Properties of Logistic Function

Symmetry

σ(z) + σ(-z) = 1

Monotonic

Always increasing, never decreases

Smooth

Differentiable everywhere (no jumps)

Bounded

Output always in (0,1), never exactly 0 or 1

Interpretable

Steepness indicates confidence

Derivative

Simple form for optimization

Symmetry Property:

$$\sigma(z) + \sigma(-z) = 1$$

Derivative:

$$\sigma'(z) = \sigma(z)(1 - \sigma(z))$$

(useful for gradient descent)

Comparison to Step Function

Logistic (Smooth)

vs

Step (Hard Threshold)

Smooth transition vs. Abrupt change