Problem 2 – 25 points

Say that a scalar observation x is related to a source θ as:

$$x = \frac{4\theta}{w}$$

where w is an independent random variable, described by an **exponential *pdf***:

$$p(w) = \begin{cases} \lambda_w e^{-\lambda_w w} & \text{for } w \geq 0 \\ 0 & \text{for } w < 0 \end{cases}$$

The *apriori* knowledge of θ is described by:

$$p(\theta) = \begin{cases} \lambda_\theta e^{-\lambda_\theta \theta} & \text{for } \theta \geq 0 \\ 0 & \text{for } \theta < 0 \end{cases}$$

where $\lambda_\theta > 0$ and $\lambda_w > 0$.

1. Determine the **MLE estimate** $\hat{\theta}_{mle}(x)$.

2. Determine the **MAP estimate** $\hat{\theta}_{map}(x)$.

3. **Evaluate** this MAP estimate for the case where:

$$\lim_{\lambda_\theta \to 0} \hat{\theta}_{map}$$

Explain this result.

4. Now **evaluate** this MAP estimate **instead** for the case where:

$$\lim_{\lambda_w \to 0} \hat{\theta}_{map}$$

Explain this result.

Hint1: Remember EECS 861 !

Hint2: Recall that the **expected value** of a random variable with an exponential distribution is λ^{-1}, and its **variance** is λ^{-2} (these facts will be helpful for your **explanations**!).