EECS 965 Exam I

Fall 2025 - 75 Points

Name_____

Instructions

1) Your solutions must be detailed, unambiguous, and organized. I require that you show **all** your mathematics (i.e., no Matlab unless explicitly allowed).

2) Mathematically reduce your solutions to their **simplest** and most fundamental form.

2) Please **underline or circle** all numeric answers (e.g., $|\boldsymbol{A}| = 11.2$), or in some way make your final answer **clear**.

4) If you feel that a problem is unclear, contradictory, incomplete, or ambiguous, **ask for clarification**.

5) This is an exam; it must reflect **your** knowledge and effort—and yours only!

Please **sign** this statement: "*As an honorable scholar and human being, I pledge that this exam is a reflection of my knowledge only. I hereby pledge that I committed no act that a reasonable person could construe as academic misconduct.*"

Signed:_____

Problem 1 – 25 points

Consider a **real-valued** random variable x.

Under hypothesis $\mathcal{H}_0$, x is described by the *pdf*:

$$p(x|\mathcal{H}_0) = \begin{cases} (3/8)(2-x)^2 & for\ 0 < x < 2 \\ 0 & for\ x < 0,\ x > 2 \end{cases}$$

While under hypothesis $\mathcal{H}_1$, x is described by the *pdf*:

$$p(x|\mathcal{H}_1) = \begin{cases} (3/8)x^2 & for\ 0 < x < 2 \\ 0 & for\ x < 0,\ x > 2 \end{cases}$$

The *a priori* probabilities of these two hypotheses are:

$$P(\mathcal{H}_0) = \frac{3}{5} \qquad \text{and} \qquad Pr(\mathcal{H}_1) = \frac{2}{5}$$

From a **single** observation x, we must (attempt to) **choose** the correct hypothesis ($\mathcal{H}_0$ or $\mathcal{H}_1$).

1. Determine the Likelihood Ratio Test (i.e., **determine** $L(x)$ **and** γ) using the **MAP** detection criteria.

2. Simplify the LRT, such that the decision rule can be expressed in terms of this **decision statistic**:

$$T_d(x) = x$$

For **this** decision statistic, determine the value of threshold γ' for MAP criteria.

3. Say you now instead desire a threshold γ' that would result in a **probability of detection** of:

$$P_D = 0.90$$

Determine:

a) the value of this **threshold** γ', and

b) the resulting **probability of false alarm**.