

Semantics

Outline

What is semantics?

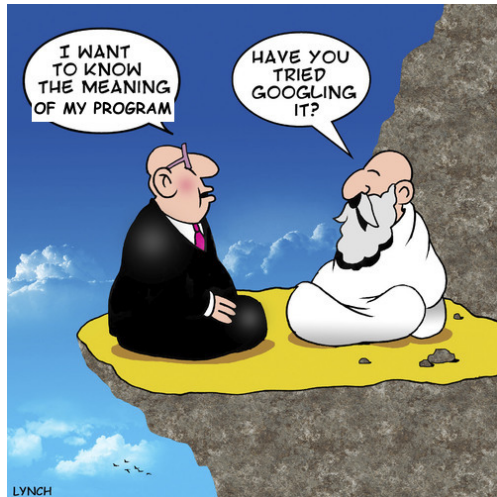
Denotational semantics

Semantics of naming

What is the meaning of a program?

Recall: aspects of a language

- **syntax**: the structure of its programs
- **semantics**: the meaning of its programs



How to define the meaning of a program?

Formal specifications

- **denotational semantics**: relates terms directly to values
- **operational semantics**: describes how to evaluate a term
- **axiomatic semantics**: describes the effects of evaluating a term
- ...

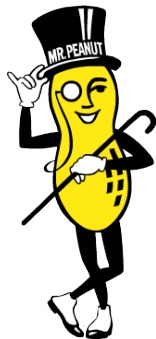
Informal/non-specifications

- **reference implementation**: execute/compile program in some implementation
- **community/designer intuition**: how people “think” a program should behave

Advantages of a formal semantics

A formal semantics ...

- is **simpler** than an implementation, **more precise** than intuition
 - can answer: is this implementation correct?
- supports the definition of analyses and transformations
 - prove properties about the language
 - prove properties about programs
- promotes better language design
 - better understand impact of design decisions
 - apply semantic insights to improve the language design (e.g. *compositionality*)



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Denotational semantics

A denotational semantics relates each **term** to a **denotation**

an abstract syntax tree



a value in some
semantic domain



Valuation function

$\llbracket \cdot \rrbracket : \text{abstract syntax} \rightarrow \text{semantics domain}$

Valuation function in Haskell

`sem :: Term -> Value`

Semantic domains

Semantic domain: captures the set of possible meanings of a program/term

what is a meaning? – it depends on the language!

Example semantic domains

Language	Meaning
Boolean expressions	Boolean value
Arithmetic expressions	Integer
Imperative language	State transformation
SQL query	Set of relations
MiniLogo program	Drawing

Defining a language with denotational semantics

Example encoding in Haskell:

1. Define the **abstract syntax**, T
the set of abstract syntax trees
2. Identify or define the **semantics domain**, V
the representation of semantic values
3. Define the **valuation function**, $\llbracket \cdot \rrbracket : T \rightarrow V$
the mapping from ASTs to semantic values

```
data Term = ...
```

```
type Value = ...
```

```
sem :: Term -> Value
```

Example: simple arithmetic expressions

1. Define abstract syntax

```
data Exp = Add Exp Exp
         | Mul Exp Exp
         | Neg Exp
         | Lit Int
```

2. Identify semantic domain

Use the set of all integers, **Int**

3. Define the valuation function

```
sem :: Exp -> Int
sem (Add l r) = sem l + sem r
sem (Mul l r) = sem l * sem r
sem (Neg e)   = negate (sem e)
sem (Lit n)   = n
```

Desirable properties of a denotational semantics

Compositionality: a program's denotation is built from the denotations of its parts

- supports modular reasoning, extensibility
- supports proof by structural induction

Completeness: every value in the semantics domain is denoted by some program

- ensures that semantics domain and language align
- if not, language has expressiveness gaps, or semantics domain is too general

Soundness: two programs are “equivalent” iff they have the same denotation

- equivalence: have the same observable effects
- ensures that the denotational semantics is correct

More on compositionality

Compositionality: a program's denotation is built from the denotations of its parts

an AST 

sub-ASTs 

Example: What is the meaning of **op** e_1 e_2 e_3 ?

1. Determine the meaning of e_1 , e_2 , e_3
2. Combine these submeanings in some way specific to **op**

Implications:

- The valuation function is probably **recursive**
- We need different valuation functions for **each syntactic category** (type of AST)

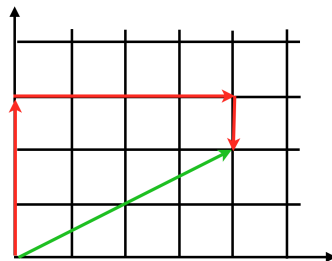
Example: move language

A language describing movements on a 2D plane

- a **step** is an n -unit horizontal or vertical movement
- a **move** is described by a sequence of steps

Abstract syntax

```
data Dir  = N | S | E | W
data Step = Go Dir Int
type Move = [Step]
```



```
[Go N 3, Go E 4, Go S 1]
```

Semantics of move language

1. Abstract syntax

```
data Dir  = N | S | E | W
data Step = Go Dir Int
type Move = [Step]
```

2. Identify semantic domain

```
type Pos = (Int,Int)
```

Domain: **Pos** -> **Pos**

3. Valuation function (**Step**)

```
step :: Step -> Pos -> Pos
step (Go N k) = \ (x,y) -> (x,y+k)
step (Go S k) = \ (x,y) -> (x,y-k)
step (Go E k) = \ (x,y) -> (x+k,y)
step (Go W k) = \ (x,y) -> (x-k,y)
```

3. Valuation function (**Move**)

```
move :: Move -> Pos -> Pos
move []      = \p -> p
move (s:m) = move m . step s
```

Alternative semantics

Often multiple **interpretations** (semantics) of the same language

Example: Database schema

One declarative spec, used to:

- initialize the database
- generate APIs
- validate queries
- normalize layout
- ...

Distance traveled

```
type Dist = Int
dstep :: Step -> Int
dstep (Go _ k) = k

dmove :: Move -> Int
dmove []      = 0
dmove (s:m)   = dstep s + dmove m
```

Combined trip information

```
trip :: Move -> Pos -> (Dist, Pos)
trip m = \p -> (dmove m, move m p)
```

Picking the right semantic domain

Simple semantics domains can be combined in two ways:

- **product**: contains a value from both domains
 - e.g. combined trip information for move language
 - use Haskell **(a, b)** or define a new data type
- **sum**: contains a value from one domain or the other
 - e.g. IntBool language can evaluate to **Int** or **Bool**
 - use Haskell **Either a b** or define a new data type

Can errors occur?

- use Haskell **Maybe a** or define a new data type

Does the language manipulate state or use naming?

- use a **function type**

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What is naming?

Most languages provide a way to **name** and **reuse** stuff

Naming concepts

declaration	introduce a new name
binding	associate a name with a thing
reference	use the name to stand for the bound thing

C/Java variables

```
int x; int y;  
x = slow(42);  
y = x + x + x;
```

In Haskell:

Local variables

```
let x = slow 42  
in x + x + x
```

Type names

```
type Radius = Float  
data Shape = Circle Radius
```

Function parameters

```
area r = pi * r * r
```

Semantics of naming

Environment: a map associating names with things

type `Env = [(Name,Thing)]`

Naming concepts

declaration	add a new name to the environment
binding	set the thing associated with a name
reference	get the thing associated with a name

Example semantic domains for expressions with ...

immutable vars (Haskell)	<code>Env -> Val</code>
mutable vars (C/Java/Python)	<code>Env -> (Env, Val)</code>

We'll come back to
mutable variables in
unit on **scope**