

# Eigenvalues and Eigenvectors

## Eigenvector $v$

$$Av = \lambda v$$

Direction unchanged by matrix A

## Eigenvalue $\lambda$

Scaling factor for the eigenvector

## Characteristic Equation

$$\det(A - \lambda I) = 0$$

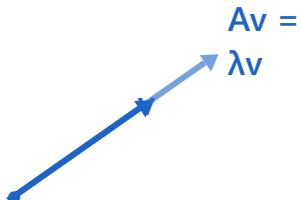
## Number of Eigenvalues

$n \times n$  matrix has  $n$  eigenvalues  
(counting multiplicity)

## Spectral Theorem

Symmetric matrices have  
orthogonal eigenvectors

## Geometric Interpretation



Matrix A **stretches**  $v$  by factor  $\lambda$   
but keeps the **same direction**

## Eigendecomposition

$$A = Q\Lambda Q^T$$

For symmetric matrix A  
Q: orthogonal eigenvectors  
 $\Lambda$ : diagonal matrix of eigenvalues

## ML Applications

- Understanding data variance and covariance structure
- PCA (Principal Component Analysis)
- Regression diagnostics use eigenanalysis

## Example: Finding Eigenvalues and Eigenvectors

### Given Matrix A

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

**Step 1:** Find eigenvalues using  $\det(A - \lambda I) = 0$

$$\det([4-\lambda \ 1] [2 \ 3-\lambda]) = 0$$

$$(4-\lambda)(3-\lambda) - 2 = 0$$
$$\lambda^2 - 7\lambda + 10 = 0$$
$$(\lambda-5)(\lambda-2) = 0$$

$$\lambda_1 = 5, \lambda_2 = 2$$

### Find Eigenvectors

For  $\lambda_1 = 5$ : Solve  $(A - 5I)v = 0$

$$[-1 \ 1] [v_1] = [0]$$

$$[2 \ -2] [v_2] = [0]$$

$$-v_1 + v_2 = 0 \rightarrow v_2 = v_1$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For  $\lambda_2 = 2$ : Solve  $(A - 2I)v = 0$

$$[2 \ 1] [v_1] = [0]$$

$$[2 \ 1] [v_2] = [0]$$

$$2v_1 + v_2 = 0 \rightarrow v_2 = -2v_1$$

$$v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$