Central Limit Theorem and Law of Large Numbers

Law of Large Numbers

Sample mean converges to population mean

$$\bar{X}_n \to \mu \text{ as } n \to \infty$$

Convergence in probability

Central Limit Theorem

Sum of random variables approaches normal distribution

$$\sqrt{n}(\bar{X}_n - \mu) / \sigma \to N(0,1)$$

as $n \to \infty$

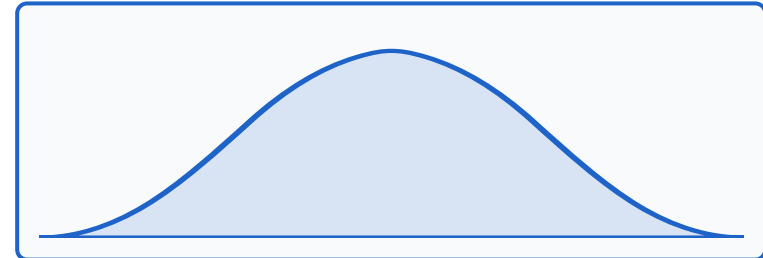## CLT: Any Distribution → Normal Distribution

Original Distribution
(any shape)

Distribution of Means
(normal)

$\to$

Take means
$n \to \infty$

Key Insight
Averages are approximately normal with large n

Regression Use
Justifies normal assumption in residuals

Enables
Confidence intervals & hypothesis tests

Real-World Applications

Quality Control

A/B Testing

Opinion Polls

Manufacturing defect rates follow normal distribution with large samples.

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

Use: *Setting acceptable quality limits in production lines*

Website conversion rates comparison uses CLT for statistical significance.

$$\hat{p} \sim N(p, p(1-p)/n)$$

Use: *Deciding which website design performs better*

Survey results become normally distributed with sufficient sample size.

$$\text{Margin of Error} = 1.96 \times \sigma/\sqrt{n}$$

Use: *Predicting election outcomes with confidence intervals*

Financial Risk

Portfolio returns approximate normal distribution over time.

$$R\_portfolio \sim N(\mu\_p, \sigma\_p^2)$$

Use: *Value-at-Risk (VaR) calculations for investments*

Medical Trials

Average treatment effects tested using normal approximation.

$$t = (\bar{X} - \mu_0)/(s/\sqrt{n})$$

Use: *Determining if new drugs are effective*

Machine Learning

Bootstrap confidence intervals rely on CLT for model evaluation.

$$\text{Accuracy} \sim N(\mu\_acc, \sigma^2\_acc/n)$$

Use: *Estimating model performance reliability*