

## Inner Product and Orthogonality

### Inner Product (Dot Product)

$$\langle x, y \rangle = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

### Geometric Interpretation

$$\langle x, y \rangle = \|x\| \|y\| \cos(\theta)$$

Measures alignment between vectors

### Norm (Length)

$$\|x\| = \sqrt{(\langle x, x \rangle)} = \sqrt{(x_1^2 + x_2^2 + \dots + x_n^2)}$$

### Orthogonality

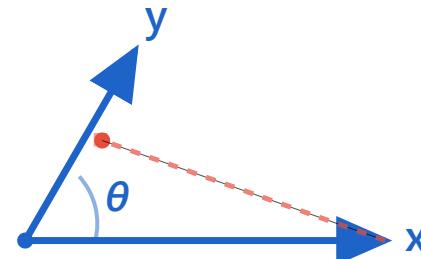
$$x \perp y \text{ when } \langle x, y \rangle = 0$$

Vectors are perpendicular ( $\theta = 90^\circ$ )

### Orthonormal Basis

Basis vectors with unit length, mutually orthogonal

### Geometric Interpretation



Inner product  $\langle x, y \rangle$  measures how much x "projects onto" y  
Dashed line shows the projection of x onto y

Key for Regression

## Projection

$$\text{proj}_y(x) = (\langle x, y \rangle / \langle y, y \rangle) \cdot y$$

Component of x in direction of y

- Projecting data onto subspaces
- Residuals are orthogonal to fitted values
- Minimizing distance = maximizing projection