

Problem 2 - 25 points

Under hypothesis \mathcal{H}_a , an observation $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$ is equal to a known signal \mathbf{s}_a with additive Gaussian noise \mathbf{w} :

$$\mathbf{x} = \mathbf{s}_a + \mathbf{w}$$

Under hypothesis \mathcal{H}_b , an observation \mathbf{x} is equal to a known signal \mathbf{s}_b with additive Gaussian noise \mathbf{w} :

$$\mathbf{x} = \mathbf{s}_b + \mathbf{w}$$

The two known signals are:

$$\mathbf{s}_a = [3, -1, 1, -3]^T \quad \text{and} \quad \mathbf{s}_b = [-3, 3, -5, 5]^T$$

while the Gaussian noise vector $\mathbf{w} = [w_1, w_2, w_3, w_4]^T$ is distributed as:

$$\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_w)$$

where:

$$\mathbf{C}_w = E\{\mathbf{w}\mathbf{w}^T\} = \sum_{n=1}^4 \lambda_n \hat{\mathbf{v}}_n \hat{\mathbf{v}}_n^T$$

And also where:

$$\lambda_1 = 6.0 \quad \hat{\mathbf{v}}_1 = [+0.5, -0.5, +0.5, -0.5]^T$$

$$\lambda_2 = 4.0 \quad \hat{\mathbf{v}}_2 = [+0.5, -0.5, -0.5, +0.5]^T$$

$$\lambda_3 = 2.0 \quad \hat{\mathbf{v}}_3 = [+0.5, +0.5, -0.5, -0.5]^T$$

$$\lambda_4 = 1.0 \quad \hat{\mathbf{v}}_4 = [+0.5, +0.5, +0.5, +0.5]^T$$

1. To decorrelate the elements of the noise vector, apply the **Karhunen-Loeve transform**.

Specifically, determine

- a) the **transformed** signal vectors \mathbf{s}'_a and \mathbf{s}'_b , and
- b) the **covariance matrix** of the transformed noise vector \mathbf{w}' .

2. Now, using a LRT threshold of $\gamma = e^2$, determine a LRT for this detection problem, expressed in terms of **transformed measurement** $\mathbf{x}' = [x'_1, x'_2, x'_3, x'_4]^T$ (i.e., in terms of variables x'_1, x'_2, x'_3, x'_4).

3. Simplify this LRT into a **decision rule** of the form:

$$T_d(\mathbf{x}') > \gamma'$$

I.E., provide **explicitly**—and in their **simplest possible form**—the statistic $T_d(\mathbf{x}')$ and threshold γ' .