Problem 3 - 25 points

Under hypothesis $\mathcal{H}_a$, a scalar measurement x is equal to random **variable** w_a:

$$x = w_a$$

Random variable w_a is described by the **Gaussian distribution**:

$$w_a \sim \mathcal{N}\left(\mu = 0, \sigma^2 = 1\right)$$

i.e.,:

$$p(w_a) = \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-w_a^2}{2}\right] \qquad for\ -\infty < w_a < \infty$$

Under hypothesis $\mathcal{H}_b$, the scalar measurement x is equal to random **variable** w_b:

$$x = w_b$$

Random variable w_b is described by a **uniform distribution:**

$$p(w_b) = \begin{cases} 0.2 & for\ -2.5 \le w_b \le 2.5 \\ 0 & for\ w_b \le -2.5,\ w_b \ge 2.5 \end{cases}$$

The two hypotheses are **equally probable**, *a priori* .

1. Use the **MAP** criterion to **determine a decision rule** using the detection statistic:

$$T_d(x) = x$$

Describe carefully and **completely** the **decision regions** $\mathcal{R}_a$ and $\mathcal{R}_b$ for this decision rule.

2. Determine the **probability of error** for this decision rule (you **may** use **MatLab** or other cumulative probability solver for this calculation).