Matrix Operations and Properties

Matrix $A \in \mathbb{R}^{m \times n}$

Rectangular array of numbers

$$\begin{bmatrix} a_{1\,1} & a_{1\,2} & \cdots & a_{1\,n} \\ a_{2\,1} & a_{2\,2} & \cdots & a_{2\,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Matrix-Vector Multiplication

Ax represents linear transformation

Matrix Multiplication

$(AB)_{ij} = \Sigma_k A_{ik} B_{kj}$

Transpose

A^T swaps rows and columns

Symmetric Matrix

$A = A^T$ (important in regression)

Identity Matrix

Matrix Multiplication Example

$$\underset{m \times k}{A} \times \underset{k \times n}{B} = \underset{m \times n}{AB}$$

Identity Matrix I

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Key Properties

$$(AB)^T = B^T A^T$$

$$(AB)C = A(BC)$$

ML Application

Matrices encode systems of linear equations

$AI = IA = A$