

# Image Registration

## Rigid vs non-rigid

Translation/rotation vs deformation

## Similarity metrics

Mutual information, correlation

## Optimization methods

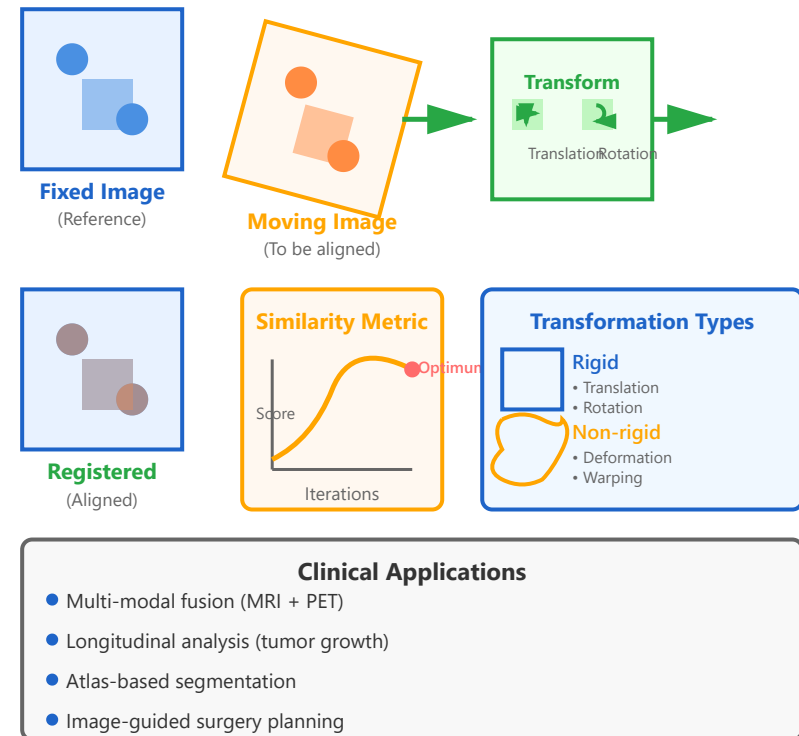
Gradient descent, genetic algorithms

## Multi-modal registration

Aligning different imaging modalities

## Validation approaches

Fiducial markers, Dice coefficient



# 1. Rigid vs Non-rigid Transformations



## Rigid Transformations

Rigid transformations preserve the shape and size of objects, only changing their position and orientation in space. These are the simplest form of geometric transformations.

### Key Characteristics:

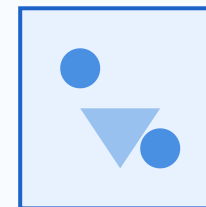
- **Translation:** Moving the image along x, y, or z axes
- **Rotation:** Rotating the image around specific axes
- **Preservation:** Maintains distances, angles, and volumes
- **Degrees of Freedom:** 6 DOF in 3D (3 translations + 3 rotations)

**Clinical Use:** Ideal for bone registration, brain registration within the same patient, and aligning images from the same imaging session.

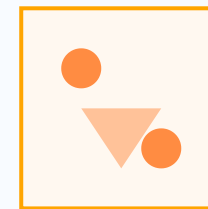
### Advantages:

- Computationally efficient and fast
- Fewer parameters to optimize
- Less prone to unrealistic deformations

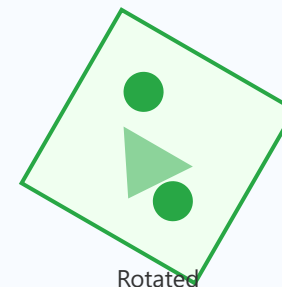
### Rigid Registration



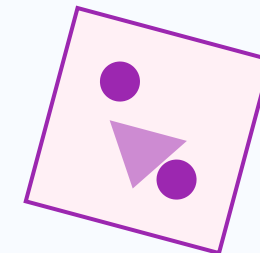
Original



Translated  
(Shifted 20px right, down)



Rotated  
(30° clockwise)



Combined  
(Translation + Rotation)

✓ Shape and size preserved in all transformations



## Non-rigid Transformations

Non-rigid (deformable) transformations allow local deformations, enabling different parts of the image to move independently. This is essential for registering soft tissues and organs that change shape.

### Key Characteristics:

- **Deformation:** Local warping of image regions
- **Flexibility:** Can model breathing, cardiac motion, soft tissue changes
- **Complexity:** Many degrees of freedom (potentially millions)
- **Control:** Regularization needed to prevent unrealistic deformations

**Clinical Use:** Soft tissue registration, cardiac imaging, respiratory motion compensation, tumor tracking during treatment.

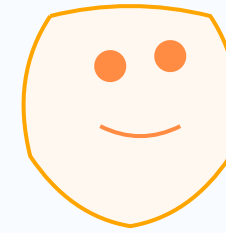
### Common Algorithms:

- B-spline free-form deformation
- Optical flow methods
- Diffeomorphic demons
- Large deformation diffeomorphic metric mapping (LDDMM)

## Non-rigid Registration

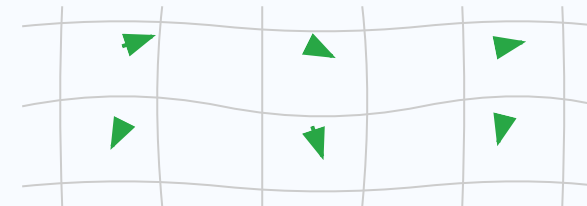


Reference  
(e.g., exhale state)



Moving  
(e.g., inhale state)

### Deformation Field



Each point moves independently  
to match the target shape

## 2. Similarity Metrics



### Measuring Image Alignment Quality

Similarity metrics quantify how well two images are aligned. The choice of metric depends on the imaging modalities and the expected relationship between intensity values.

#### 1. Sum of Squared Differences (SSD)

Measures the squared difference between corresponding pixel intensities. Works best for mono-modal registration (same imaging technique).

- Formula:  $SSD = \sum (I_1(x) - I_2(x))^2$
- Lower values indicate better alignment
- Fast to compute but sensitive to intensity variations

#### 2. Normalized Cross-Correlation (NCC)

Measures the linear relationship between image intensities. Robust to linear intensity differences.

- Values range from -1 to 1 (higher is better)
- Invariant to linear intensity transformations
- Good for mono-modal registration

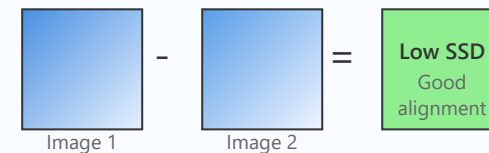
#### 3. Mutual Information (MI)

Measures statistical dependence between images. The gold standard for multi-modal registration.

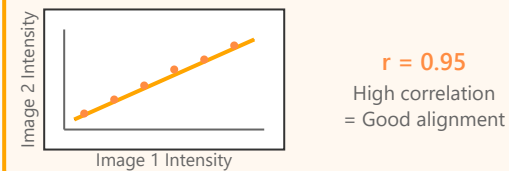
- Based on joint probability distributions

### Similarity Metrics Comparison

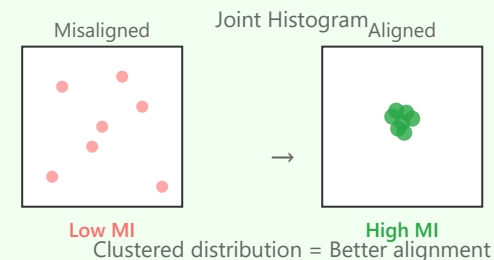
#### Sum of Squared Differences (SSD)



#### Normalized Cross-Correlation (NCC)



#### Mutual Information (MI)



- Works even when intensity relationships are non-linear
- Essential for MRI-CT, PET-MRI registration

**Key Insight:** MI can detect alignment even when the same anatomical structure appears bright in one image and dark in another.

# 3. Optimization Methods

## Finding the Best Transformation

Optimization methods search for transformation parameters that maximize the similarity metric. Different algorithms balance speed, accuracy, and robustness.

### 1. Gradient Descent

Iteratively moves in the direction of steepest improvement of the similarity metric.

- **Pros:** Fast convergence, well-understood, easy to implement
- **Cons:** Can get stuck in local minima
- **Variants:** Stochastic gradient descent, Adam optimizer
- **Use case:** Initial alignment, mono-modal registration

### 2. Powell's Method

Direction-set method that doesn't require gradient computation.

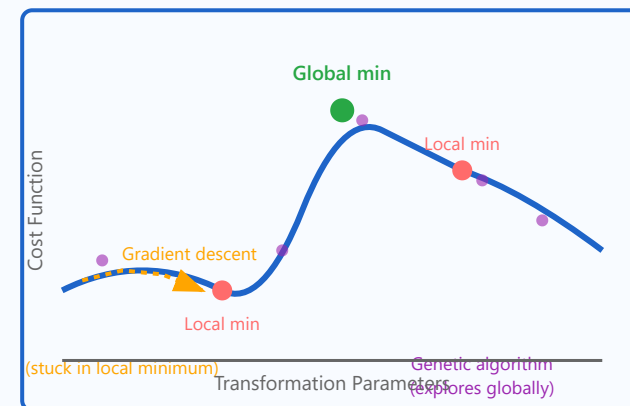
- Sequentially optimizes along different directions
- Good for metrics with discontinuous derivatives
- More robust but slower than gradient descent

### 3. Genetic Algorithms

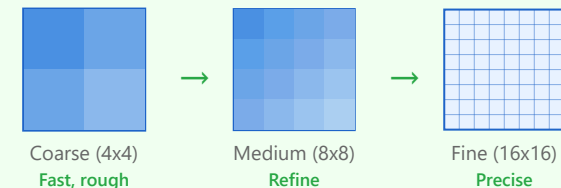
Population-based search inspired by biological evolution.

- **Pros:** Global optimization, handles multimodal landscapes

### Optimization Landscape



### Multi-resolution Strategy



#### Benefits:

- Avoids local minima at coarse scales
- Faster convergence (fewer iterations needed)
- More robust to initial misalignment
- Progressively refines the solution

- **Cons:** Computationally expensive
- **Use case:** Complex multi-modal registration with many local optima

**Multi-resolution Strategy:** Start optimization at coarse resolution and progressively refine at finer resolutions. This prevents local minima and speeds up convergence.

#### 4. L-BFGS (Limited-memory BFGS)

Quasi-Newton method that approximates the inverse Hessian matrix.

- Better convergence than gradient descent
- Memory-efficient for high-dimensional problems
- Popular for deformable registration

## 4. Multi-modal Registration



### Aligning Different Imaging Modalities

Multi-modal registration aligns images from different imaging techniques (MRI, CT, PET, ultrasound) where the same anatomy appears with different intensities and contrasts.

### Why Multi-modal Registration?

- **Complementary information:** Each modality reveals different tissue properties
- **Treatment planning:** Combine anatomical detail (CT/MRI) with functional data (PET/SPECT)
- **Diagnosis:** Correlate structural and metabolic abnormalities

### Common Multi-modal Pairs

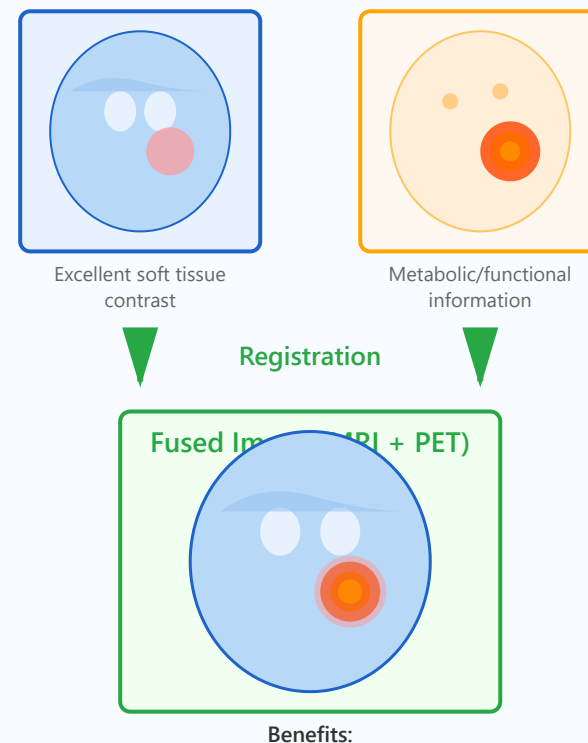
**MRI-CT:** MRI provides superior soft tissue contrast, CT shows bone structure and electron density for radiation therapy planning.

**PET-CT:** PET reveals metabolic activity (tumor detection, staging), CT provides anatomical reference and attenuation correction.

**MRI-PET:** Combines high-resolution anatomy with metabolic information for neuroimaging and oncology.

**Ultrasound-MRI:** Used in image-guided interventions, combining real-time ultrasound with pre-operative MRI.

### Multi-modal Registration Example



#### Why Mutual Information Works

Tumor appears BRIGHT on MRI (hyperintense)  
Tumor appears BRIGHT on PET (high uptake)

MI detects this co-occurrence pattern!



**Challenge:** Intensity values have no direct correspondence between modalities. Mutual Information is essential as it captures statistical dependencies rather than linear relationships.

### Pre-processing Steps

- Resampling to common resolution
- Intensity normalization or histogram matching
- Brain extraction or organ segmentation
- Removing artifacts specific to each modality

# 5. Validation Approaches

## ✓ Ensuring Registration Accuracy

Validating registration accuracy is critical for clinical applications. Different methods provide quantitative and qualitative assessment of alignment quality.

### 1. Fiducial Markers (Gold Standard)

Physical markers placed on or attached to the patient that are visible in both images.

- **Types:** Skin markers, bone-implanted markers, vitamin E capsules
- **Metric:** Target Registration Error (TRE) - distance between corresponding markers after registration
- **Pros:** Direct, interpretable measure of accuracy
- **Cons:** Invasive, marker placement errors, not always feasible

### 2. Dice Similarity Coefficient (DSC)

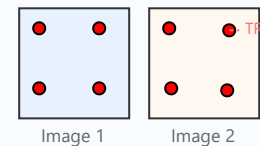
Measures overlap between corresponding anatomical structures or segmentations.

- **Formula:**  $DSC = \frac{2|A \cap B|}{(|A| + |B|)}$
- **Range:** 0 (no overlap) to 1 (perfect overlap)
- **Use:** Organ alignment validation, tumor tracking
- **Threshold:**  $DSC > 0.7$  often considered good alignment

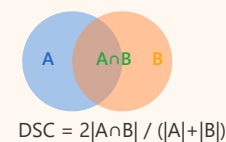
### 3. Hausdorff Distance

## Validation Methods

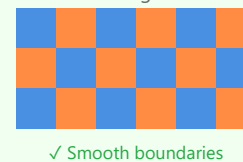
### Fiducial Markers



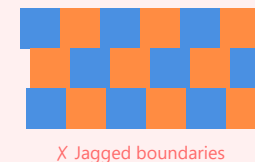
### Dice Coefficient



### Checkerboard View



### Poor Alignment



## Validation Metrics Comparison

Method	Advantages	Limitations
Fiducials	Gold standard	Invasive, costly
Dice/DSC	Quantitative overlap	Needs segmentation
Hausdorff	Edge sensitivity	Outlier sensitive
Visual	Intuitive, fast	Subjective
Landmarks	Non-invasive	Observer variability

**Recommendation: Use multiple methods**  
Combine quantitative metrics with visual inspection for comprehensive validation

Measures maximum distance between surface points of corresponding structures.

- Sensitive to outliers and local misalignments
- Useful for detecting edge mismatches
- Often used alongside DSC for comprehensive assessment

**Clinical Relevance:** For radiation therapy, TRE should be  $< 2\text{mm}$ . For surgical navigation, sub-millimeter accuracy may be required.

#### 4. Visual Inspection

Expert review using visualization tools:

- **Checkerboard:** Alternating tiles from both images
- **Overlay:** Semi-transparent overlay with adjustable opacity
- **Contour comparison:** Edge overlays to check alignment
- **Side-by-side:** Synchronized scrolling through both volumes

#### 5. Landmark-based Validation

Anatomical landmarks identified by experts in both images.

- Less invasive than fiducial markers
- Subject to inter-observer variability
- Used when physical markers not available