Autoregressive (AR)

Core Concept: AR(p)

Predicts current value from previous **p** values
Linear combination of past observations

AR Model Formula

$$X_t = c + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \ldots + \varphi_p X_{t-p} + \varepsilon_t$$

Order Selection

Via AIC/BIC criteria

Stationarity

Assumes stationary time series

Dependencies

Good for linear temporal dependencies

Trend Calculation Example

📌 Sample Data: Monthly Sales

t	t	t	t	t	t
1	2	3	4	5	6
100	110	105	115	120	125

📐 Method 1: Trend using Moving Average

- **3-period Moving Average** calculation: $MA_3 = (Y_{t-1} + Y_t + Y_{t+1}) / 3$
- t=2: **MA = (100+110+105)/3 = 105.0**
- t=3: MA = (110+105+115)/3 = 110.0
- t=4: MA = (105+115+120)/3 = 113.3
- t=5: MA = (115+120+125)/3 = 120.0

📐 Method 2: Trend using Linear Regression

- **Regression equation**: $Y_t = a + b \cdot t$
- Slope $b = \Sigma[(t - \bar{t})(Y_t - \bar{Y})] / \Sigma(t - \bar{t})^2$
- Calculation: $b \approx 5.14$ → **Average increase of 5.14 per month**
- Intercept $a = \bar{Y} - b \cdot \bar{t} \approx 94.86$

- Trend equation: $Y_t = 94.86 + 5.14 \cdot t$

Forecasting Using Trend

Trend-Based Forecasting

Use the estimated trend equation to predict future values

📈 Method 1: Forecasting with Moving Average

- **Last calculated MA** at t=5: MA = 120.0
- **Trend rate**: Calculate average change between consecutive MAs
- (110.0 - 105.0) + (113.3 - 110.0) + (120.0 - 113.3) = 15.0
- Average change ≈ 15.0 / 3 = **5.0 per period**
- **Forecast for t=7**: 125 + 5.0 = **130.0**
- **Forecast for t=8**: 130.0 + 5.0 = **135.0**

📈 Method 2: Forecasting with Linear Regression (Recommended)

- **Use trend equation**: $Y_t = 94.86 + 5.14 \cdot t$
- **Forecast for t=7**: $Y_7 = 94.86 + 5.14 \times 7 =$ **130.84**
- **Forecast for t=8**: $Y_8 = 94.86 + 5.14 \times 8 =$ **135.98**
- **Forecast for t=9**: $Y_9 = 94.86 + 5.14 \times 9 =$ **141.12**
- **Forecast for t=10**: $Y_{10} = 94.86 + 5.14 \times 10 =$ **146.26**

Summary: Forecasted Values

Period	t=7	t=8	t=9	t=10
Forecast	130.84	135.98	141.12	146.26