

# Eigenvalues and Eigenvectors

## Eigenvector $v$

$$Av = \lambda v$$

Direction unchanged by matrix  $A$

## Eigenvalue $\lambda$

Scaling factor for the eigenvector

## Characteristic Equation

$$\det(A - \lambda I) = 0$$

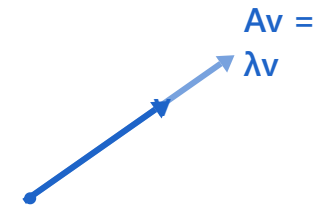
## Number of Eigenvalues

$n \times n$  matrix has  $n$  eigenvalues  
(counting multiplicity)

## Spectral Theorem

Symmetric matrices have  
orthogonal eigenvectors

## Geometric Interpretation



Matrix  $A$  **stretches**  $v$  by factor  $\lambda$   
but keeps the **same direction**

## Eigendecomposition

$$A = Q\Lambda Q^T$$

For symmetric matrix  $A$   
 $Q$ : orthogonal eigenvectors  
 $\Lambda$ : diagonal matrix of eigenvalues

## ML Applications

- Understanding data variance and covariance structure
- PCA (Principal Component Analysis)
- Regression diagnostics use eigenanalysis

## Example: Finding Eigenvalues and Eigenvectors

### Given Matrix A

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

**Step 1:** Find eigenvalues using  $\det(A - \lambda I) = 0$

$$\det \begin{pmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{pmatrix} = 0$$

$$\begin{aligned} (4-\lambda)(3-\lambda) - 2 &= 0 \\ \lambda^2 - 7\lambda + 10 &= 0 \\ (\lambda-5)(\lambda-2) &= 0 \end{aligned}$$

$$\lambda_1 = 5, \lambda_2 = 2$$

### Find Eigenvectors

**For  $\lambda_1 = 5$ :** Solve  $(A - 5I)v = 0$

$$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-v_1 + v_2 = 0 \rightarrow v_2 = v_1$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

**For  $\lambda_2 = 2$ :** Solve  $(A - 2I)v = 0$

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2v_1 + v_2 = 0 \rightarrow v_2 = -2v_1$$

$$v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$