Problem 1- 25 points

A scalar random variable x is **Gaussian** distributed as:

$$x \sim \mathcal{N}(0, \sigma^2)$$

   a)   Say we wish to estimate the **standard deviation** $\sigma = \theta$ from an observation x.

         Determine the **Maximum Likelihood** estimator $\hat{\sigma}_{mle}(x)$

   b)   Say instead we wish to estimate the **variance** $\sigma^2 = \theta$ from an observation x.

         Determine the **Maximum Likelihood** estimator $\widehat{\sigma^2}_{mle}(x)$

   c)   How is the **ML** estimate $\widehat{\sigma^2}_{mle}(x)$ mathematically related to the **ML** estimate $\hat{\sigma}_{mle}(x)$ (i.e., can one estimate be expressed in terms of the other?).

         ==Does this result surprise you? **Explain** why or why not.==

   d)   Use the **CRLB theorem** to determine whether $\widehat{\sigma^2}_{mle}(x)$ is (or is not) an **efficient** estimator.

   e)   Use the **CRLB theorem** to determine whether $\hat{\sigma}_{mle}(x)$ is (or is not) an **efficient** estimator.

f) Given the results of part **c),** do the results of **d)** and **e)** surprise you? **Explain** why or why not.

Hint: Be sure to use the CRLB **THEOREM** in parts **d)** and **e)**. In other words, do **NOT** attempt to determine the CRLB directly!