Problem 24

Consider a **real-valued** random variable x.

Under hypothesis $\mathcal{H}_0$, x is described by the *pdf*:

$$p(x|\mathcal{H}_0) = \begin{cases} 3(1-x)^2 & \text{for } 0 < x < 1 \\ 0 & \text{for } x < 0,\ x > 1 \end{cases}$$

While under hypothesis $\mathcal{H}_1$, x is described by the *pdf*:

$$p(x|\mathcal{H}_1) = \begin{cases} 3x^2 & \text{for } 0 < x < 1 \\ 0 & \text{for } x < 0,\ x > 1 \end{cases}$$

The *a priori* probabilities of these two hypotheses are:

$$P(\mathcal{H}_0) = 0.8 \qquad \text{and} \qquad P(\mathcal{H}_1) = 0.2$$

From a **single** observation x, we must (attempt to) **choose** the correct hypothesis ($\mathcal{H}_0$ or $\mathcal{H}_1$).

1. Determine the Likelihood Ratio Test (i.e., **determine** $L(x)$ **and** γ) for both the **Maximum Likelihood** (ML) and **MAP** detection criteria.

2. Simplify the LRT, such that the decision rule can be expressed in terms of this **decision statistic**:

$$T_d(x) = x$$

For **this** decision statistic, determine the values of threshold γ' for **both** the ML and MAP criteria.

3. Say that the **Neyman-Pearson** criterion is instead used, so that the probability of false alarm is

$$P_{FA} = 0.2$$

Determine the resulting **probability of detection** P_D

4. Plot the **Receiver Operating Curve** (ROC) for this detection problem (you may use Matlab to **plot** your expression).