Regression
Minimizes Sum of Squared Errors

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n$$

Normal Equation
Solution

Gradient Descent
Optimization

Linearity

Independence

Homoscedasticity

Overfitting

Multicollinearity

Noise

Goal: Extend Linear Regression Capabilities

Linear Regression: Key Assumptions

1 Linearity

Relationship between X and y is linear

2 Independence

Observations are independent of each other

3 Homoscedasticity

Constant variance of residuals

4 Normality

Residuals follow normal distribution

5 No Multicollinearity

Predictors are not highly correlated

? Violation Check

What happens when assumptions are violated?

Impact of Assumption Violations

Affects **prediction accuracy** and **interpretation** of model results

Polynomial Regression & Basis Expansion

Key Insight

Linear models can fit non-linear relationships

Transform Features:

$$x \longrightarrow x, x^2, x^3, \ldots$$

Example Equation:

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

✓ Still "linear" in parameters (coefficients)

⚖ **Trade-off:** Model flexibility vs. overfitting risk

🎯 Choosing polynomial degree: validation approach

f(x) **Other basis functions:**
Logarithmic, Exponential, Trigonometric

Extends Linear Regression Capabilities

📊 Interactive Polynomial Regression Simulator

Explore polynomial regression with different degrees using Desmos!

Launch Interactive Simulator →

Ridge Regression (L2 Regularization)

PROBLEM

Large coefficients
lead to overfitting

→

SOLUTION

Add penalty term
for large coefficients

Cost Function:

$$RSS + \lambda \sum \beta_i^2$$

(L2 Penalty)

Coefficient Shrinkage

Shrinks coefficients toward zero (but not exactly zero)

Hyperparameter λ

Controls regularization strength

Effect of λ

Larger λ → more regularization, simpler model

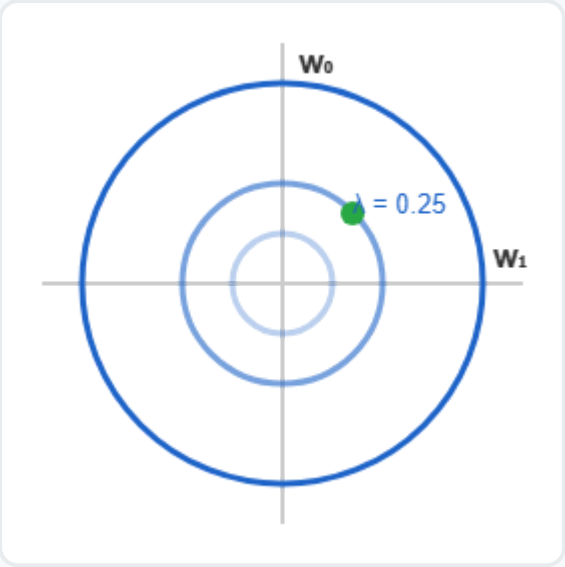
✓ Key Benefit

Helps with multicollinearity problem

2D Visual Understanding

L2 (Ridge)

L1 (Lasso)



Circle shape

$w_0^2 + w_1^2 = \text{constant}$

Gradually shrinks coefficients

Diamond shape

$|w_0| + |w_1| = \text{constant}$

Can make coefficients exactly zero

🎚️ Interactive: Effect of λ on Coefficients

λ (Lambda) = 0.0

💡 Move the slider to change λ value. As λ increases, coefficients shrink toward zero!

β_1 (original: 5.00)	β_2 (original: 3.00)	β_3 (original: 4.00)

5.00	3.00	4.00

📐 Geometric Intuition

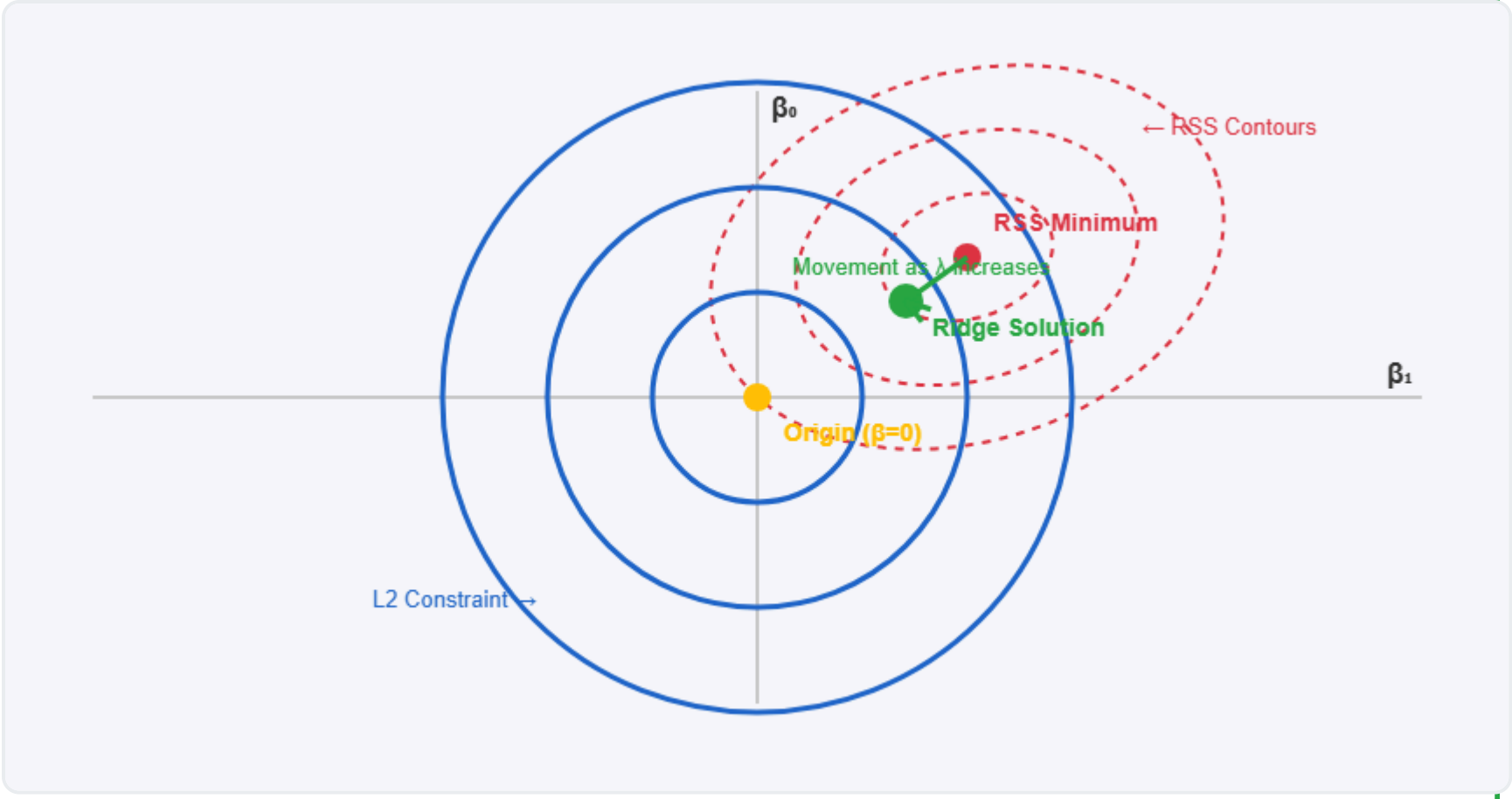
Ridge regression finds the optimal solution by balancing two objectives:

1 Minimize Prediction Error (RSS)
Want to fit the training data well

2 Minimize Coefficient Magnitude ($\lambda\sum\beta^2$)
Want to keep the model simple

$$\text{Loss} = \text{RSS} + \lambda\sum\beta_i^2$$

The solution moves from the RSS optimal point
toward the origin ($\beta=0$) as λ increases



🎯 **Key Insight:** L2 penalty creates a circular constraint. The optimal solution is where the RSS contour (ellipse) touches the L2 constraint circle. As λ increases, the circle gets smaller, pulling coefficients toward zero.

💡 Why Ridge Helps with Multicollinearity

When features are highly correlated (multicollinear), ordinary linear regression can produce very large coefficients that cancel each other out.

⚠ **Problem Example:**
If $x_1 \approx x_2$:

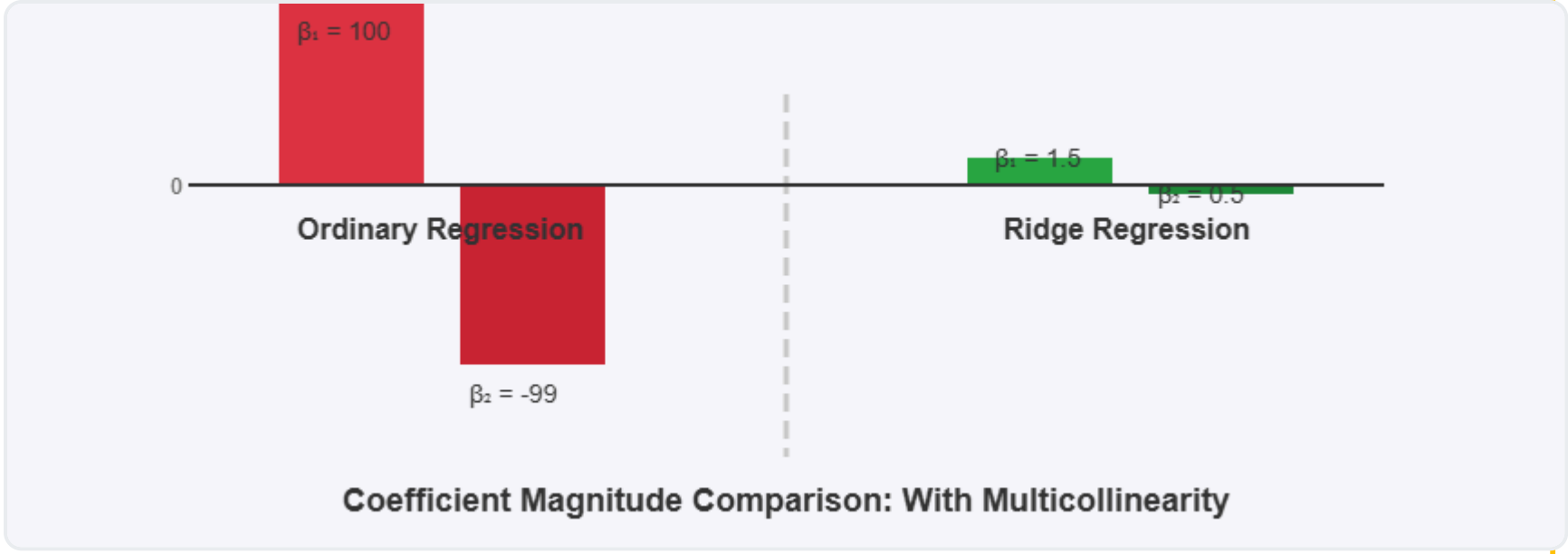
$$y = 100 \cdot x_1 - 99 \cdot x_2$$

These large coefficients are unstable and sensitive to small data changes!

✅ **Ridge Solution:**
By penalizing large coefficients:

$$y = 1.5 \cdot x_1 + 0.5 \cdot x_2$$

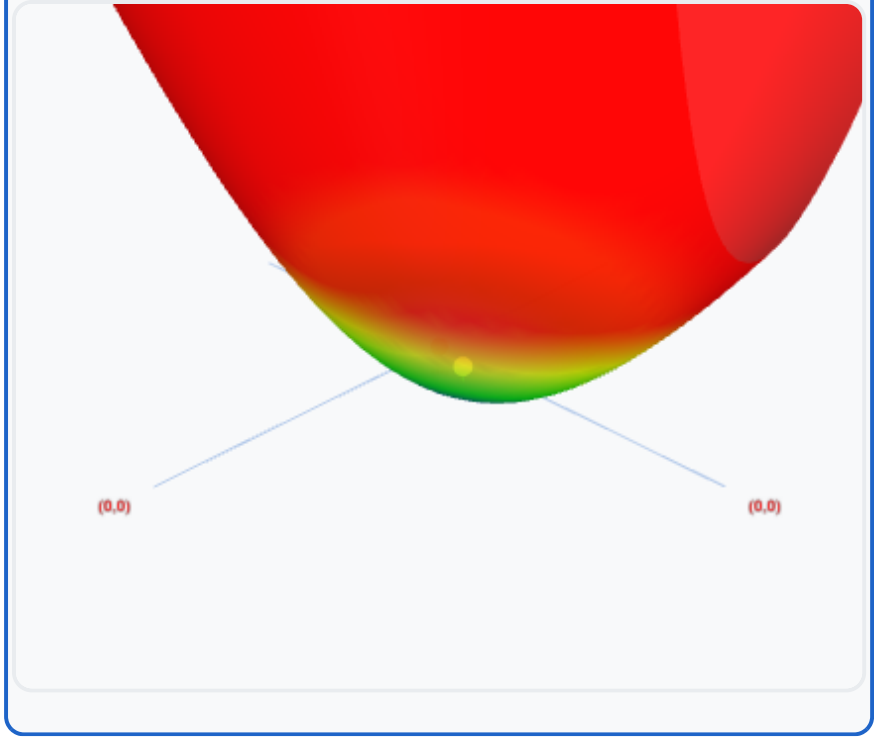
The model becomes more stable and generalizes better!

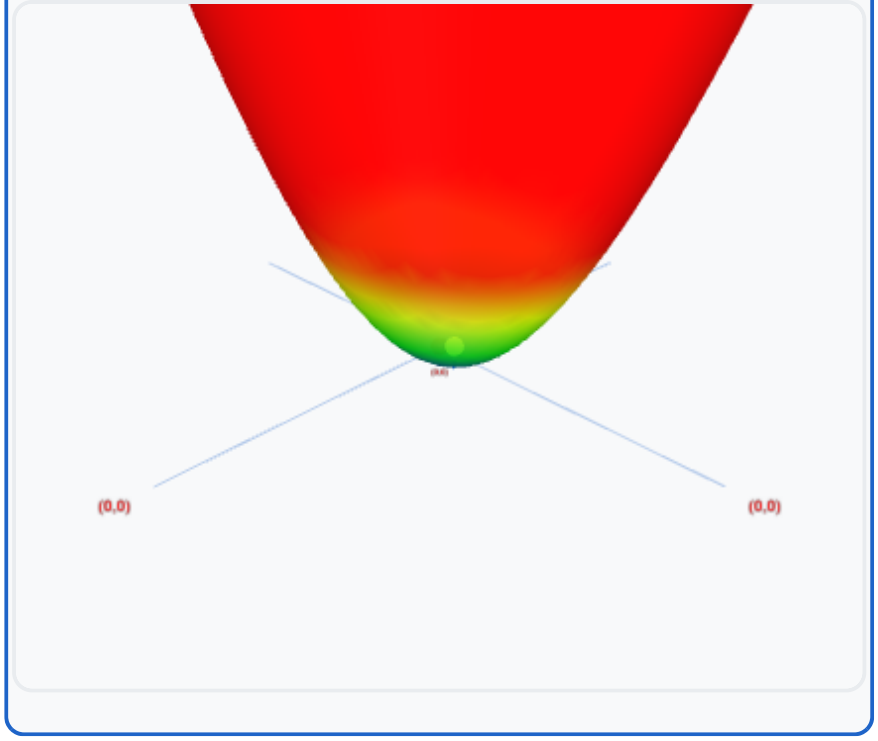


3D Loss Surface Visualization

Compare how the loss surface changes with and without L2 regularization.
The white sphere marks the optimal coefficient values.

Ordinary Least Squares (OLS)	Ridge Regression (L2)





λ (Lambda): 1.0

Rotation Speed: 1.0

💡 **Tip:** Increase λ to see how the optimal point (white sphere) moves closer to the origin (red sphere at center), representing smaller coefficient values and a simpler model.

Lasso Regression (L1 Regularization)

L1 Penalty:

$$RSS + \lambda \sum |\beta_i|$$

Sparse Solutions
Makes some coefficients exactly zero

Feature Selection
Automatically selects important features

Hyperparameter λ
Controls sparsity level

Interpretability
Creates simpler, more interpretable models

Ridge vs Lasso

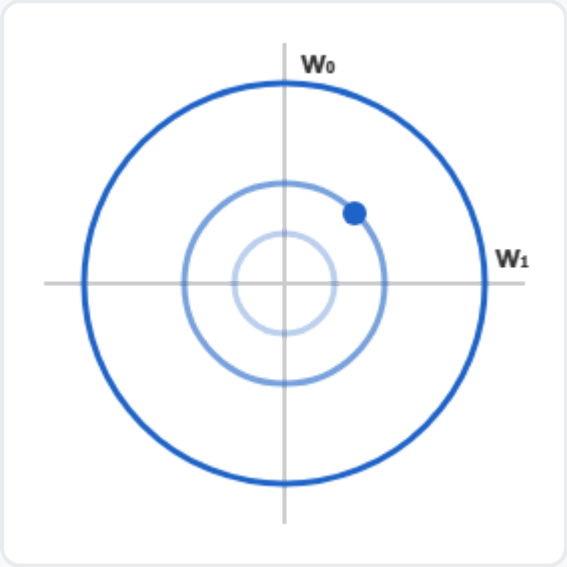
Ridge (L2) vs **Lasso (L1)**

Ridge shrinks coefficients, but Lasso can make them **exactly zero** → Feature selection!

Use Cross-Validation to choose optimal λ

🎯 2D Visual Understanding

L2 (Ridge) 🔵



Circle shape

$w_0^2 + w_1^2 = \text{constant}$

Gradually shrinks coefficients

L1 (Lasso) 🔴

Diamond shape

$|w_0| + |w_1| = \text{constant}$

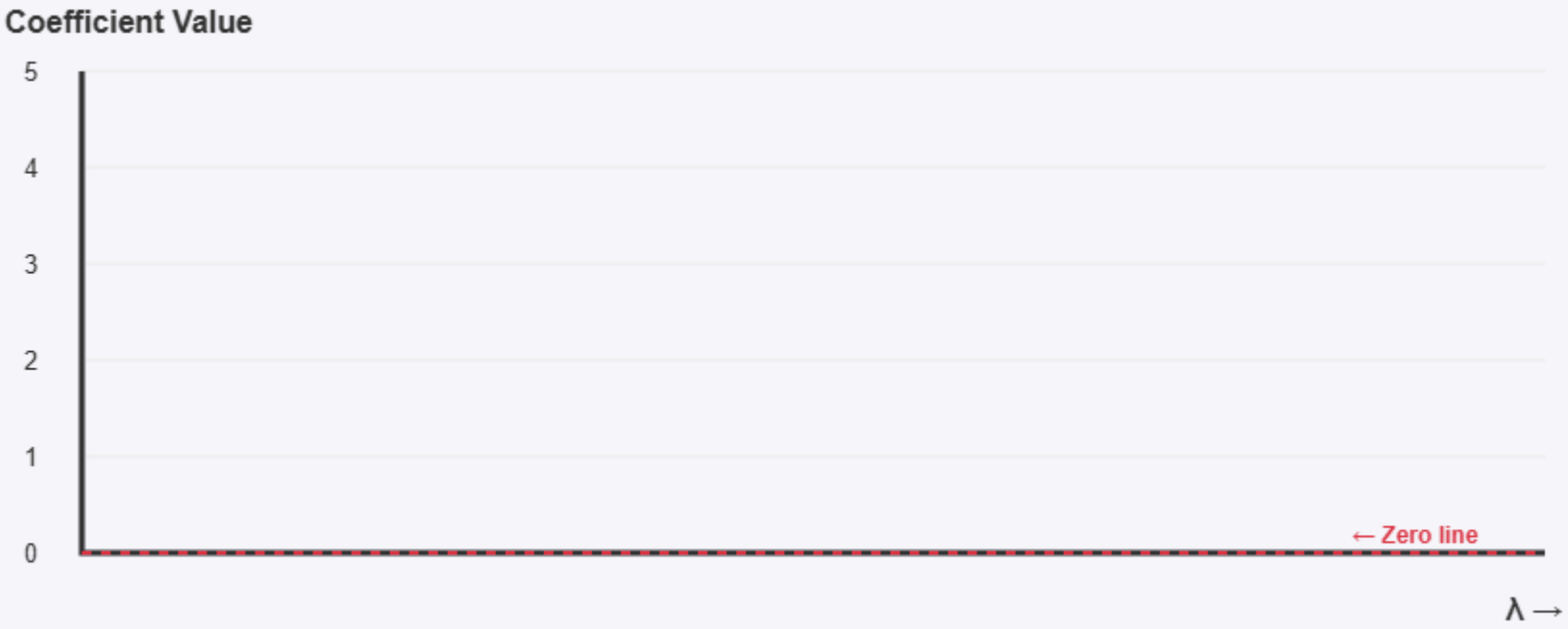
Can make coefficients exactly zero

🎚️ Interactive: Effect of λ on Lasso Coefficients

λ (Lambda) = 0.0

💡 Move the slider to change λ value. Notice how Lasso makes coefficients exactly zero!

β_1 (original: 5.00)	β_2 (original: 3.00)	β_3 (original: 4.00)
5.00	**3.00**	**4.00**



📐 Geometric Intuition: Why Lasso Creates Sparsity

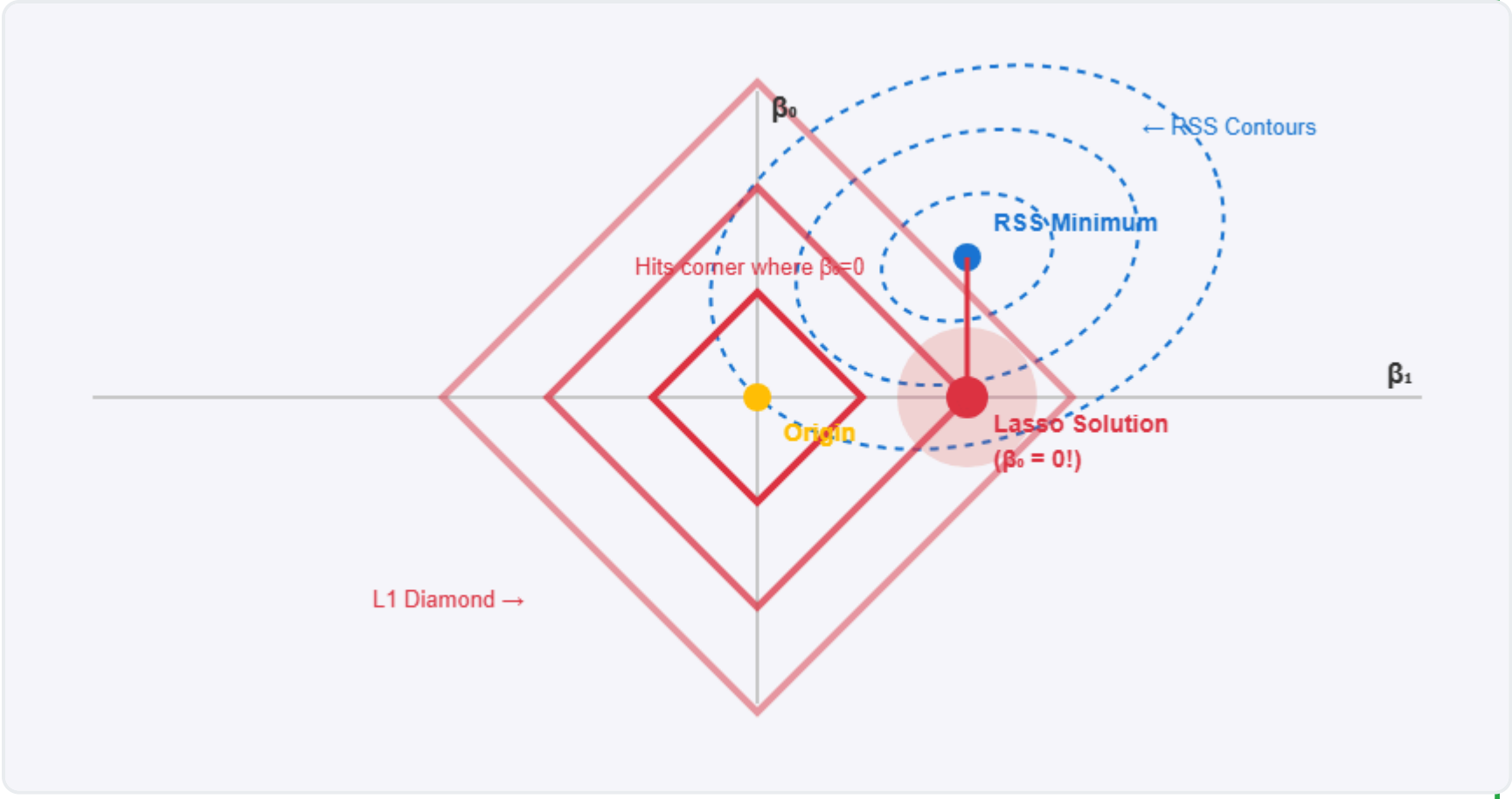
Lasso regression finds the optimal solution by balancing two objectives:

1 Minimize Prediction Error (RSS)
Want to fit the training data well

2 Minimize Coefficient Magnitude (λ∑|β|)
Want to keep the model simple with sparse coefficients

$$\text{Loss} = \text{RSS} + \lambda \sum |\beta_i|$$

The diamond shape of L1 constraint
has corners where coefficients become exactly zero



🎯 **Key Insight:** L1 penalty creates a diamond-shaped constraint. The optimal solution tends to hit the corners of the diamond, where one or more coefficients are exactly zero. This is why Lasso performs automatic feature selection!

🔍 Why Sparsity Matters: Feature Selection

When you have many features, not all of them are useful. Lasso automatically identifies and removes irrelevant features by setting their coefficients to zero.

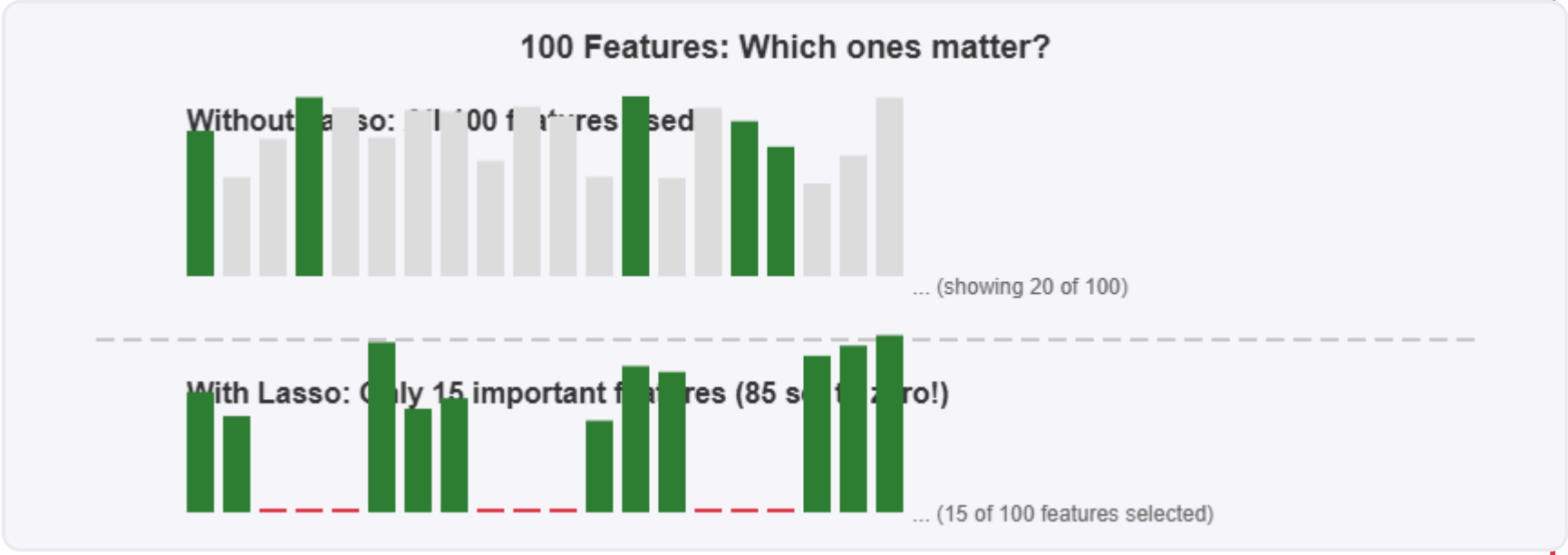
💼 Real-world Scenario:
Predicting house prices with 100 features:

```
Many features like "color of doorknob" are irrelevant
```

Lasso identifies that only 15 features truly matter and sets the rest to zero.

✅ Benefits of Sparsity:

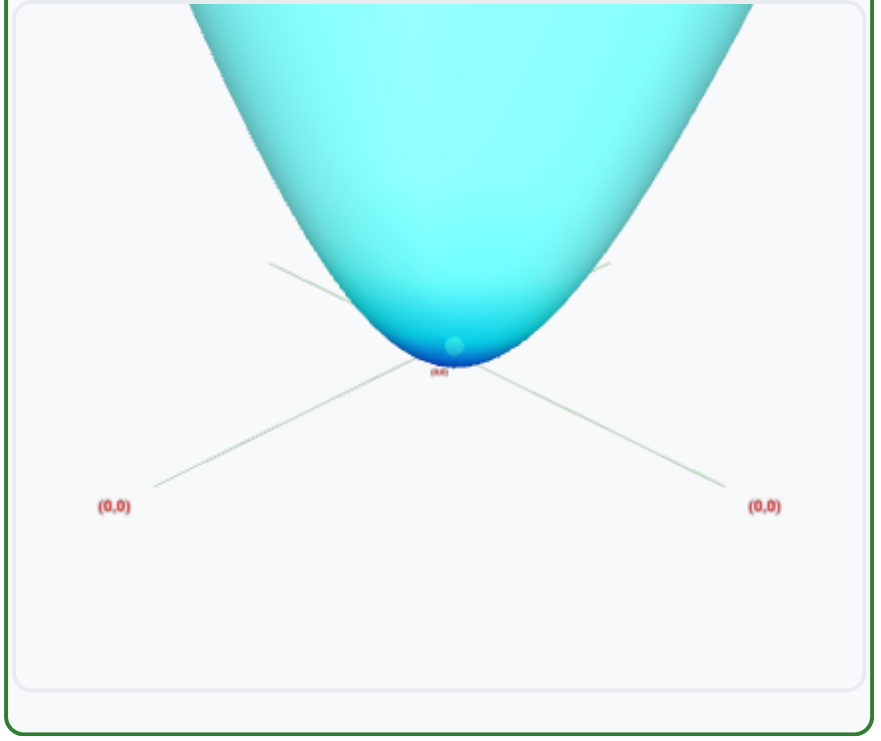
1. **Interpretability:** Simpler models are easier to understand
2. **Efficiency:** Fewer features mean faster predictions
3. **Generalization:** Removes noise, improving performance on new data
4. **Cost savings:** Don't need to collect/store irrelevant features

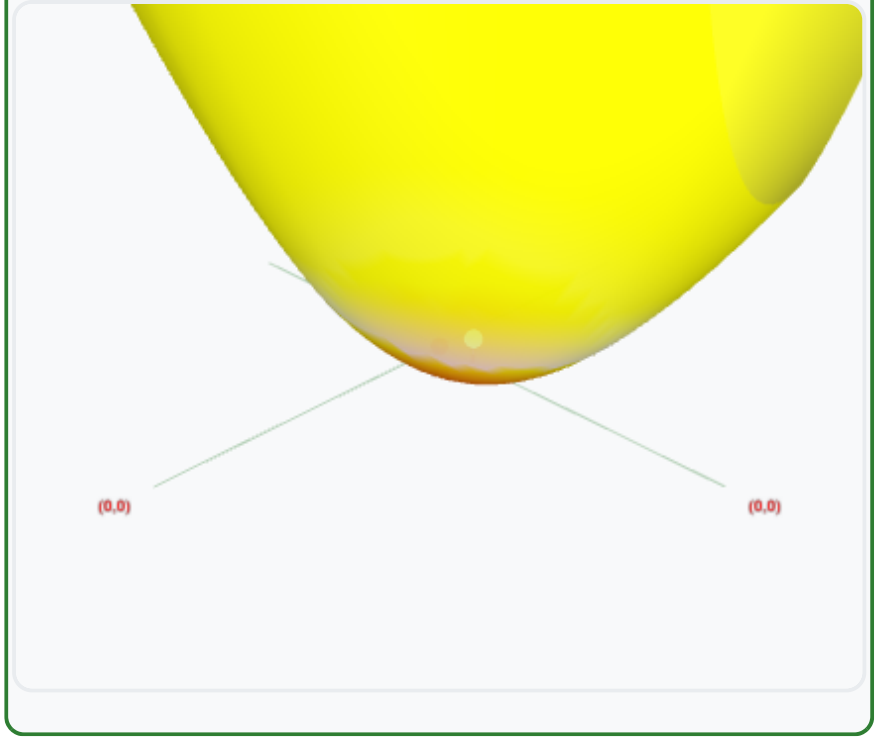


3D Loss Surface Visualization

Compare how the loss surface changes between Ridge (L2) and Lasso (L1) regularization.
Notice how Lasso's solution (white sphere) tends to align with axes, indicating zero coefficients.

Ridge Regression (L2)	Lasso Regression (L1)





λ (Lambda): 1.0

Rotation Speed: 1.0

💡 **Tip:** Increase λ to see how Lasso's optimal point (white sphere) moves toward the axes, while Ridge's point moves smoothly toward the origin. This visualizes why Lasso creates sparse solutions!

Elastic Net: Best of Both Worlds

Ridge (L2) + Lasso (L1) = Elastic Net Combined Power

Cost Function:

$$RSS + \lambda_1 \sum \beta_i^2 + \lambda_2 \sum |\beta_i|$$

- ✓ Gets benefits of **both methods**
- 🎯 **Feature selection** like Lasso
- ⚖️ **Stability** like Ridge
- 💡 Useful when predictors are **highly correlated**

Hyperparameters: α (mixing) and λ (strength)

α = 0	0 < α < 1	α = 1
Ridge	Elastic Net	Lasso

Feature Selection and Importance

Why Feature Selection Matters

Interpretability + Efficiency

Selection Methods

Filter
Correlation-based

Wrapper
RFE (Recursive Feature Elimination)

Embedded
Lasso regularization

Balance: Model Simplicity ⚖ Predictive Power

Coefficient Magnitude
Indicates feature importance

Standardization
Standardize features before comparing coefficients

Beware
Multicollinearity affects importance interpretation

Permutation Importance
Alternative robust method

Example: Applying Feature Selection Methods

Filter Method

Step 1: Calculate Correlation

Compute correlation between each feature and target

Wrapper Method (RFE)

Step 1: Train Full Model

Fit model with all features

Embedded Method (Lasso)

Step 1: Set Regularization

Choose penalty parameter λ

```
corr_matrix = X.corrwith(y)
```

Step 2: Set Threshold

Select features with |correlation| > 0.5

```
threshold = 0.5
```

Step 3: Select Features

Keep features above threshold

```
selected = corr_matrix[abs(corr_matrix) > 0.5]
```

✓ Result

Fast, independent of model
Good for initial screening

```
model.fit(X_all, y)
```

Step 2: Rank Features

Identify least important feature

```
importance = model.coef_
```

Step 3: Remove & Repeat

Eliminate feature, retrain model

```
X_new = X.drop(worst_feature)
model.fit(X_new, y)
```

✓ Result

Model-specific selection
More accurate but slower

```
model = Lasso(alpha=0.1)
```

Step 2: Train Model

L1 penalty shrinks coefficients

```
model.fit(X, y)
```

Step 3: Extract Features

Select features with non-zero coefficients

```
selected = X.columns[model.coef_ != 0]
```

✓ Result

Automatic selection
Built into training process

Limitations of Linear Regression

- Cannot handle non-linear decision boundaries naturally
- Assumes continuous output (not suitable for classification)
- Sensitive to outliers without robust variants
- Feature engineering required for complex patterns
- Interpretation difficult with many interaction terms
- Limited for categorical outcomes

Why We Need Classification Algorithms

Critical Question:

What if output is categorical (yes/no)?

Part 2/3:

Transition to Classification

9. Regression vs Classification Problems
10. Linear Classifier Concepts
11. Perceptron Algorithm
12. Decision Boundaries and Linear Separability
13. Why Linear Regression Fails for Classification
14. Odds and Log Odds
15. Introduction to Sigmoid Function
16. Properties of Logistic Function

Regression vs Classification Problems

Regression

Predict continuous values

Examples:
Price, Temperature, Age

VS

Classification

Predict discrete categories

Examples:
Spam/Ham, Disease/Healthy

Key Difference:

Output Space: $\mathbb{R}$ (continuous) vs. Finite Set (discrete)

Regression Output

$$y \in \mathbb{R}$$

Scalar/Vector: $\hat{y}$ = 25.7°C
Multi-output: $\hat{y}$ = [150.2, 175.8, 163.4] kg

Classification Output

$$y \in \{0, 1, ..., K\text{-}1\}$$

Binary: $\hat{y}$ = 1 (Spam), $\hat{y}$ = 0 (Ham)
Multi-class: $\hat{y}$ = 2 (Cat), or $\hat{y}$ = [0, 0, 1, 0]

Why linear regression fails:
Predictions outside [0,1] range

Need for probability:
P(y=1|x) interpretation

Today's Focus:

Binary Classification (Two Classes)

Linear Classifier Concepts

Goal:

Find a line (hyperplane) that separates two classes

Decision Function:

$$f(x) = w^T x + b$$

→

Classification Rule:

if f(x) > 0 → **Class 1**

else → **Class 0**

Geometric View

Distance from decision boundary

Weight Vector w

Perpendicular to decision boundary

Bias Term b

Shifts boundary position

Multiple Features

Hyperplane in high-dimensional space

Perceptron Algorithm

Simplest Linear Classifier

Frank Rosenblatt

1957

Update Rule:

$$w \leftarrow w + \eta(y - \hat{y})x$$

Activation Function:

Step Function

(0 or 1)

Iteratively adjusts weights for misclassified points

Converges if data is linearly separable

Limitation: No convergence for non-separable data

Historical Significance: Foundation of neural networks

Decision Boundaries and Linear Separability

Linearly Separable

Classes can be separated by straight line/plane

VS

Non-Linearly Separable

Requires curved boundary

⊕ XOR Problem: Classic example of non-linear separability

Real-world: Most datasets are not perfectly separable

Solutions:

Feature Transformation | Kernel Methods | Non-Linear Models

✓ **Linear Classifiers**

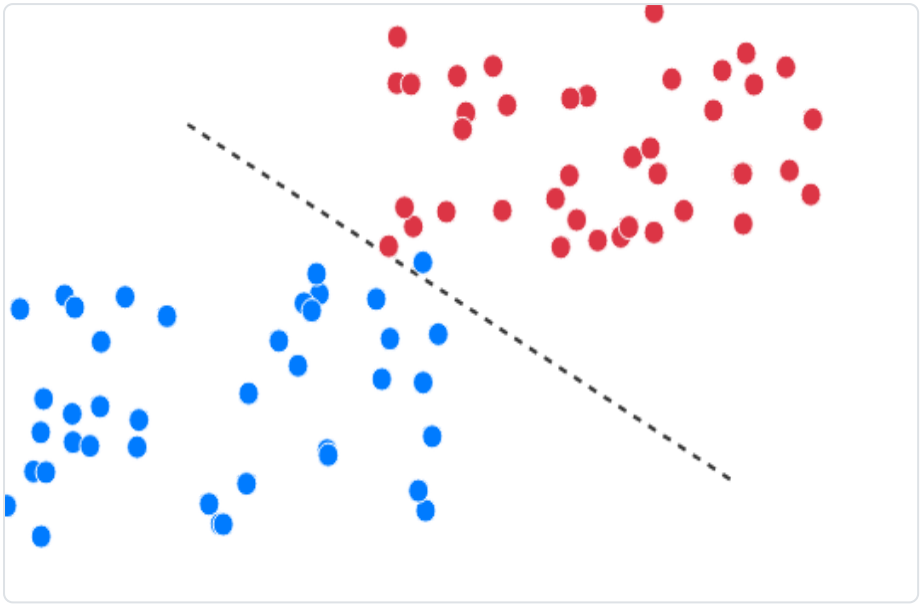
Fast, interpretable, good baseline

When to Use

High-dimensional data, large datasets

2D Decision Boundary Visualization

✓ Linearly Separable (2D)

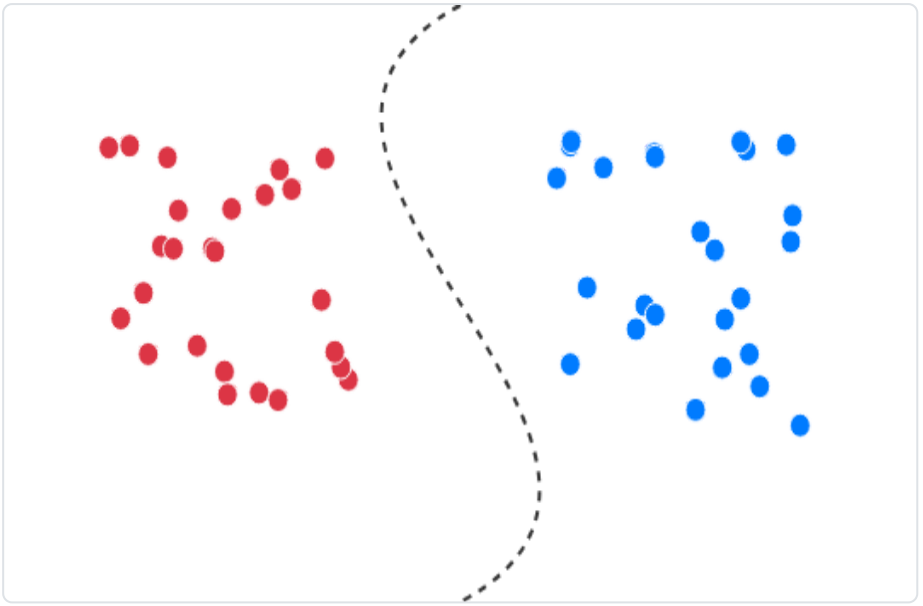


Class A Class B

Generate Data Clear

A single straight line can perfectly separate the two classes

✗ Non-Linearly Separable (XOR)



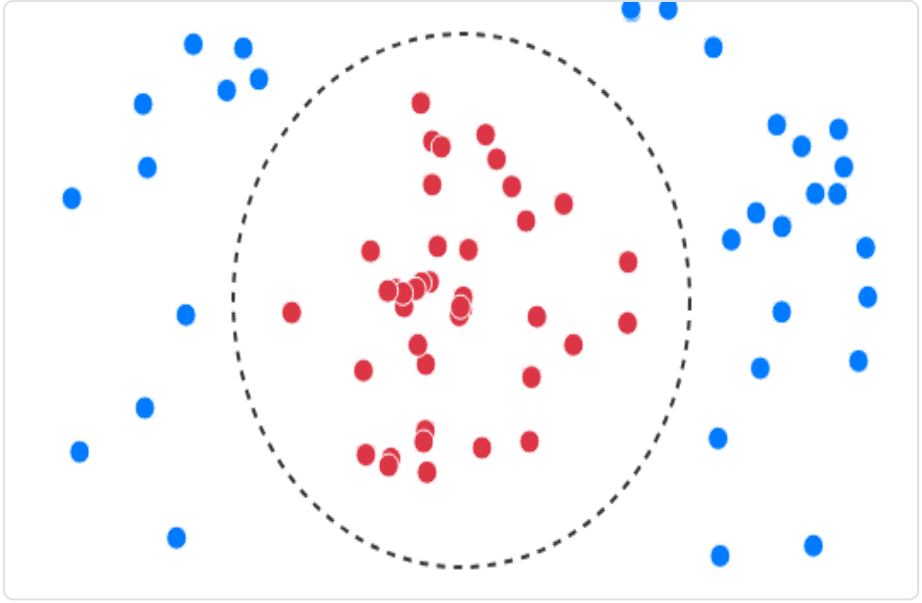
Class A Class B

Generate XOR Clear

XOR pattern requires a curved (non-linear) boundary

✗ Non-Linearly Separable (Circles)

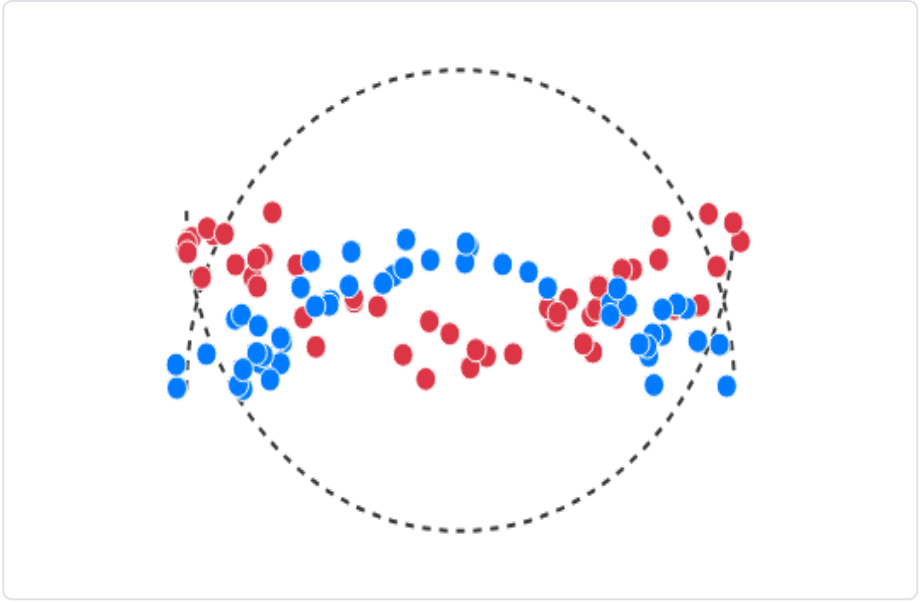
✗ Non-Linearly Separable (Moons)



Inner Class Outer Class

Generate Circles Clear

Concentric circles require a circular boundary



Class A Class B

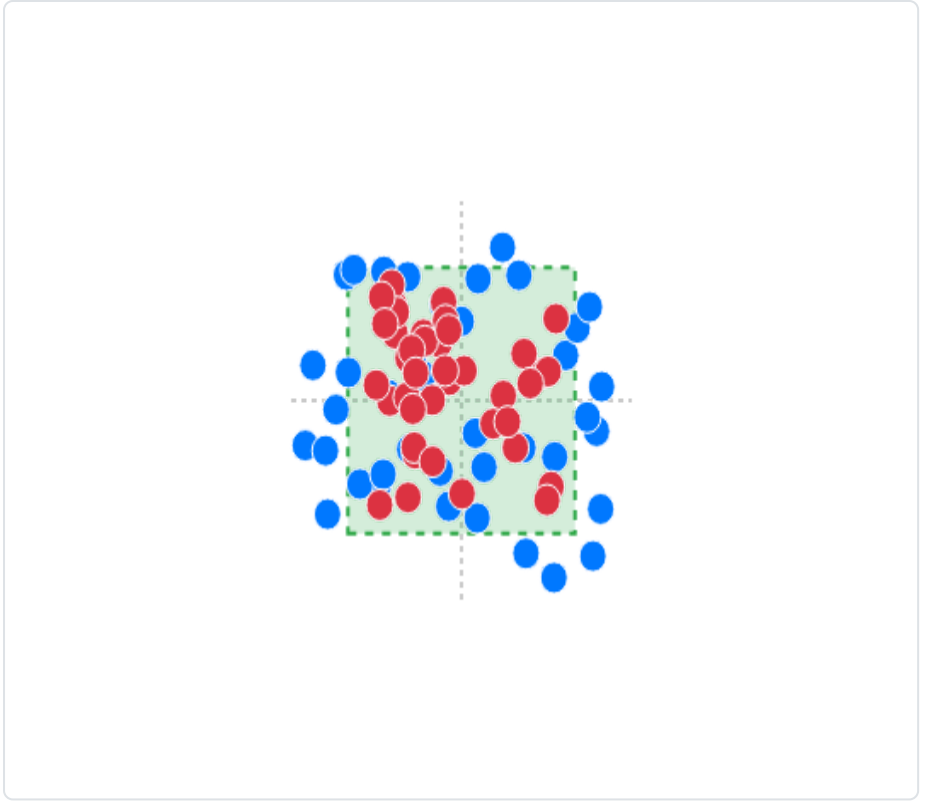
Generate Moons Clear

Interleaved crescent shapes require a complex boundary

3D Decision Boundary Visualization

✓ Linearly Separable (3D)

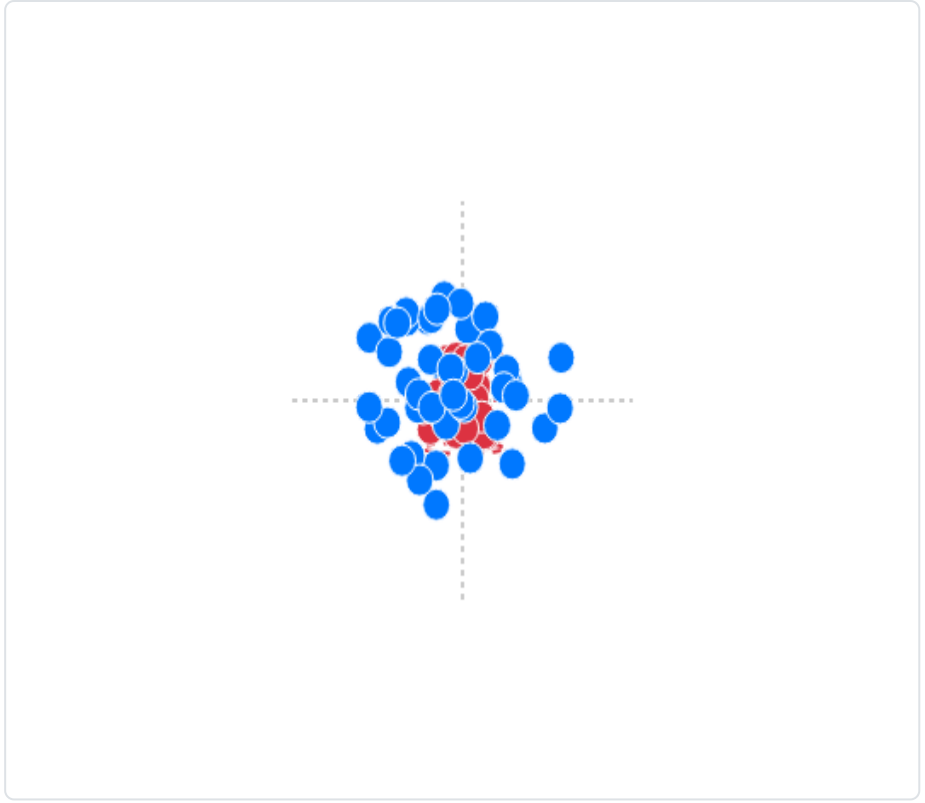
✗ Non-Linearly Separable (3D Spheres)



Class A Class B

Generate Data | Rotate View | Stop | Clear

A single plane can separate classes in 3D space



Inner Class Outer Class

Generate Spheres | Rotate View | Stop | Clear

Concentric spheres require a spherical boundary surface

Why Linear Regression Fails for Classification

1. Predictions can be < 0 or > 1
2. Treats classes as ordered numerical values
3. Sensitive to outliers in feature space
4. Squared loss inappropriate for binary outcomes

Example:

Predict probability of disease (should be in [0,1])

Linear Regression Output:

Any real number

$\in \mathbb{R}$

$\rightarrow$

NEED

Required Output:

Bounded [0,1]

Probability

Solution Requirement:

Output bounded to [0,1] representing probability

Odds and Log Odds

Probability

P(event)

$\in [0, 1]$

→

Odds

$p / (1 - p)$

$\in [0, \infty)$

→

Log Odds (Logit)

$log(p / (1 - p))$

$\in (-\infty, \infty)$

Key Insight:

Log odds can take any real value!

Linear model for log odds instead of probability

Example:

P = 0.8 → Odds = 4 → Log odds = 1.39

Bridge to Logistic Regression Formulation

Introduction to Sigmoid Function

Definition:

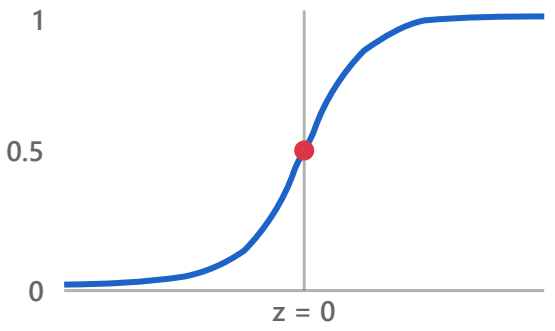
$$\sigma(z) = 1 / (1 + e^{-z})$$

Output:

$$P(y=1|x) = \sigma(w^T x + b)$$

- Maps any real number to (0,1) range
- Perfect for probability interpretation
- S-shaped curve: Smooth transition between 0 and 1
- ✓ **Solves** the "prediction outside [0,1]" problem

S-Shaped Curve



Input z=0 → Output σ=0.5 (decision threshold)

Properties of Logistic Function

Symmetry

σ(z) + σ(-z) = 1

Monotonic

Always increasing, never decreases

Smooth

Differentiable everywhere (no jumps)

Bounded

Output always in (0,1), never exactly 0 or 1

Interpretable

Steepness indicates confidence

Derivative

Simple form for optimization

Symmetry Property:

$$\sigma(z) + \sigma(-z) = 1$$

Derivative:

$$\sigma'(z) = \sigma(z)(1 - \sigma(z))$$

(useful for gradient descent)

Comparison to Step Function

Logistic (Smooth)

vs

Step (Hard Threshold)

Smooth transition vs. Abrupt change

Part 3/3:

Completing Logistic Regression

17. Defining the Logistic Regression Model
18. Maximum Likelihood Estimation (MLE)
19. Binary Cross-Entropy Loss
20. Applying Gradient Descent
21. Multiclass - One-vs-Rest Strategy
22. Softmax Regression
23. Categorical Cross-Entropy
24. Regularized Logistic Regression
25. Real-World Cases and Implementation

Defining the Logistic Regression Model

Complete Formulation:

$$P(y=1|x) = \sigma(w^T x + b)$$

$$= 1 / (1 + e^{-(w^T x + b)})$$

Step 1

$w^T x + b$

Linear combination

→

Step 2

$\sigma(...)$

Sigmoid function

→

Output

$P \in [0,1]$

Probability

Decision Rule:

Predict Class 1 if
P(y=1|x) ≥ 0.5

Threshold can be adjusted based on application needs

w

Weight Vector w

Model parameters to learn

b

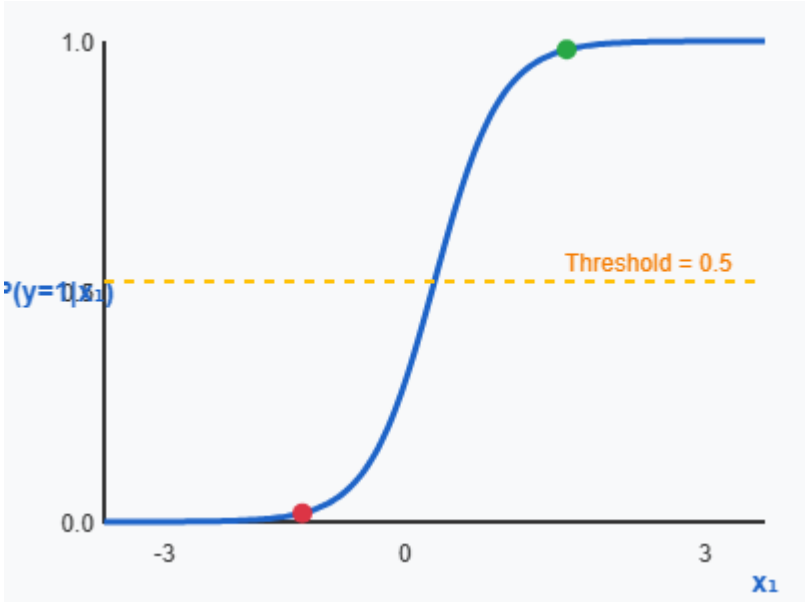
Bias b

Intercept term to learn

Training Goal: Find optimal w and b from data

Input-Output Visualization

2D: Single Feature (x_1)



Sigmoid curve: $P(y=1|x_1) = \sigma(w_1x_1 + b)$

3D: Two Features (x_1, x_2)

Decision surface: $P(y=1|x_1,x_2) = \sigma(w_1x_1 + w_2x_2 + b)$

Drag to rotate, scroll to zoom

Maximum Likelihood Estimation (MLE)

Goal:

Find parameters that maximize likelihood of observed data

1

Likelihood Function:

$$L(w,b) = \prod P(y_i|x_i)$$

2

Log-Likelihood (easier to optimize):

$$\ell = \sum log\ P(y_i|x_i)$$

3

For Binary Classification:

$$\ell = \sum[y_i\ log(p) + (1-y_i)log(1-p)]$$

Maximizing log-likelihood = Minimizing negative log-likelihood

?

Why MLE?

Principled probabilistic approach

Connection:

Links to cross-entropy loss function

Binary Cross-Entropy Loss

Loss Function:

$$L = -[y \log(\hat{y}) + (1-y)\log(1-\hat{y})]$$

⚠ Penalizes wrong predictions heavily

∪ Convex function: Guaranteed global minimum

⚖ Asymmetric penalty: Confident wrong predictions costly

If y = 1

Loss:

$$-\log(\hat{y})$$

Minimized when $\hat{y} \to 1$

If y = 0

Loss:

$$-\log(1-\hat{y})$$

Minimized when $\hat{y} \to 0$

💡 Asymmetric penalty structure

💰 Confident wrong predictions are costly

Total Loss: Average over all training examples

📊 Interactive 3D Loss Surface

Drag to rotate • Scroll to zoom

Reset View Toggle Wireframe Auto Rotate

Interact with the 3D surface: Click and drag to rotate, scroll to zoom in/out

3D Surface Visualization: This interactive plot shows the Binary Cross-Entropy loss landscape with **X-axis: y (True Label, 0 or 1)** **Z-axis: ŷ (Predicted Value, 0-1)** **Y-axis: Loss (BCE)**

The surface is **not convex** in the traditional sense when visualized this way. Note the two "valleys" at (0,0) and (1,1) where predictions match true labels. Colors: **Blue (low loss)** → **Yellow** → **Red (high loss)**

Applying Gradient Descent

No closed-form solution like linear regression

Solution:

Iterative Optimization: Gradient Descent

Update Rule:

$$w \leftarrow w - \eta \nabla L(w)$$

Gradient:

$$\partial L/\partial w = (\hat{y} - y)x$$

(similar form to linear regression!)

Learning Rate η:

Controls step size (typically 0.001-0.1)

Variants:

- Batch Gradient Descent
- Mini-batch Gradient Descent
- Stochastic Gradient Descent

Convergence

Monitor on validation set:

Loss | Accuracy

Iterative Process

Initialize → Compute Gradient → Update ↻

Step-by-Step: Gradient Descent in Action

1

Initialize Parameters

Movement in Loss Space

Start with initial weights (often random or zeros)

$$w_0 = 0.5$$

2 Compute Prediction

Forward pass through the model

$$\hat{y} = \sigma(w_0 x) = \sigma(0.5 \times 2.0) = 0.73$$

3 Calculate Loss

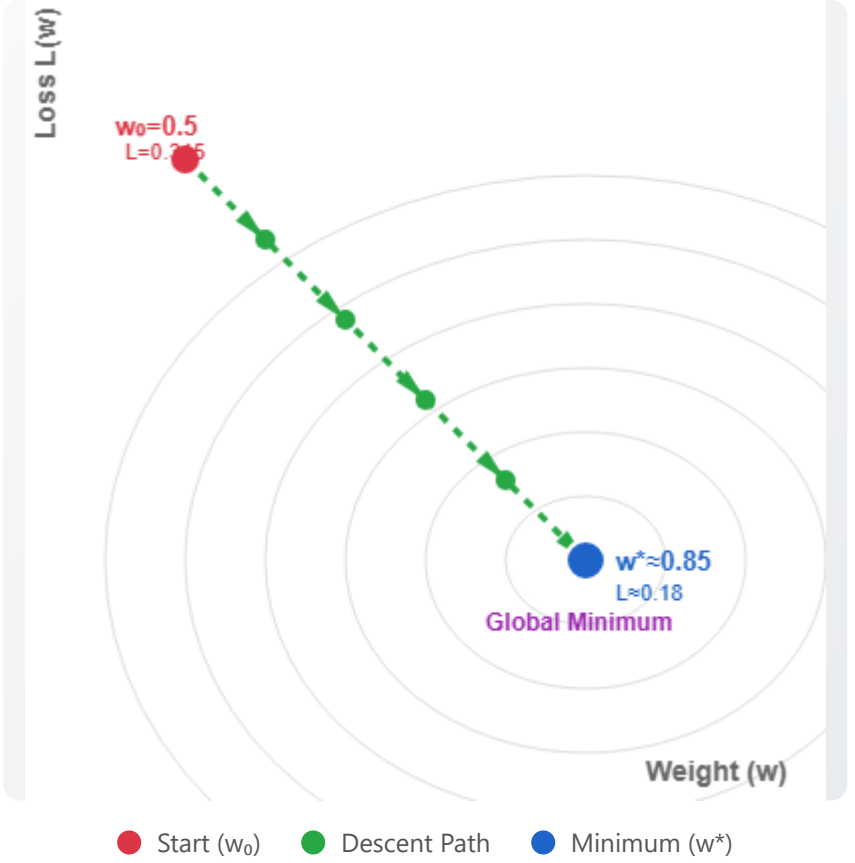
Measure prediction error

$$L = -[y \log(\hat{y}) + (1-y) \log(1-\hat{y})]$$
$$L = 0.315$$

4 Compute Gradient

Calculate direction of steepest ascent

$$\partial L/\partial w = (\hat{y} - y)x = (0.73 - 1) \times 2.0$$
$$\nabla L = -0.54$$



5

Update Parameters

Move in opposite direction of gradient

$$w_1 = w_0 - \eta \nabla L$$

$$w_1 = 0.5 - 0.1 \times (-0.54) = 0.554$$

6

Repeat

Continue until convergence

Iterate steps 2-5 until $\Delta L < \varepsilon$

Concrete Example

Training Data:
$x = 2.0$, $y = 1$ (positive class)

Learning Rate:
$\eta = 0.1$

Initial Weight:
$w_0 = 0.5$

After Update:
$w_1 = 0.554$ (↓ Loss: 0.315 → 0.289)

Multiclass - One-vs-Rest Strategy

Extension to K > 2 classes

K Train **K binary classifiers**: Class k vs. all others

For each class: Separate logistic regression model

Architecture

Class 1 vs Rest	Class 2 vs Rest	Class 3 vs Rest
... ...	Class K vs Rest	

↓

Choose class with highest probability

Advantages:

- ✓ Simple
- ✓ Interpretable
- ✓ Parallelizable

Disadvantages:

- ✗ Probabilities may not sum to 1

Works well when classes are well-separated

Softmax Regression

Multinomial Logistic Regression (Native Multiclass)

Softmax Function:

$$P(y=k|x) = e^{w_k^T x} / \sum_j e^{w_j^T x}$$

- Σ Outputs: **K probabilities** that sum to 1
- σ Generalizes sigmoid to multiple classes
- w Each class has its own weight vector $\mathbf{w_k}$
- ✓ More elegant than one-vs-rest for multiclass
- 🧠 Widely used: Neural network output layer

Weight Vectors per Class

- w_1 for Class 1
- w_2 for Class 2
- w_3 for Class 3
- ... ...
- w_K for Class K

Softmax → $\sum P = 1$

vs. Sigmoid

Sigmoid: Binary (2 classes)
Softmax: Multiclass (K classes)

Softmax Regression - Calculation Example

🔢 3-Class Classification Example

1 Given Input & Weights

입력 벡터: **x = [2, 3]**

Weight vectors:

$w_1 = [0.5, 0.3] \rightarrow$ Class 1 (Cat)

$w_2 = [0.8, 0.4] \rightarrow$ Class 2 (Dog)

$w_3 = [0.2, 0.9] \rightarrow$ Class 3 (Bird)

2 Calculate Logits ($z = w^T x$)

$z_1 = w_1^T x = 0.5 \times 2 + 0.3 \times 3 = 1.0 + 0.9 = \mathbf{1.9}$

$z_2 = w_2^T x = 0.8 \times 2 + 0.4 \times 3 = 1.6 + 1.2 = \mathbf{2.8}$

$z_3 = w_3^T x = 0.2 \times 2 + 0.9 \times 3 = 0.4 + 2.7 = \mathbf{3.1}$

3 Apply Exponential Function

$e^{z_1} = e^{1.9} \approx \mathbf{6.686}$

$e^{z_2} = e^{2.8} \approx \mathbf{16.445}$

$e^{z_3} = e^{3.1} \approx \mathbf{22.198}$

$$\text{Sum} = 6.686 + 16.445 + 22.198 = \mathbf{45.329}$$

4 Calculate Softmax Probabilities

$$P(y=1|x) = e^{z_1} / \text{Sum} = 6.686 / 45.329 \approx \mathbf{0.147\ (14.7\%)}$$

$$P(y=2|x) = e^{z_2} / \text{Sum} = 16.445 / 45.329 \approx \mathbf{0.363\ (36.3\%)}$$

$$P(y=3|x) = e^{z_3} / \text{Sum} = 22.198 / 45.329 \approx \mathbf{0.490\ (49.0\%)}$$

✅ Final Result

Prediction: Class 3 (Bird) with 49.0% probability

Note: All probabilities sum to 1.0 (0.147 + 0.363 + 0.490 = 1.000)

Categorical Cross-Entropy

Loss Function for Multiclass Classification

Formula:

$$L = -\sum_k y_k \log(\hat{y}_k)$$

One-hot Encoding:

```
y = [0, 0, 1, 0, ...]
```

for true class

Combined with Softmax:

Softmax + CCE

Differentiable end-to-end

Σ Generalizes binary cross-entropy to **K classes**

⚠ Penalizes deviation from true class distribution

Gradient:

$$\hat{y}_k - y_k$$

(elegant and simple!)

Binary → 2 classes
Categorical → K classes

Regularized Logistic Regression

Prevent Overfitting with Penalty Terms

L2 (Ridge)

$Loss + \lambda \sum w_i^2$

Smooth coefficient shrinkage

L1 (Lasso)

$Loss + \lambda \sum |w_i|$

Feature selection

Elastic Net

$L1 + L2$

Combines L1 and L2 penalties

λ Hyperparameter **λ**: Controls regularization strength

⚠ Especially important with **high-dimensional data**

Choosing λ:

Cross-validation to choose optimal λ

Real-World Cases and Implementation

Applications:

- Spam Detection
- Medical Diagnosis
- Credit Scoring

Scikit-learn Implementation:

```
LogisticRegression(
  penalty='l2',
  C=1.0
)
```

Key Hyperparameters:

- Regularization (C)
- Solver
- max_iter

Evaluation Metrics:

- Accuracy
- Precision
- Recall
- F1-score
- ROC-AUC

Important Considerations:

- **Class imbalance:** Use class_weight='balanced'
- **Feature scaling:** Standardize for better convergence

Best Practices:

- Cross-validation
- Calibration
- Threshold tuning

Thank you

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