Introduction

Newton's Method

Quasi-Newton Methods

Homotopy Methods

Final Remarks

Example: Economic Growth

- Augmented Solow Growth Model (Mankiw et al. (1992))

$$\begin{aligned}
K_{t+1} &= s_K\ F(K_t, H_t, A_t L_t) + (1-\delta_K)K_t \\
H_{t+1} &= s_H\ F(K_t, H_t, A_t L_t) + (1-\delta_H)H_t \\
L_{t+1} &= (1+n)\ L_t \\
A_{t+1} &= (1+g)\ A_t
\end{aligned}$$

- K_t, H_t, L_t, A_t: physical capital, human capital, labor, technology
- $F(K_t, H_t, A_t L_t)$: CRS production technology
- $s_K, s_H \in (0,1)$: physical capital, human capital saving rates
- $\delta_K, \delta_H \in [0,1]$: physical capital, human capital depreciation rates
- n, g: population and productivity growth rates

Example: Economic Growth, Cont.

- Intensive form

$$k_{t+1} = \frac{1}{(1+n)(1+g)} \left(s_K \; f(k_t, h_t) + (1 - \delta_K) k_t \right)$$
$$h_{t+1} = \frac{1}{(1+n)(1+g)} \left(s_H \; f(k_t, h_t) + (1 - \delta_H) h_t \right)$$

- $k_t = \frac{K_t}{A_t L_t}$: physical capital per efficiency unit of labor
- $h_t = \frac{H_t}{A_t L_t}$: human capital per efficiency unit of labor
- $f(k_t, h_t) = \frac{F(K_t, H_t, A_t L_t)}{A_t L_t}$: output per efficiency unit of labor
- Details, e.g., Acemoglu (2009, ch. 3.3)

Example: Economic Growth, Cont.

- Balanced growth path: constant p. c. growth, $(k_t, h_t) = (k^*, h^*)$

$$s_k f(k^*, h^*) - (\delta_k + g + n + ng) k^* = 0 \qquad (1)$$
$$s_h f(k^*, h^*) - (\delta_h + g + n + ng) h^* = 0 \qquad (2)$$

- Production technology
  - Cobb-Douglas (used by Mankiw et al. (1992))

$$F(K_t, H_t, A_t L_t) = K_t^\alpha H_t^\beta (A_t L_t)^{1-\alpha-\beta}$$

  $\Rightarrow$ analytical solution for (k^*, h^*).
  - Constant elasticity of substitution (CES)

$$F(K_t, H_t, A_t L_t) = (\alpha K_t^\rho + \beta H_t^\rho + (1 - \alpha - \beta)(A_t L_t)^\rho)^{1/\rho}$$

  $\Rightarrow$ (1,2) system of nonlinear eqs without analytical solution