

Regression Loss Functions Comparison

MSE

Mean Squared Error (L2)

✓ Strong penalty for large errors

✗ Sensitive to outliers

Characteristics from squaring operation

Huber Loss

Hybrid Approach

✓ Combines MSE and MAE advantages

✓ Small errors: Quadratic function

✓ Large errors: Linear function

MAE

Mean Absolute Error (L1)

✓ Robust to outliers

Uniform penalty regardless of error size



Use MSE when

Outliers are important

Use Huber when

Balanced approach is needed

Use MAE when

Robustness is required



Calculation Examples with 5 Data Points

Sample	Actual (y)	Predicted (\hat{y})	Error (y - \hat{y})
1	100	98	2
2	150	153	-3

Sample	Actual (y)	Predicted (\hat{y})	Error (y - \hat{y})
3	200	199	1
4	180	182	-2
5	120	140	-20

MSE Calculation

Formula: $MSE = (1/n)\sum(y - \hat{y})^2$

Steps:

- $(2)^2 = 4$
- $(-3)^2 = 9$
- $(1)^2 = 1$
- $(-2)^2 = 4$
- $(-20)^2 = \mathbf{400}$

Sum = 418

MSE = $418 \div 5$

MSE = 83.6

Huber Loss ($\delta=5$)

Formula:

If $|\text{error}| \leq \delta$: $0.5 \times \text{error}^2$

If $|\text{error}| > \delta$: $\delta \times (|\text{error}| - 0.5\delta)$

Steps:

- $|2| \leq 5$: $0.5 \times 4 = 2$
- $|3| \leq 5$: $0.5 \times 9 = 4.5$
- $|1| \leq 5$: $0.5 \times 1 = 0.5$
- $|2| \leq 5$: $0.5 \times 4 = 2$
- $|20| > 5$: $5 \times (20 - 2.5) = \mathbf{87.5}$

Sum = 96.5

Huber = $96.5 \div 5$

Huber = 19.3

MAE Calculation

Formula: $MAE = (1/n)\sum|y - \hat{y}|$

Steps:

- $|2| = 2$
- $|-3| = 3$
- $|1| = 1$
- $|-2| = 2$
- $|-20| = \mathbf{20}$

Sum = 28

MAE = $28 \div 5$

MAE = 5.6

💡 Key Insight:

Notice sample #5 with error of -20 (outlier): **MSE = 83.6** (heavily penalized), **Huber = 19.3** (balanced penalty), **MAE = 5.6** (moderate penalty). Huber Loss provides a middle ground between MSE and MAE!

