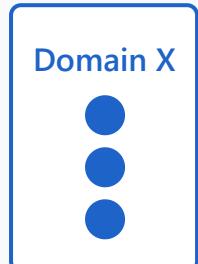


## Functions and Mapping Concepts

Function:  $f: X \rightarrow Y$



Each input maps to **exactly one** output

**Domain X:** Set of all possible inputs

**Codomain Y:** Set of all possible outputs

**Range:** Actual outputs achieved by f

**One-to-one (Injective)**

Different inputs → Different outputs  
No two inputs map to the same output

**Onto (Surjective)**

Every output is reached  
Range = Codomain

**Bijection**

Both injective and surjective  
Perfect one-to-one correspondence

**ML Application**

Regression models are functions

$$f(x) = y$$

## Function Mapping Examples

**Natural Number Division**

$$f: \mathbb{N} \times \mathbb{N} \setminus \{0\} \rightarrow \mathbb{Q}$$

- $f(6, 2) = 3$
- $f(7, 2) = 3.5$

**Square Function**

$$f: \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$$

- $f(-3) = 9$
- $f(0) = 0$

**Absolute Value**

$$f: \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$$

- $f(-5) = 5$
- $f(0) = 0$

- $f(10, 4) = 2.5$

✓ Well-defined function

Each pair maps to unique rational

- $f(4) = 16$

✓ Surjective onto non-negative reals

✗ Not injective ( $\pm x \rightarrow$  same output)

- $f(3) = 3$

✓ Surjective onto non-negative reals

✗ Not injective ( $\pm x \rightarrow$  same output)