# Stacks & Queues

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# Template

# Abstracted Bag Container

```
class Bag
{
public:
    Bag(int bagCapacity = 10);  // Constructor
   ~Bag();                      // Destructor

    int Size() const;           // Return the number of elements
    bool IsEmpty() const;       // Check if bag is empty
    int Element() const;        // Return an element in the bag

    void Push(const int);       // Insert an integer into the bag
    void Pop()                  // Delete an integer from the bag

private:
    int *array;                 // Integer array that stores the data
    int capacity;               // Capacity of array
    int top;                    // Position of top element
};
```

# Bag Implementation

```
Bag::Bag( int bagCapacity):capacity( bagCapacity ) {
   if(capacity < 1) throw "Capacity must be > 0";
   array = new int [ capacity ];
   top = -1;}

Bag::~Bag(){ delete [] array; }

inline int Bag::Size() const { return top + 1; }

inline bool Bag::IsEmpty() const { return Size() == 0; }

inline int Bag::Element() const {
   if(IsEmpty()) throw "Bag is empty";
   return array [0];  // Always return the first element
}

void Bag::Push(const int x) {
   if(capacity == top+1) ChangeSize1D(array,capacity,2* capacity);
   capacity *= 2;
   array[++top]=x;}

void Bag::Pop( ) {
   if(IsEmpty()) throw "Bag is empty, cannot delete";
   int deletePos = top / 2; // Always delete the middle element
   copy (array+deletePos+1, array+top+1, array+deletePos);
   top--;
}
```

# Abstracted Bag Container

```
template<class T>
class Bag
{
public:
    Bag(int bagCapacity = 10);  // Constructor
   ~Bag();                      // Destructor

    int Size() const;           // Return the number of elements
    bool IsEmpty() const;       // Check if bag is empty
    T& Element() const;         // Return an element in the bag

    void Push(const T&);        // Insert an element into the bag
    void Pop()                  // Delete an element from the bag

private:
    T *array;                   // Data array
    int capacity;               // Capacity of array
    int top;                    // Position of top element
};
```

# Template Bag Implementation

```
template<class T>
Bag<T>::Bag( int bagCapacity):capacity( bagCapacity ) {
   if(capacity < 1) throw "Capacity must be > 0";
   array = new T [ capacity ];
   top = -1;
}

template<class T>
void Bag<T>::Push(const T& x) {
   if(capacity == top+1) ChangeSize1D(array,capacity,2* capacity);
   capacity *= 2;
   array[++top]=x;
}

template<class T>
void Bag<T>::Pop() {
   if(IsEmpty()) throw "Bag is empty, cannot delete";
   int deletePos = top / 2; // Always delete the middle emelent
   copy (array+deletePos+1, array+top+1, array+deletePos);
   array[top--].~T();
}
```

# Subtype

- Inheritance is used to express subtype relationships
  - A Data object of Type B IS-A data object of Type A
  - Type B is more specialized than Type A
  - E.g., Chair IS-A Furniture

- Bag is a data structure, where
  - Elements can be inserted and deleted
- Stack is a data structure, where
  - Elements can be inserted and deleted
- Stack is more specialized
  - Stack IS-A Bag

# Generic Bag ADT

```
Class Bag
{
public:
  Bag(int bagCapacity=10);
  virtual ~Bag();
  virtual int Size() const;
  virtual bool IsEmpty() const;
  virtual int Element() const;
  virtual void Push(const int);
  virtual void Pop();
protected:
  int *array;
  int capacity;
  int top;
};
```

Implement operations not exist in the Bag class

```
class Stack : public Bag
{
public:
  Stack(int stackCapacity=10);
  virtual ~Stack();
  int Top()const;
  virtual void Pop();
};
```

# The Stack

# Stack

- A stack is an ordered list, in which
  - insertions (or called additions or pushes)
  - deletions (or called removals or pops)
  - Both made at one end called the top
- Operate in ***Last-In-First-Out (LIFO)*** order

# Stack: ADT

```
template < class T >
class Stack // A finite ordered list
{
public:
        // Constructor
        Stack (int stackCapacity = 10);

        // Check if the stack is empty
        bool IsEmpty ( ) const;

        // Return the top element
        T& Top ( ) const;

        // Insert a new element at top
        void Push (const T& item);

        // Delete one element from top
        void Pop ( );
private:
        T* stack;
        int top;   // init. value = -1
        int capacity;
};
```

# Stack Operations: Push & Pop

```
template < class T >
void Stack < T >::Push (const T& x)
{   // Add x to stack
    if(top == capacity - 1)
    {
       ChangeSize1D(stack, capacity, 2*capacity);
       capacity *= 2;
    }
    stack [ ++top ] = x;
}
```

```
template < class T >
void Stack < T >::Pop ( )
{   // Delete top element from stack
    if(IsEmpty()) throw "Stack is empty. Cannot delete.";
    stack [ top-- ].~T();  // Delete the element
}
```

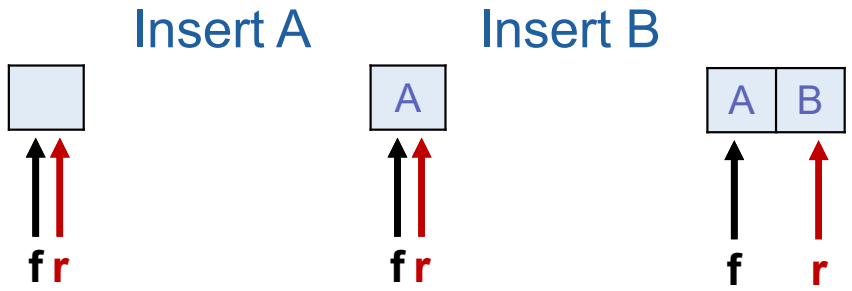
# The Queue

# Queue

- A queue is an ordered list
  - in which
    - insertions (or called additions or pushes)
    - deletions (or called removals or pops)
    - Made at different ends
  - New elements are inserted at rear end
  - Old elements are deleted at front end
  - Operate in ***First-In-First-Out (FIFO)*** order

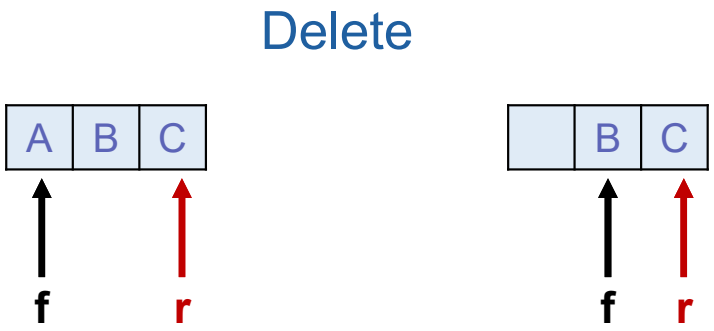
# Queue Insertion

- Insert a new element into queue
  - f: front position
  - r: rear position



# Queue Deletion

- Delete an old element from queue
  - f: front position
  - r: rear position



# Circular Queue

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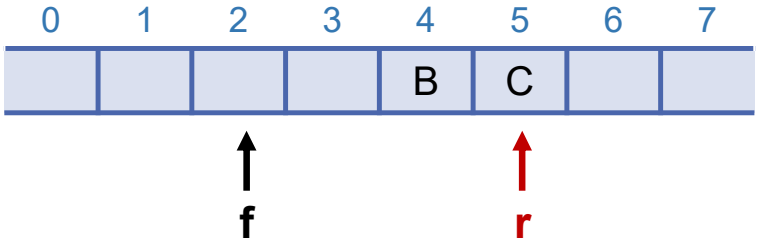
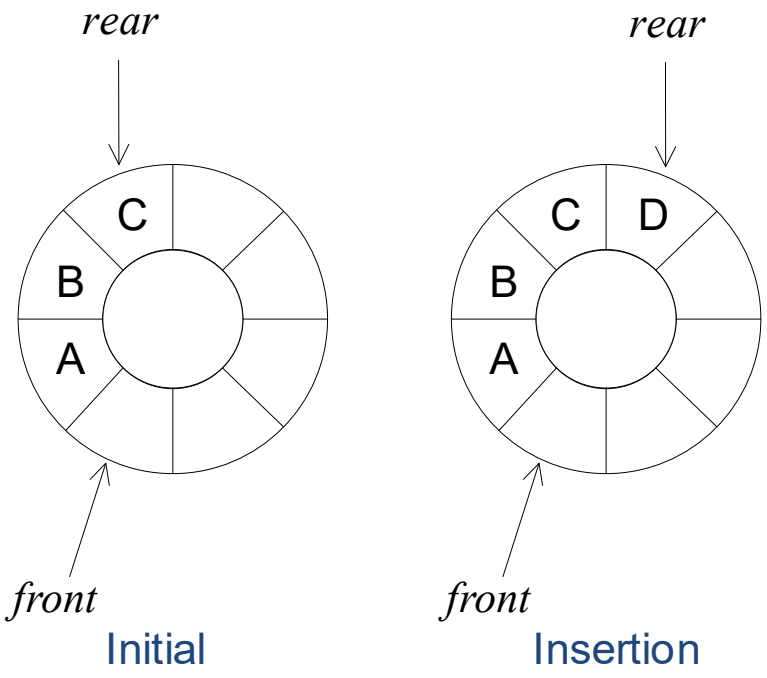


# Queue: ADT

```
template < class T >
class Queue // A finite ordered list
{
public:
        // Constructor
        Queue (int queueCapacity = 10);

        // Check if the stack is empty
        bool IsEmpty ( ) const;

        // Return the front element
        T& Front ( ) const;

        // Return the rear element
        T& Rear ( ) const;

        // Insert a new element at rear
        void Push (const T& item);

        // Delete one element from front
        void Pop ( );
private:
    T* queue;
    int front, rear; // init. value = -1
    int capacity;
};
```

# Queue Operations: Front & Rear

```
template < class T >
void Queue < T >::IsEmpty() const { return front==rear; }

template < class T >
T& Queue < T >::Front() const {
   if(IsEmpty()) throw “Queue is empty!”;
   return queue[(front+1)%capacity];
}

template < class T >
T& Queue < T >::Rear() const {
   if(IsEmpty()) throw “Queue is empty!”;
   return queue[rear];
}
```

# Queue Operations: Push & Pop

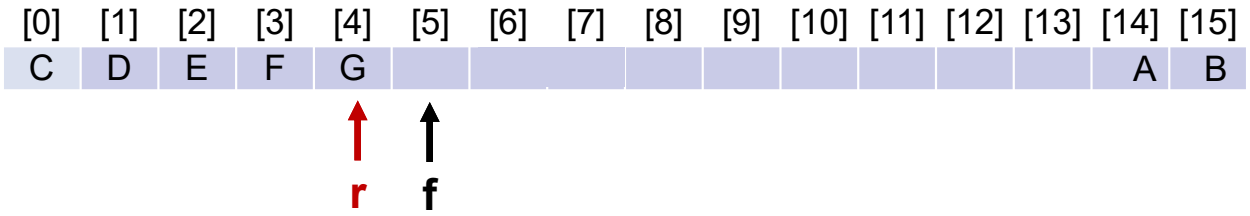
```
template < class T >
void Queue< T >::Push (const T& x)
{   // Add x at rear of queue
    if((rear+1)%capacity == front)
    {
       // queue is going to full, double the capacity!
    }
    rear = (rear+1)%capacity;
    queue [rear] = x;
}
```

```
template < class T >
void Queue < T >::Pop ( )
{   // Delete front element from queue
    if(IsEmpty()) throw "Queue is empty. Cannot delete.";
    front = (front+1)%capacity;
    queue[front].~T(); // Delete the element
}
```

# Doubling Queue Capacity

# Doubling Queue Capacity: Scenario 1

capacity=16

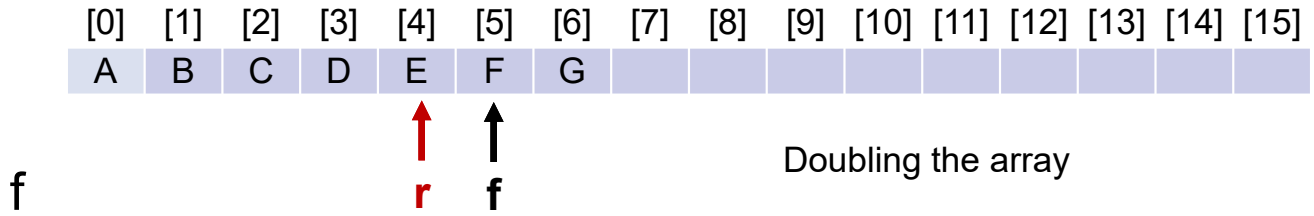



Scenario 1: After shifting right segment

front = 13, rear = 4

# Doubling Queue Capacity: Scenario 2

capacity=16




Scenario 2: Alternative configuration

front = 15, rear = 6

# Evaluation of Expressions

# Expression

$$X = A/B - C + D * E - A * C$$

- Operators
  - +,-,*,/,…,etc
- Operands
  - A,B,C,D,E,F
- Execution order might affect the final result

# Expression Evaluation

- For $X = A/B - C + D * E - A * C$
- If A = 4, B=C=2, D=E=3
- 

- For $X = (A/(B - C + D)) * (E - A) * C$
- If A = 4, B=C=2, D=E=3
- 

# Evaluation Rules

- Operators have priority
- Operator with higher priority is evaluated first
- Operators of equal priority are evaluated from left to right
- Unary operators are evaluated from right to left

# Priority of Operators in CPP

| Priority | Operators |
|---|---|
| 1 | Minus, ! |
| 2 | *, /, % |
| 3 | +, - |
| 4 | <, <=, >=, > |
| 5 | = =, != |
| 6 | && |
| 7 | \|\| |

# Infix and Postfix Notation

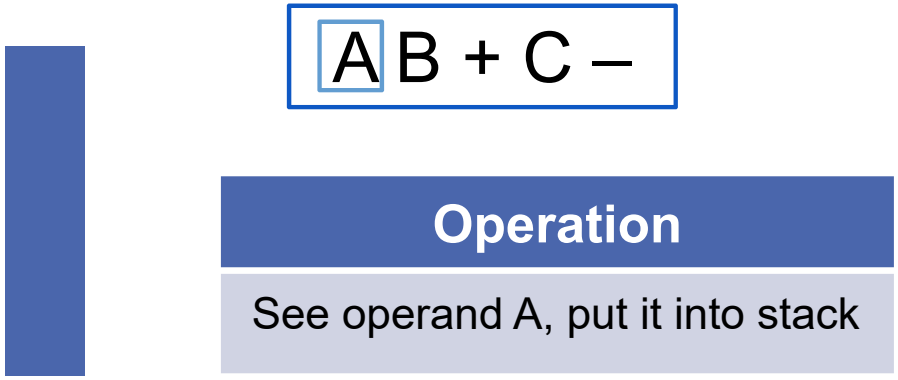
- Infix notation
  - Operator comes in–between the operands
  - E.g., A+B*C
  -
- Postfix notation
  - Each operator appears after its operands
  - E.g., ABC*+

# Advantages of Postfix Notation

- You don’t need parentheses
- Priority of operators is no longer relevant!
- Expression can be efficiently evaluated by
  - Making a left to right scan
  - Stacking operands
  - Evaluating operators
  - Push the result into stack

# Example 1

- Infix : A+B – C => Postfix : A B + C –
- Suppose A = 4, B = 3, C = 2



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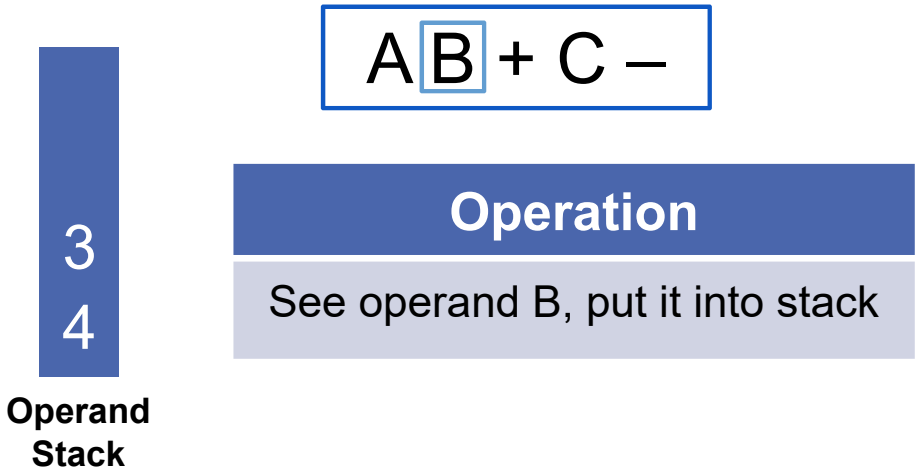

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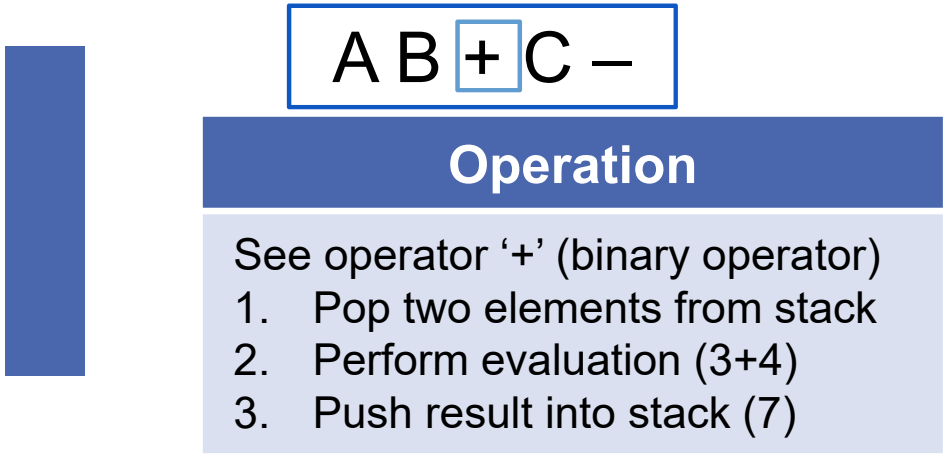

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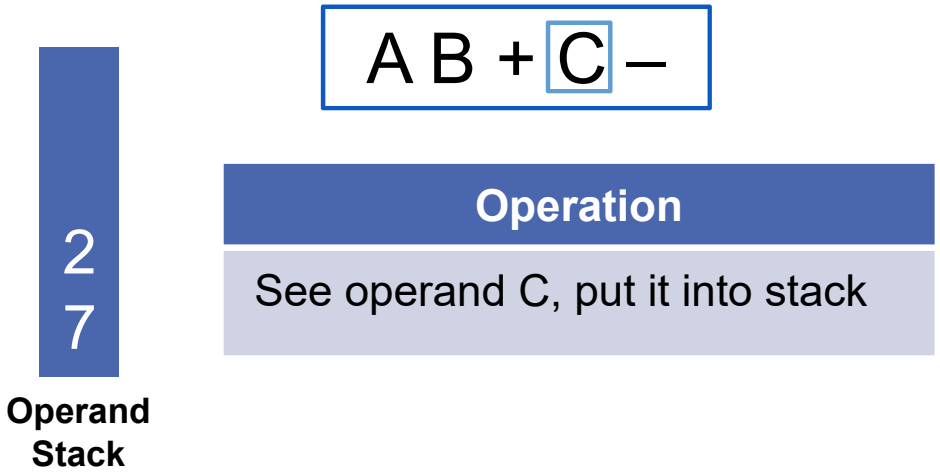

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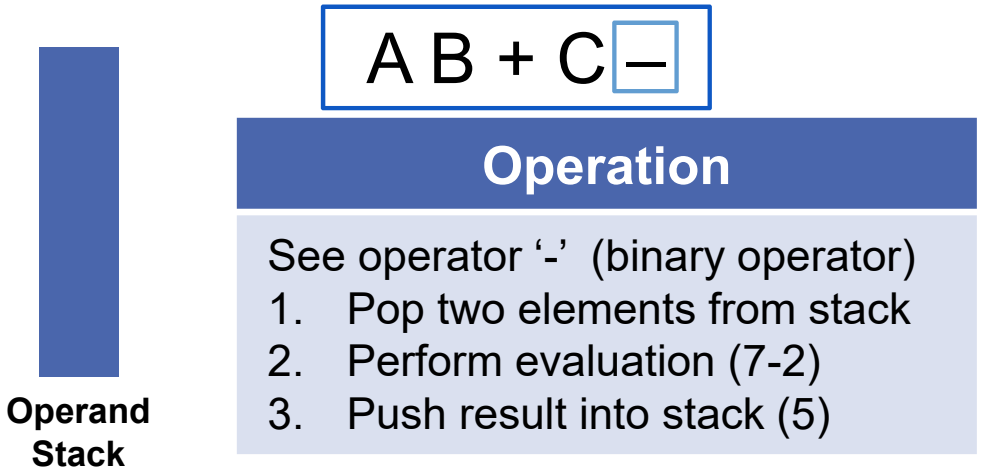

# Example 2

- Infix : $X = A/B - C + D * E - A * C$
- Postfix : $X = AB/C - DE * +AC * -$

# Evaluation Pseudo Codes

```
void Eval(Expression e)
{  // Assume the last token of e is '#'
   // A function NextToken is used to get next token in e
   Stack<Token> stack; // initialize stack
   for (Token x = NextToken(e); x != '#'; x = NextToken(e)){
     if(x is an operand) stack.Push(x);
     else{
         // Remove the correct number of operands from stack
         // Perform the evaluation
         // Push the result back to stack
         // ***Try to fill up the codes by your own***
     }
   }
};
```

# Infix to Postfix

- Fully parenthesize algorithm:
  - Fully parenthesize the expression
  - Move all operators so the they replace the corresponding right parentheses
  - Delete all parentheses

$$((((A / B ) – C ) + ( D * E ) ) - ( A * C ) )$$

# Infix to Postfix

- Smarter algorithm
  - Scan the expression only once
  - Utilize stack
- The order of operands dose not change
- Output every visiting operand directly
- Use stack to store visited operators
  - Pop them out at the right moment
    - The ***priority*** of operator on top of stack is *higher or equal to* that of the incoming operator
    - left-to-right associativity

# Example 1

- Infix : A + B * C

| Next token | Stack | Output |
|---|---|---|
| None | Empty | None |
| A | Empty | A |
| + | + | A |
| B | + | AB |
| * | +* | AB |
| C | +* | ABC |
| | + | ABC* |
| | Empty | ABC*+ |

# Notes

- Expression with ( )
  - ‘(‘ has the highest priority, always push to stack.
  - Once pushed, ‘(’ get lowest priority.
  - Pop the operators until you see the matched ‘)’