1. Use the MAP criterion to determine a decision rule using the detction statistic: $T_d(x) = x$. ***Describe carefully and completely the decision regions*** $\mathcal{R}_A$ ***and*** $\mathcal{R}_B$ ***for this decision rule.***

Following the intiution built in *Problem 1*, the maximum a posteriori probability density function is found via its relation to the likelihood and a priori densities through Bayes' theorm.

$$p(H_i|x) = \frac{p(x|H_i)p(H_i)}{p(x)} = \frac{p(x|H_i)p(H_i)}{\sum_i p(x|H_i)p(x)dx}$$

Substituing in the given likelihood and a priori densities into the above equation, the MAP densities for hypotheses H_A and H_B are:

$$p(H_0|x) = \begin{cases} \dfrac{\frac{1}{2\pi}\exp\left(\frac{-x^2}{2}\right)(0.5)}{\frac{1}{2\pi}\exp\left(\frac{-x^2}{2}\right)(0.5) + (0.2)(0.5)} & \text{for } -2.5 \le x \le 2.5 \\\\ \dfrac{\frac{1}{2\pi}\exp\left(\frac{-x^2}{2}\right)(0.5)}{\frac{1}{2\pi}\exp\left(\frac{-x^2}{2}\right)(0.5) + 0} & \text{otherwise} \end{cases} ,$$

$$p(H_0|x) = \begin{cases} \dfrac{(0.2)(0.5)}{\frac{1}{2\pi}\exp\left(\frac{-x^2}{2}\right)(0.5) + (0.2)(0.5)} & \text{for } -2.5 \le x \le 2.5 \\\\ 0 & \text{otherwise} \end{cases} ,$$

which can be further simplified to

$$p(H_0|x) = \begin{cases} \dfrac{\frac{1}{2\pi}\exp\left(\frac{-x^2}{2}\right)}{\frac{1}{2\pi}\exp\left(\frac{-x^2}{2}\right) + (0.2)} & \text{for } -2.5 \le x \le 2.5 \\\\ 1 & \text{otherwise} \end{cases} ,$$

$$p(H_0|x) = \begin{cases} \dfrac{(0.2)}{\frac{1}{2\pi}\exp\left(\frac{-x^2}{2}\right) + (0.2)} & \text{for } -2.5 \le x \le 2.5 \\\\ 0 & \text{otherwise} \end{cases} .$$

2. Determine the probability of error for this decision rule (MatLab allowed for this calculation).