

## Dynamic Time Warping

DTW Algorithm



Warps time axis to align sequences



Finds optimal alignment path



Allows one-to-many matching



DTW distance = sum of aligned distances



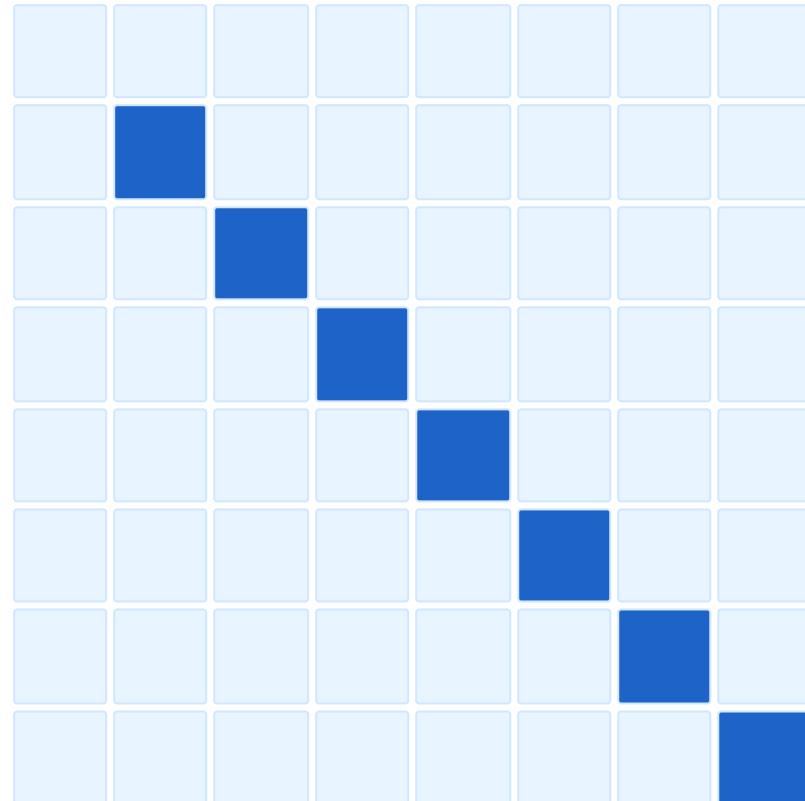
Handles speed variations & phase shifts

Time Complexity

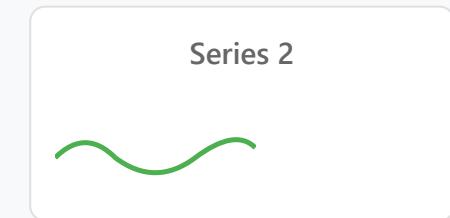
$O(n^2)$

## DTW Alignment Matrix & Warping Path

Series 1



Series 2



Alignment Constraints

Sakoe-Chiba Band

Itakura Parallelogram

# DTW Calculation Example

Step-by-Step Computation

Sequence X

1 3 4 5

Sequence Y

2 3 5

📊 DTW Matrix (Cumulative Cost)

	0	2	3	5
0	$ 1-2 $ 1	$\infty$	$\infty$	$\infty$
1	$ 3-2 $ 2	$ 3-3 $ 2	$\infty$	$\infty$
3	$ 4-2 $ 4	$ 4-3 $ 3	$ 4-5 $ 4	$\infty$
4	$ 5-2 $ 7	$ 5-3 $ 5	$ 5-5 $ 4	4

12  
34

Calculation Steps

- 1 Initialize first cell with cost  $|x_1-y_1|$

$$D(1,1) = |1-2| = 1$$

- 2 Fill remaining cells using:

$$D(i,j) = \text{cost}(i,j) + \min(D(i-1,j), D(i,j-1), D(i-1,j-1))$$

- 3 Example:  $D(2,2)$

$$|3-3| + \min(1, 2, 1) = 0 + 2 = 2$$

- 4 Continue until reaching  $D(n,m)$

5

Backtrack from end to find optimal path

Final DTW Distance

4