Mathematical Properties

Part 2/7: Forward Process

📊 Mean Preservation

$$E[x_t] = \sqrt{\bar{\alpha}_t} \cdot x_0$$

Expected value scaled by $\sqrt{\bar{\alpha}_t}$

📈 Variance Growth

$$Var[x_t] = (1-\bar{\alpha}_t) \cdot I$$

Variance increases as noise accumulates

🔔 Gaussian Distribution

$$q(x_t|x_0) = N(x_t; \sqrt{\bar{\alpha}_t} \cdot x_0, (1-\bar{\alpha}_t) \cdot I)$$

Complete distributional characterization

🔙 Posterior Distribution

$$q(x_{t-1}|x_t, x_0)$$

Also Gaussian with closed-form solution

🧮 Tractable Likelihood

Can compute exact log-likelihood for model evaluation and comparison

🔄 Reversibility

Theoretical foundation enables the reverse generative process

🎲 Noise Independence

Noise at different timesteps is independent, simplifying analysis

💡 All properties maintain Gaussian structure, enabling tractable inference and generation