

Inner Product and Orthogonality

Inner Product (Dot Product)

$$\langle x, y \rangle = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

Geometric Interpretation

$$\langle x, y \rangle = \|x\| \|y\| \cos(\theta)$$

Measures alignment between vectors

Norm (Length)

$$\|x\| = \sqrt{\langle x, x \rangle} = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

Orthogonality

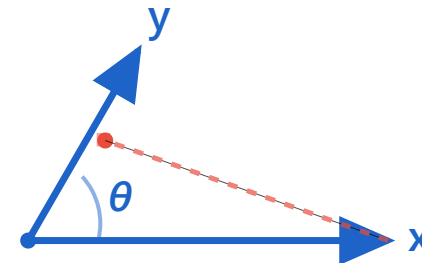
$$x \perp y \text{ when } \langle x, y \rangle = 0$$

Vectors are perpendicular ($\theta = 90^\circ$)

Orthonormal Basis

Basis vectors with unit length, mutually orthogonal

Geometric Interpretation



Inner product $\langle x, y \rangle$ measures how much x "projects onto" y
Dashed line shows the projection of x onto y

Key for Regression

Projection

$$\text{proj}_y(\mathbf{x}) = (\langle \mathbf{x}, \mathbf{y} \rangle / \langle \mathbf{y}, \mathbf{y} \rangle) \cdot \mathbf{y}$$

Component of \mathbf{x} in direction of \mathbf{y}

- Projecting data onto subspaces
- Residuals are orthogonal to fitted values
- Minimizing distance = maximizing projection