EGOR ZAITSEV - 07 452 541

XIAO XU - 07 498 897

In [181]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

import statsmodels.api as sm
```

Exercise 1: Univariate Problems

Code the bisection algorithm for any function $f(x)$

In [65]:
```
def bisection(function, low, high, epsilon = 1e-4, sigma = 1e-4, numIter = 100
0):
    """
    Finds a root of function in an interval [low, high] such that function(lo
w) < 0 and function(high) > 0

    Arguments:

    function - continuous input function
    low - lower bound of domain
    high - upper bound of domain
    epsilon - distance betwee refined low and high
    sigma - function value at mid point
    numIter - maximum number of iterations

    Returns:

    mid - root of function such that f(mid) ~ 0 in interval [low, high]
    """
    if function(low) * function(high) < 0:
        for i in range(numIter):
            # check whether function(low) and function(high) have different si
gns
            mid = (low + high) / 2

            # shorten search domain by moving to mid point either from below o
r above
            if function(low) * function(mid) > 0:
                low = mid
            else:
                high = mid

            # check epsilon convergence or sigma convergence
            if high - low <= epsilon * (1 + abs(low) + abs(high)) or abs(funct
ion(mid)) <= sigma:
                return mid
    else:
        print('function(low) * function(high) > 0 => Define a better interval'
)
```

Use bisection to compute the zeros of the functions

1. $f(x) = x^3 + 4 - \frac{1}{x}$
2. $f(x) = -\exp(-x) + \exp(-x^2)$

In [24]:
```
def function1(x):
    return x**3 + 4 - 1/x

def function2(x):
    return -np.exp(-x) + np.exp(-x**2)
```

In [50]:
```
mid1 = bisection(function1, -2, -1)
mid2 = bisection(function2, 1/2, 2)

print('ROOT: {} <=> Bisection on function1 in an interval [-2, -1]'.format(mid
1))
print('ROOT: {} <=> Bisection on function2 in an interval [1/2, 2]'.format(mid
2))
```

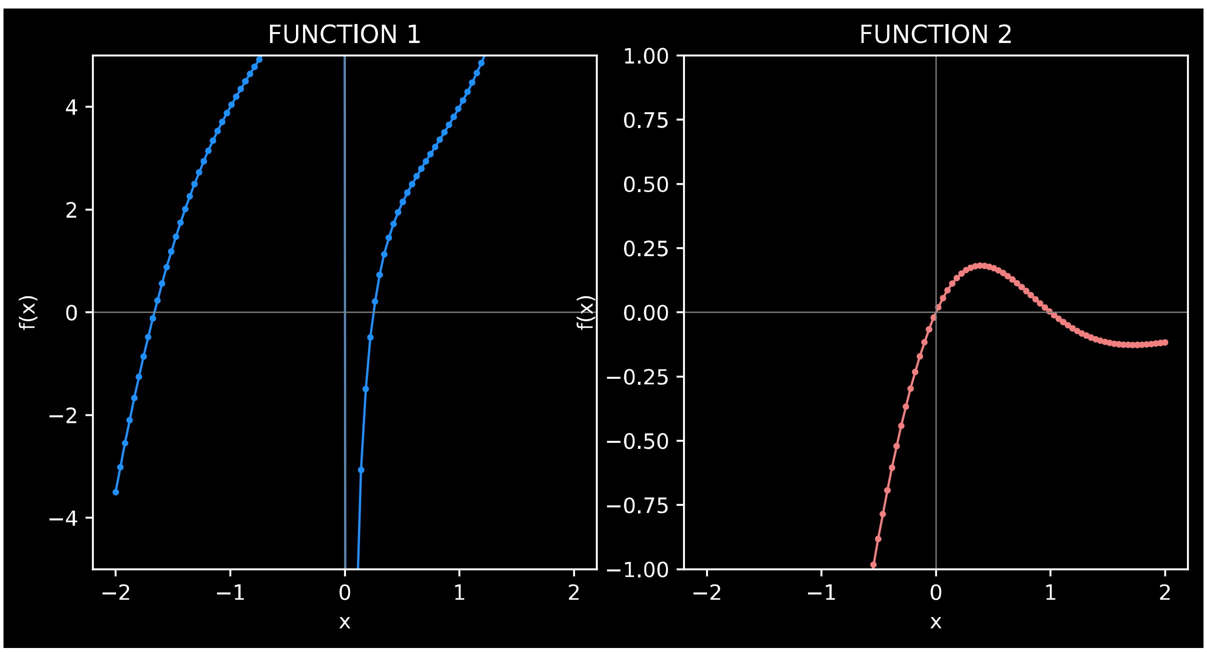
```
ROOT: -1.663330078125 <=> Bisection on function1 in an interval [-2, -1]
ROOT: 1.000244140625 <=> Bisection on function2 in an interval [1/2, 2]
```

In [64]:
```
x = np.linspace(-2, 2, 100)
y1 = function1(x)
y2 = function2(x)

fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(8, 4))
fig.tight_layout()
ax1.plot(x, y1, color = 'dodgerblue', marker = 'o', markersize = 2, linewidth
= 1)
ax1.axhline(y = 0, color = 'gray', linewidth = 0.5)
ax1.axvline(x = 0, color = 'gray', linewidth = 0.5)
ax1.title.set_text('FUNCTION 1')
ax1.set_xlabel('x')
ax1.set_ylabel('f(x)')
ax1.set_ylim(-5, 5)

ax2.plot(x, y2, color = 'lightcoral', marker='o', markersize = 2, linewidth =
1)
ax2.axhline(y = 0, color = 'gray', linewidth = 0.5)
ax2.axvline(x = 0, color = 'gray', linewidth = 0.5)
ax2.title.set_text('FUNCTION 2')
ax2.set_xlabel('x')
ax2.set_ylabel('f(x)')
ax2.set_ylim(-1, 1)
```

Out[64]: (-1.0, 1.0)



Code the secant method for any function $f(x)$

In [73]:
```
def secant(function, x1, x2, tol = 1e-4, numIter = 1000):
    """
    Finds a root of function given two points x1 and x2

    Arguments:

    function - continuous input function
    x1 - initial point 1
    x2 - initial point 2
    tol - breakpoint tolerance
    numIter - maximum number of iterations

    Returns:
    z - root of function
    """
    fx1 = function(x1)
    fx2 = function(x2)

    for i in range(numIter):
        z = (x1 * fx2 - x2 * fx1) / (fx2 - fx1)

        if abs(function(z)) <= tol * (1 + function(z)):
            return z

        x1, x2  = x2, z
        fx1, fx2 = fx2, function(z)

    else:
        print('numIter exceeded. Current z is {}'.format(z))
```

Use secant to compute the zeros of the functions

1. $f(x) = x^2 + 10 - \frac{1}{x}$
2. $f(x) = \exp(-x^2)$

In [68]:
```
def function3(x):
    return x**2 + 10 - 1/x

def function4(x):
    return np.exp(-x**2)
```

In [75]:
```
z3 = secant(function3, 0.1, 1)
z4 = secant(function4, 1/2, 2)

print('ROOT: {} <=> Secant on function3 given x1 = 0.1 and x2 = 1'.format(z3))
print('ROOT: {} <=> Secant on function4 in an interval [1/2, 2]'.format(z4))
```

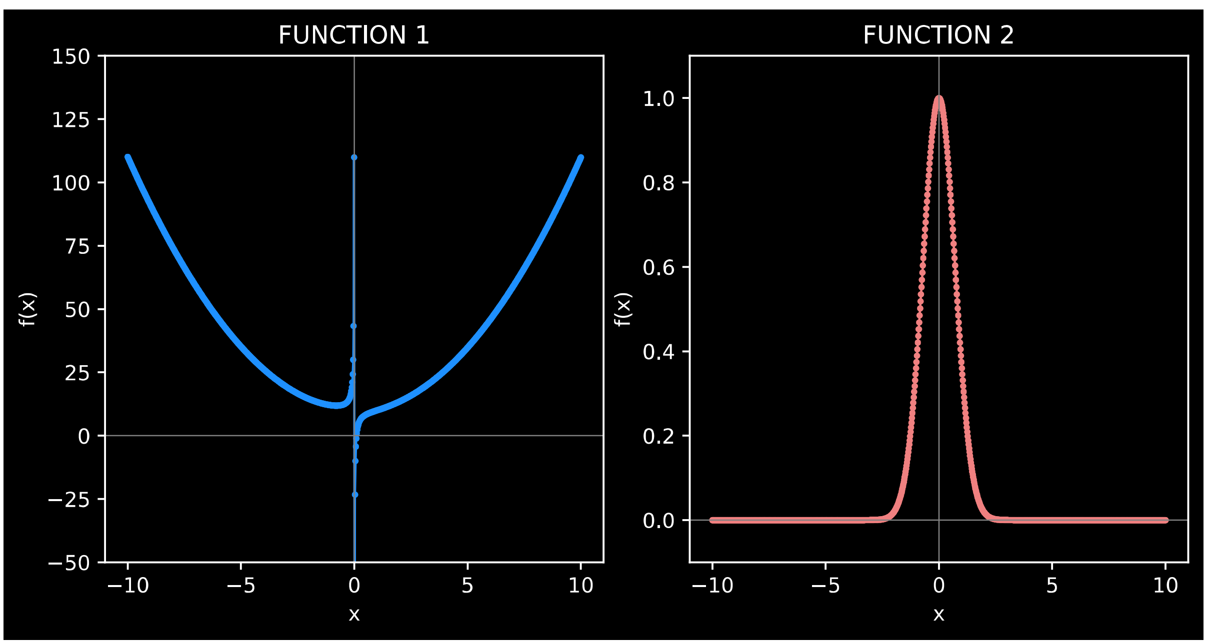
```
ROOT: 0.09990030050935535 <=> Secant on function3 given x1 = 0.1 and x2 = 1
ROOT: 3.063995107434631 <=> Secant on function4 in an interval [1/2, 2]
```

In [98]:
```
x3 = np.linspace(-10, 10, 1000)
x4 = np.linspace(-10, 10, 1000)
y3 = function3(x)
y4 = function4(x)

fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(8, 4))
fig.tight_layout()
ax1.plot(x3, y3, color = 'dodgerblue', marker = 'o', markersize = 2, linewidth
= 1)
ax1.axhline(y = 0, color = 'gray', linewidth = 0.5)
ax1.axvline(x = 0, color = 'gray', linewidth = 0.5)
ax1.title.set_text('FUNCTION 1')
ax1.set_xlabel('x')
ax1.set_ylabel('f(x)')
ax1.set_ylim(-50, 150)

ax2.plot(x4, y4, color = 'lightcoral', marker='o', markersize = 2, linewidth =
1)
ax2.axhline(y = 0, color = 'gray', linewidth = 0.5)
ax2.axvline(x = 0, color = 'gray', linewidth = 0.5)
ax2.title.set_text('FUNCTION 2')
ax2.set_xlabel('x')
ax2.set_ylabel('f(x)')
ax2.set_ylim(-0.1, 1.1)
```

Out[98]: (-0.1, 1.1)



Revisit the demand-supply example of problem set 1

$$\mathbf{D}: \quad p = q - bq$$
$$\mathbf{S}: \quad p = c + dq^{\phi}$$

where p is the price, q is the quantity, ab, c, d, ϕ are some parameters.

1. Write it as a univariate problem

$$bq + dq^{\phi} - (a - c) = 0$$

In [100]:
```
def supplyDemand(q, a = 3, b = 1/2, c = 1, d = 1, phi = 1/2):
    return b * q + d * q**phi - (a - c)
```

1. Parametrize the model with $a = 3, b = 0.5, c = d = 1, \phi = 1/2$. Compute the solution analytically.

Let $\tilde{q} = q^{\frac{1}{2}}$ hence

$$\frac{1}{2}\tilde{q}^2 + \tilde{q} - 2 = 0$$

Solutions to this quadratic equations are

$$\tilde{q}_1 = \sqrt{5} - 1 \quad \tilde{q}_2 = -\sqrt{5} - 1$$

Hence, the only solution q_1 is given by

$$q_1 = (\sqrt{5} - 1)^2 = 6 - 2\sqrt{5}$$

1. Compute the solutions with your bisection algorithm

In [112]:
```
q = bisection(supplyDemand, 1, 4)
```

In [113]:
```
print('ROOT: {} <=> Bisection on supplyDemand in interval [1, 3]'.format(q))
```

```
ROOT: 1.5277099609375 <=> Bisection on supplyDemand in interval [1, 3]
```

Exercise 2: A Contribution to the Empirics of Economic Growth

1. Load the data set and delete countries with missing values

In [128]:
```
data = pd.read_excel('../Helpers/MRW92QJE-data.xls', header = 0)
data = data.dropna(axis = 0, how = 'any') # drop rows with missing values
data.head(10)
```

Out[128]:

	country number	country name	Non-oil	intermediate	oecd	gdp/adult 1960	gdp/adult 1985	growth gdp	growth working age pop	I/y	s(
0	1	Algeria	1	1	0	2485.0	4371.0	4.8	2.6	24.1	
1	2	Angola	1	0	0	1588.0	1171.0	0.8	2.1	5.8	
2	3	Benin	1	0	0	1116.0	1071.0	2.2	2.4	10.8	
3	4	Botswana	1	1	0	959.0	3671.0	8.6	3.2	28.3	
4	5	Burkina Faso	1	0	0	529.0	857.0	2.9	0.9	12.7	
5	6	Burundi	1	0	0	755.0	663.0	1.2	1.7	5.1	
6	7	Cameroon	1	1	0	889.0	2190.0	5.7	2.1	12.8	
7	8	Central African Republic	1	0	0	838.0	789.0	1.5	1.7	10.5	
8	9	Chad	1	0	0	908.0	462.0	-0.9	1.9	6.9	
9	10	Congo, Peoples Republic	1	0	0	1009.0	2624.0	6.2	2.4	28.8	

1. Generate sub-samples for non-oil countries, intermediate countries and OECD countries

In [129]:
```
nonOil = data[data['Non-oil'] == 1]
intermediate = data[data['intermediate'] == 1]
oecd = data[data['oecd'] == 1]
```

1. For each sub-sample compute the regression coefficients and respective standard error for the following regression model

$$
\begin{aligned}
\log(\text{gdp1985}_j) - \log(\text{gdp1960}_j) = \beta_0 &+ \beta_1 \log(\text{gdp1960}_j) \\
&+ \beta_2 \log\left(\frac{\text{investment}}{\text{gdp}_j}\right) \\
&+ \beta_3 \log(\text{popgrowth}_j + g + \delta) \\
&+ \beta_4 \log(\text{schoolenrol}_j) + \epsilon_j
\end{aligned}
$$

where $g + \delta = 0.05$.

In [183]:
```
gdelta = 0.05
y = np.log(data['gdp/adult 1985']) - np.log(data['gdp/adult 1960'])
y = np.array(y).reshape(-1, 1)

intercept = np.ones(len(y))
x1 = np.log(data['gdp/adult 1960'])
x2 = np.log(data['I/y'])
x3 = np.log(data['growth working age pop'] + gdelta)
x4 = np.log(data['school'])
X = np.matrix([intercept, x1, x2, x3, x4])

model = sm.OLS(y, X.T)
results = model.fit()
```

In [185]:
```
results.summary()
```

Out[185]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.520
Model:	OLS	**Adj. R-squared:**	0.501
Method:	Least Squares	**F-statistic:**	26.85
Date:	Mon, 01 Feb 2021	**Prob (F-statistic):**	4.32e-15
Time:	16:38:56	**Log-Likelihood:**	-31.975
No. Observations:	104	**AIC:**	73.95
Df Residuals:	99	**BIC:**	87.17
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	1.1083	0.404	2.745	0.007	0.307	1.909
x1	-0.3116	0.051	-6.166	0.000	-0.412	-0.211
x2	0.5527	0.087	6.365	0.000	0.380	0.725
x3	-0.1415	0.063	-2.229	0.028	-0.267	-0.016
x4	0.2204	0.059	3.728	0.000	0.103	0.338

Omnibus:	1.583	**Durbin-Watson:**	2.230
Prob(Omnibus):	0.453	**Jarque-Bera (JB):**	1.081
Skew:	-0.077	**Prob(JB):**	0.583
Kurtosis:	3.475	**Cond. No.**	104.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.