EECS 965 Exam II

Fall 2025 - 75 Points

Name__________________________________

Instructions

1) Your solutions must be detailed, unambiguous, and organized. I require that you show **all** your mathematics (i.e., no Matlab unless explicitly allowed).

2) Mathematically reduce your solutions to their **simplest** and most fundamental form.

2) Please **underline or circle** all numeric answers (e.g., |**A**| = 11.2), or in some way make your final answer **clear**.

4) If you feel that a problem is unclear, contradictory, incomplete, or ambiguous, **ask for clarification**.

5) This is an exam; it must reflect **your** knowledge and effort—and yours only!

Please **sign** this statement: "*As an honorable scholar and human being, I pledge that this exam is a reflection of my knowledge only. I hereby pledge that I committed no act that a reasonable person could construe as academic misconduct.*"

Signed:__________________________________

Problem 1- 25 points

A scalar random variable x is **Gaussian** distributed as:

$$x \sim \mathcal{N}\left(0, \sigma^2\right)$$

a) Say we wish to estimate the **standard deviation** $\sigma = \theta$ from an observation x.

Determine the **Maximum Likelihood** estimator $\hat{\sigma}_{mle}(x)$

b) Say instead we wish to estimate the **variance** $\sigma^2 = \theta$ from an observation x.

Determine the **Maximum Likelihood** estimator $\widehat{\sigma^2}_{mle}(x)$

c) How is the **ML** estimate $\widehat{\sigma^2}_{mle}(x)$ mathematically related to the **ML** estimate $\hat{\sigma}_{mle}(x)$ (i.e., can one estimate be expressed in terms of the other?).

Does this result surprise you? **Explain** why or why not.

d) Use the **CRLB theorem** to determine whether $\widehat{\sigma^2}_{mle}(x)$ is (or is not) an **efficient** estimator.

e) Use the **CRLB theorem** to determine whether $\hat{\sigma}_{mle}(x)$ is (or is not) an **efficient** estimator.

f) Given the results of part **c),** do the results of **d)** and **e)** surprise you? **Explain** why or why not.

Hint: Be sure to use the CRLB **THEOREM** in parts **d)** and **e).** In other words, do **NOT** attempt to determine the CRLB directly!

Problem 2 – 25 points

Say that a scalar observation x is related to a source θ as:

$$x = \frac{4\theta}{w}$$

where w is an independent random variable, described by an **exponential *pdf***:

$$p(w) = \begin{cases} \lambda_w e^{-\lambda_w w} & \text{for } w \geq 0 \\ 0 & \text{for } w < 0 \end{cases}$$

The *apriori* knowledge of θ is described by:

$$p(\theta) = \begin{cases} \lambda_\theta e^{-\lambda_\theta \theta} & \text{for } \theta \geq 0 \\ 0 & \text{for } \theta < 0 \end{cases}$$

where $\lambda_\theta > 0$ and $\lambda_w > 0$.

1. Determine the **MLE estimate** $\hat{\theta}_{mle}(x)$.

2. Determine the **MAP estimate** $\hat{\theta}_{map}(x)$.

3. **Evaluate** this MAP estimate for the case where:

$$\lim_{\lambda_\theta \to 0} \hat{\theta}_{map}$$

Explain this result.

4. Now **evaluate** this MAP estimate **instead** for the case where:

$$\lim_{\lambda_w \to 0} \hat{\theta}_{map}$$

Explain this result.

Hint1: Remember EECS 861 !

Hint2: Recall that the **expected value** of a random variable with an exponential distribution is λ^{-1}, and its **variance** is λ^{-2} (these facts will be helpful for your **explanations**!).

Problem 3 - 25 points

Say the elements x_n of an **N-dimensional** vector $\mathbf{x}$ are **independent** and identically distributed random variables, each with an Exponential *pdf* :

$$p(x_n|\lambda) = \lambda\, exp(-\lambda\, x_n) \qquad for\ x_n > 0$$

Therefore,

$$\begin{aligned} p(\mathbf{x}|\lambda) &= \prod_{n=1}^{N} p(x_n|\lambda) \\ &= \prod_{n=1}^{N} \lambda\, exp(-\lambda\, x_n) \qquad for\ x_n > 0 \end{aligned}$$

Determine the **MLE estimate** of λ (i.e., determine $\hat{\lambda}(\mathbf{x})$).

Hint: Recall that $e^x\, e^y = e^{x+y}$.