Hypothesis Testing and Confidence Intervals

Hypothesis Testing

H_0: Null Hypothesis

Default assumption

e.g., β = 0

H_1: Alternative Hypothesis

Claim to test

e.g., β ≠ 0

Test Statistic

Measure computed from data (t-stat, z-stat)

p-value

P(observe data or more extreme | H_0 true)

Decision Process

Set significance level α
(typically 0.05)

↓

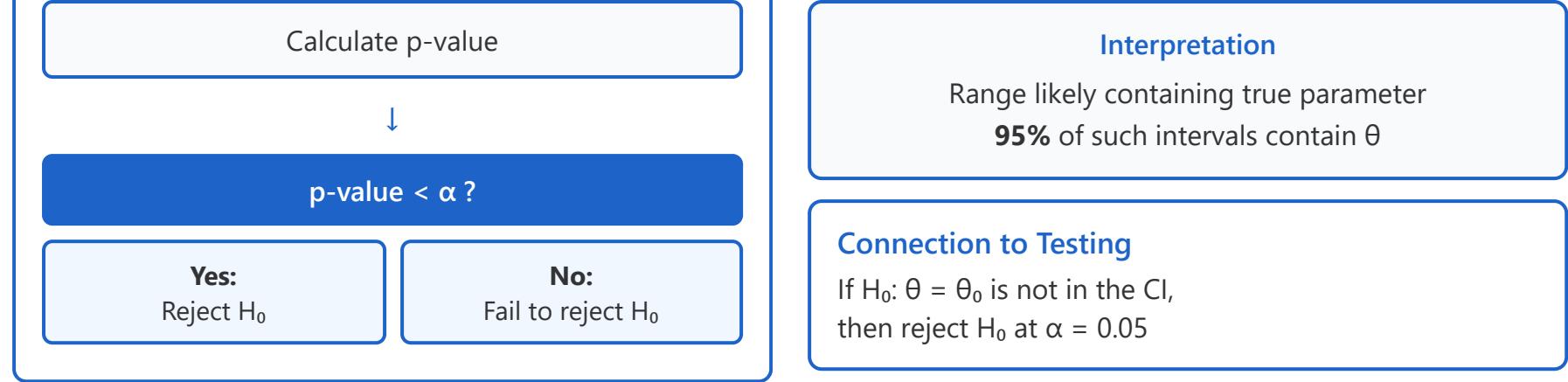Compute test statistic

↓

Confidence Intervals

95% Confidence Interval

$\hat{\theta}$ - 1.96SE $\hat{\theta}$ $\hat{\theta}$ + 1.96SE

[θ^- 1.96SE, θ^+ 1.96SE]

Calculate p-value

↓

p-value < α ?

Yes:
Reject H_0

No:
Fail to reject H_0

Interpretation
Range likely containing true parameter
95% of such intervals contain θ

Connection to Testing
If H_0: $\theta = \theta_0$ is not in the CI,
then reject H_0 at $\alpha = 0.05$

Real-World Examples: Hypothesis Testing

Case 1: Drug Efficacy

A pharmaceutical company tests whether a new drug reduces blood pressure more than placebo.

H_0: $\mu\_drug = \mu\_placebo$
H_1: $\mu\_drug < \mu\_placebo$

Data:
n = 100 patients
Mean difference = -8.5 mmHg
SE = 2.1 mmHg
t-stat = -4.05

p-value: 0.0001
α level: 0.05

Reject H_0
Drug is effective!

Case 2: A/B Testing

An e-commerce site tests whether a new checkout design increases conversion rate.

H_0: p_new = p_old
H_1: p_new ≠ p_old

Data:
Control: 450/10000 = 4.5%
Treatment: 520/10000 = 5.2%
z-stat = 2.28

p-value: 0.023
α level: 0.05

Reject H_0
New design works!

Case 3: Salary Regression

Testing whether years of education significantly predict salary (β_1 coefficient).

H_0: $\beta_1 = 0$
H_1: $\beta_1 \neq 0$

Data:
n = 500 employees
β_1 = 5,200 ($/year)
SE(β_1) = 800
t-stat = 6.5

p-value: < 0.001
95% CI: [3,632, 6,768]

Reject H_0
Education matters!