

Properties of Logistic Function



Symmetry

$$\sigma(z) + \sigma(-z) = 1$$



Monotonic

Always increasing, never decreases



Smooth

Differentiable everywhere (no jumps)



Bounded

Output always in (0,1), never exactly 0 or 1



Interpretable

Steepness indicates confidence



Derivative

Simple form for optimization

Symmetry Property:

$$\sigma(z) + \sigma(-z) = 1$$

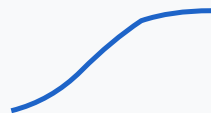
Derivative:

$$\sigma'(z) = \sigma(z)(1 - \sigma(z))$$

(useful for gradient descent)

Comparison to Step Function

Logistic (Smooth)



vs

Step (Hard Threshold)



Smooth transition vs. Abrupt change