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In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

import statsmodels.api as sm
```

Exercise 1: Univariate Problems

Code the bisection algorithm for any function $f(x)$

In [65]:
```
def bisection(function, low, high, epsilon = 1e-4, sigma = 1e-4, numIter = 100
0):
    """
    Finds a root of function in an interval [low, high] such that function(lo
w) < 0 and function(high) > 0

    Arguments:

    function - continuous input function
    low - lower bound of domain
    high - upper bound of domain
    epsilon - distance betwee refined low and high
    sigma - function value at mid point
    numIter - maximum number of iterations

    Returns:

    mid - root of function such that f(mid) ~ 0 in interval [low, high]
    """
    if function(low) * function(high) < 0:
        for i in range(numIter):
            # check whether function(low) and function(high) have different si
gns
            mid = (low + high) / 2

            # shorten search domain by moving to mid point either from below o
r above
            if function(low) * function(mid) > 0:
                low = mid
            else:
                high = mid

            # check epsilon convergence or sigma convergence
            if high - low <= epsilon * (1 + abs(low) + abs(high)) or abs(funct
ion(mid)) <= sigma:
                return mid
    else:
        print('function(low) * function(high) > 0 => Define a better interval'
)
```

Use bisection to compute the zeros of the functions

1. $f(x) = x^3 + 4 - \frac{1}{x}$
2. $f(x) = -\exp(-x) + \exp(-x^2)$

In [24]:
```
def function1(x):
    return x**3 + 4 - 1/x

def function2(x):
    return -np.exp(-x) + np.exp(-x**2)
```

In [50]:
```
mid1 = bisection(function1, -2, -1)
mid2 = bisection(function2, 1/2, 2)

print('ROOT: {} <=> Bisection on function1 in an interval [-2, -1]'.format(mid
1))
print('ROOT: {} <=> Bisection on function2 in an interval [1/2, 2]'.format(mid
2))
```

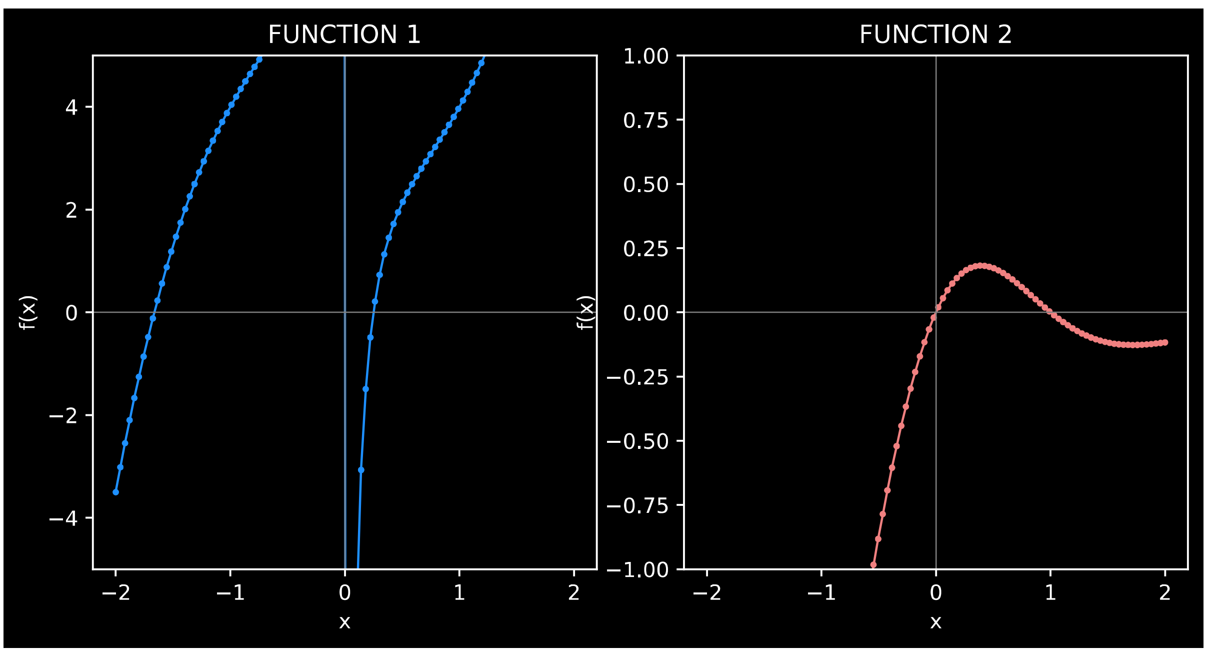
```
ROOT: -1.663330078125 <=> Bisection on function1 in an interval [-2, -1]
ROOT: 1.000244140625 <=> Bisection on function2 in an interval [1/2, 2]
```

In [64]:
```
x = np.linspace(-2, 2, 100)
y1 = function1(x)
y2 = function2(x)

fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(8, 4))
fig.tight_layout()
ax1.plot(x, y1, color = 'dodgerblue', marker = 'o', markersize = 2, linewidth
= 1)
ax1.axhline(y = 0, color = 'gray', linewidth = 0.5)
ax1.axvline(x = 0, color = 'gray', linewidth = 0.5)
ax1.title.set_text('FUNCTION 1')
ax1.set_xlabel('x')
ax1.set_ylabel('f(x)')
ax1.set_ylim(-5, 5)

ax2.plot(x, y2, color = 'lightcoral', marker='o', markersize = 2, linewidth =
1)
ax2.axhline(y = 0, color = 'gray', linewidth = 0.5)
ax2.axvline(x = 0, color = 'gray', linewidth = 0.5)
ax2.title.set_text('FUNCTION 2')
ax2.set_xlabel('x')
ax2.set_ylabel('f(x)')
ax2.set_ylim(-1, 1)
```

Out[64]: (-1.0, 1.0)



Code the secant method for any function $f(x)$

In [73]:
```
def secant(function, x1, x2, tol = 1e-4, numIter = 1000):
    """
    Finds a root of function given two points x1 and x2

    Arguments:

    function - continuous input function
    x1 - initial point 1
    x2 - initial point 2
    tol - breakpoint tolerance
    numIter - maximum number of iterations

    Returns:
    z - root of function
    """
    fx1 = function(x1)
    fx2 = function(x2)

    for i in range(numIter):
        z = (x1 * fx2 - x2 * fx1) / (fx2 - fx1)

        if abs(function(z)) <= tol * (1 + function(z)):
            return z

        x1, x2  = x2, z
        fx1, fx2 = fx2, function(z)

    else:
        print('numIter exceeded. Current z is {}'.format(z))
```

Use secant to compute the zeros of the functions

1. $f(x) = x^2 + 10 - \frac{1}{x}$
2. $f(x) = \exp(-x^2)$

In [68]:
```
def function3(x):
    return x**2 + 10 - 1/x

def function4(x):
    return np.exp(-x**2)
```

In [75]:
```
z3 = secant(function3, 0.1, 1)
z4 = secant(function4, 1/2, 2)

print('ROOT: {} <=> Secant on function3 given x1 = 0.1 and x2 = 1'.format(z3))
print('ROOT: {} <=> Secant on function4 in an interval [1/2, 2]'.format(z4))
```

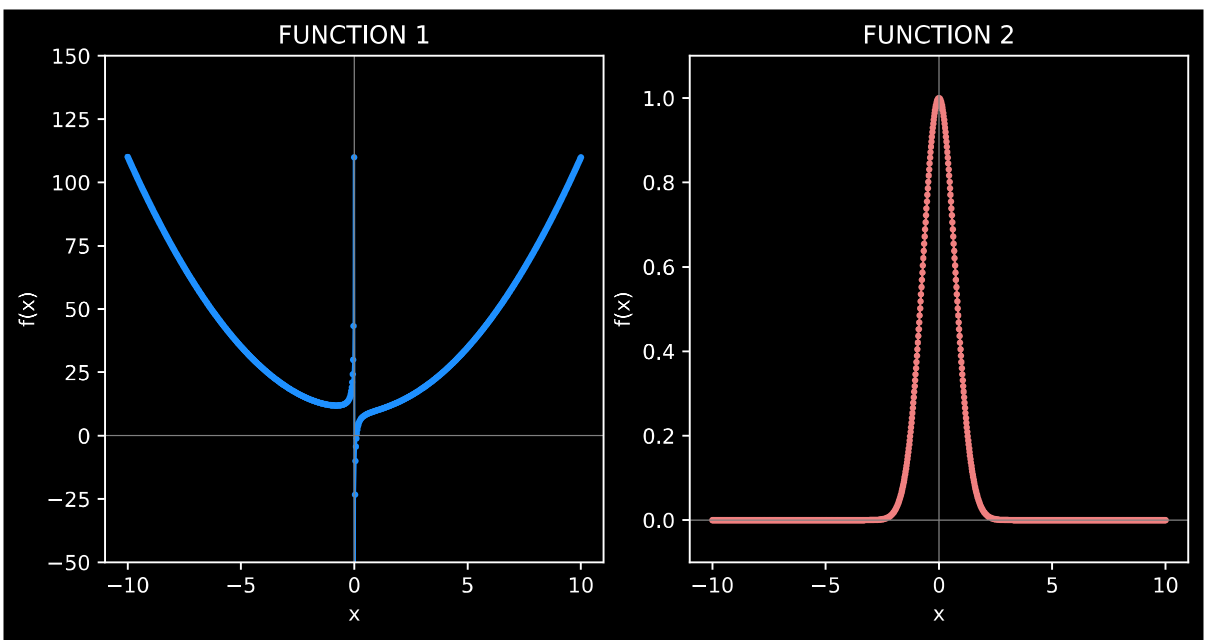
```
ROOT: 0.09990030050935535 <=> Secant on function3 given x1 = 0.1 and x2 = 1
ROOT: 3.063995107434631 <=> Secant on function4 in an interval [1/2, 2]
```

In [98]:
```
x3 = np.linspace(-10, 10, 1000)
x4 = np.linspace(-10, 10, 1000)
y3 = function3(x)
y4 = function4(x)

fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(8, 4))
fig.tight_layout()
ax1.plot(x3, y3, color = 'dodgerblue', marker = 'o', markersize = 2, linewidth
= 1)
ax1.axhline(y = 0, color = 'gray', linewidth = 0.5)
ax1.axvline(x = 0, color = 'gray', linewidth = 0.5)
ax1.title.set_text('FUNCTION 1')
ax1.set_xlabel('x')
ax1.set_ylabel('f(x)')
ax1.set_ylim(-50, 150)

ax2.plot(x4, y4, color = 'lightcoral', marker='o', markersize = 2, linewidth =
1)
ax2.axhline(y = 0, color = 'gray', linewidth = 0.5)
ax2.axvline(x = 0, color = 'gray', linewidth = 0.5)
ax2.title.set_text('FUNCTION 2')
ax2.set_xlabel('x')
ax2.set_ylabel('f(x)')
ax2.set_ylim(-0.1, 1.1)
```

Out[98]: (-0.1, 1.1)



Revisit the demand-supply example of problem set 1

$$\mathbf{D}: \quad p = q - bq$$
$$\mathbf{S}: \quad p = c + dq^{\phi}$$

where p is the price, q is the quantity, ab, c, d, ϕ are some parameters.

1. Write it as a univariate problem

$$bq + dq^{\phi} - (a - c) = 0$$

In [100]:
```
def supplyDemand(q, a = 3, b = 1/2, c = 1, d = 1, phi = 1/2):
    return b * q + d * q**phi - (a - c)
```

1. Parametrize the model with $a = 3, b = 0.5, c = d = 1, \phi = 1/2$. Compute the solution analytically.

Let $\tilde{q} = q^{\frac{1}{2}}$ hence

$$\frac{1}{2}\tilde{q}^2 + \tilde{q} - 2 = 0$$

Solutions to this quadratic equations are

$$\tilde{q}_1 = \sqrt{5} - 1 \quad \tilde{q}_2 = -\sqrt{5} - 1$$

Hence, the only solution q_1 is given by

$$q_1 = (\sqrt{5} - 1)^2 = 6 - 2\sqrt{5}$$

1. Compute the solutions with your bisection algorithm

In [112]:
```
q = bisection(supplyDemand, 1, 4)
```

In [113]:
```
print('ROOT: {} <=> Bisection on supplyDemand in interval [1, 3]'.format(q))
```

```
ROOT: 1.5277099609375 <=> Bisection on supplyDemand in interval [1, 3]
```

Exercise 2: A Contribution to the Empirics of Economic Growth

1. Load the data set and delete countries with missing values

In [2]:
```
df = pd.read_excel('../Helpers/MRW92QJE-data.xls', header = 0)
df = df.dropna()
df.head(10)
```

Out[2]:

	country number	country name	Non-oil	intermediate	oecd	gdp/adult 1960	gdp/adult 1985	growth gdp	growth working age pop	I/y	s
0	1	Algeria	1	1	0	2485.0	4371.0	4.8	2.6	24.1	
1	2	Angola	1	0	0	1588.0	1171.0	0.8	2.1	5.8	
2	3	Benin	1	0	0	1116.0	1071.0	2.2	2.4	10.8	
3	4	Botswana	1	1	0	959.0	3671.0	8.6	3.2	28.3	
4	5	Burkina Faso	1	0	0	529.0	857.0	2.9	0.9	12.7	
5	6	Burundi	1	0	0	755.0	663.0	1.2	1.7	5.1	
6	7	Cameroon	1	1	0	889.0	2190.0	5.7	2.1	12.8	
7	8	Central African Republic	1	0	0	838.0	789.0	1.5	1.7	10.5	
8	9	Chad	1	0	0	908.0	462.0	-0.9	1.9	6.9	
9	10	Congo, Peoples Republic	1	0	0	1009.0	2624.0	6.2	2.4	28.8	

1. Generate sub-samples for non-oil countries, intermediate countries and OECD countries

In [3]:
```
nonOil = df[df['Non-oil'] == 1]
intermediate = df[df['intermediate'] == 1]
oecd = df[df['oecd'] == 1]
```

1. For each sub-sample compute the regression coefficients and respective standard error for the following regression model

$$
\begin{aligned}
\log(\text{gdp1985}_j) - \log(\text{gdp1960}_j) = \beta_0 &+ \beta_1 \log(\text{gdp1960}_j) \\
&+ \beta_2 \log\left(\frac{\text{investment}}{\text{gdp}_j}\right) \\
&+ \beta_3 \log(\text{popgrowth}_j + g + \delta) \\
&+ \beta_4 \log(\text{schoolenrol}_j) + \epsilon_j
\end{aligned}
$$

where $g + \delta = 0.05$.

In [11]:
```
results = []
gdelta = 0.05
for data in [nonOil, intermediate, oecd]:

    y = np.log(data['gdp/adult 1985'] / 100) - np.log(data['gdp/adult 1960'] / 
100) # log
    y = np.array(y).reshape(-1, 1)

    intercept = np.ones(len(y))
    x1 = np.log(data['gdp/adult 1960'] / 100)
    x2 = np.log(data['I/y'] / 100)
    x3 = np.log(data['growth working age pop'] / 100 + gdelta)
    x4 = np.log(data['school'] / 100)
    X = np.matrix([intercept, x1, x2, x3, x4])

    model = sm.OLS(y, X.T)
    results.append(model.fit())
```

In [12]:
```
print('RESULTS FOR NON-OIL')
results[0].summary()
```

```
RESULTS FOR NON-OIL
```

Out[12]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.485
Model:	OLS	**Adj. R-squared:**	0.463
Method:	Least Squares	**F-statistic:**	21.94
Date:	Mon, 01 Feb 2021	**Prob (F-statistic):**	8.99e-13
Time:	20:33:42	**Log-Likelihood:**	-26.952
No. Observations:	98	**AIC:**	63.90
Df Residuals:	93	**BIC:**	76.83
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	1.6935	0.760	2.228	0.028	0.184	3.203
x1	-0.2884	0.062	-4.683	0.000	-0.411	-0.166
x2	0.5237	0.087	6.029	0.000	0.351	0.696
x3	-0.5057	0.289	-1.752	0.083	-1.079	0.067
x4	0.2311	0.059	3.887	0.000	0.113	0.349

Omnibus:	2.061	**Durbin-Watson:**	2.135
Prob(Omnibus):	0.357	**Jarque-Bera (JB):**	1.518
Skew:	-0.153	**Prob(JB):**	0.468
Kurtosis:	3.528	**Cond. No.**	135.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [13]:
```
print('RESULTS FOR INTERMEDIATE')
results[1].summary()
```

```
RESULTS FOR INTERMEDIATE
```

Out[13]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.465
Model:	OLS	**Adj. R-squared:**	0.435
Method:	Least Squares	**F-statistic:**	15.23
Date:	Mon, 01 Feb 2021	**Prob (F-statistic):**	5.26e-09
Time:	20:33:43	**Log-Likelihood:**	-14.635
No. Observations:	75	**AIC:**	39.27
Df Residuals:	70	**BIC:**	50.86
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	2.0235	0.808	2.505	0.015	0.412	3.635
x1	-0.3660	0.067	-5.427	0.000	-0.500	-0.231
x2	0.5376	0.102	5.255	0.000	0.334	0.742
x3	-0.5450	0.288	-1.890	0.063	-1.120	0.030
x4	0.2705	0.080	3.365	0.001	0.110	0.431

Omnibus:	6.278	**Durbin-Watson:**	2.452
Prob(Omnibus):	0.043	**Jarque-Bera (JB):**	9.147
Skew:	-0.208	**Prob(JB):**	0.0103
Kurtosis:	4.659	**Cond. No.**	132.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [14]:
```
print('RESULTS FOR OECD')
results[2].summary()
```

```
RESULTS FOR OECD
```

Out[14]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.718
Model:	OLS	**Adj. R-squared:**	0.651
Method:	Least Squares	**F-statistic:**	10.80
Date:	Mon, 01 Feb 2021	**Prob (F-statistic):**	0.000152
Time:	20:33:45	**Log-Likelihood:**	13.798
No. Observations:	22	**AIC:**	-17.60
Df Residuals:	17	**BIC:**	-12.14
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	0.9239	1.051	0.879	0.392	-1.293	3.141
x1	-0.3977	0.070	-5.668	0.000	-0.546	-0.250
x2	0.3318	0.173	1.914	0.073	-0.034	0.698
x3	-0.8634	0.338	-2.557	0.020	-1.576	-0.151
x4	0.2277	0.145	1.570	0.135	-0.078	0.534

Omnibus:	1.879	**Durbin-Watson:**	1.695
Prob(Omnibus):	0.391	**Jarque-Bera (JB):**	1.098
Skew:	0.547	**Prob(JB):**	0.578
Kurtosis:	3.009	**Cond. No.**	202.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [ ]: